Assignment 13
Waves

1. Consider the wave y = 4 cos (3z – 6t). Units are in meters and seconds. Find the following.

a) Amplitude

Amplitude = 4 units

b) Frequency

Angular Frequency $\omega = 6$ rad/sec

Frequency $\nu = \frac{\omega}{2\pi} = 0.95$ Hz

c) Period

Period $T = \frac{1}{\nu} = 1.05$ sec.

d) Wavelength

Wave Vector $k = 3$ unit^{-1}

Wavelength $\lambda = \frac{2\pi}{k} = 2.09$ units

e) Direction

Observer on wave crest sees $3z - 6t = \text{constant}$

$\therefore$ wave goes in $\hat{z}$ direction.

$$3\frac{dz}{dt} - 6 = 0$$

$$\frac{dz}{dt} = 2$$

f) Phase Velocity

Phase Velocity $v_p = \frac{\omega}{k}$

$v_p = 2$ units/sec.

2. Consider an organ pipe of length L. The pipe is closed at one end and open at the other allowing a standing wave where a node exists at the closed end and a maximum exists at the open end.

a) What are the resonant wavelengths?

Pipe length = odd # of quarter wavelengths

$$L = (2n+1)\frac{\lambda}{4}$$

$$\lambda = \frac{4L}{2n+1} \qquad n = 0, 1, 2, 3, \ldots$$

b) Sketch the resonant nodes corresponding to the 3 longest wavelengths.

$n = 0$

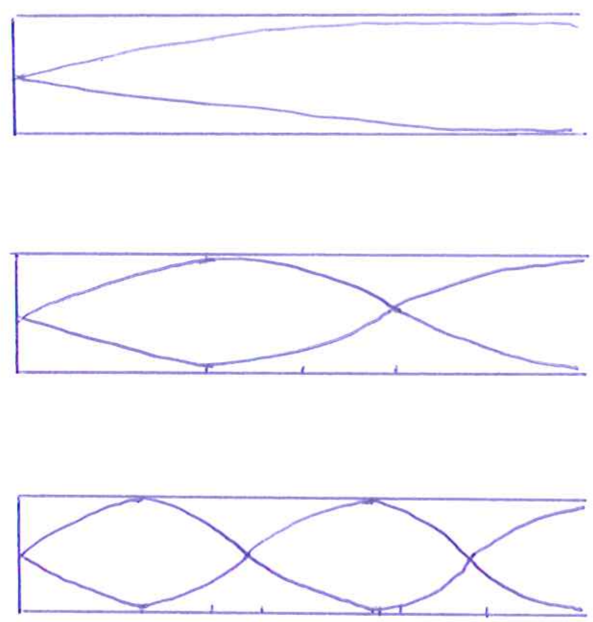
$\lambda = 4L$

$n = 1$

$\lambda = \frac{4L}{3}$

$n = 2$

$\lambda = \frac{4L}{5}$



c) If L = 2 meters, what is the lowest frequency?

λ_{max} occurs for $n=0 \Rightarrow \lambda_{max} = 4 \times 2 = 8$ m.

$$\nu_{min} = \frac{v_{air}}{\lambda_{max}} = \frac{330 \text{ m/sec}}{8 \text{ m}} = 41 \text{ Hz}.$$

ν_{min} corresponds to E nearly 3 octaves below middle C.

3. An orchestra wishes to have a listener in front of the conductor hear notes at the same time, played by the violinist located next to the conductor and from a drummer located 50 meters further back.

a) Assuming that the drummer plays as soon as he sees the conductor give the command, how long should the violinist wait before playing her note?

$$\Delta t = \frac{50 \text{ meters}}{330 \text{ m/sec}}$$

$$= 0.15 \text{ sec.}$$

b) Why may one assume that the two players see the conductor command at the same time?

Conductor command travels at speed of light.

$$\Rightarrow \Delta t_{light} = \frac{50 \text{ m}}{3 \times 10^{8} \text{ m/sec}}$$

$$= 1.67 \times 10^{-7} \text{ sec}$$

$$\Delta t_{light} << \Delta t_{sound}$$

4. A singer sings middle C. She then gulps helium and tries to sing the same note i.e. her vocal chords remain the same. What note will the listener hear?

Vocal Chords control wavelength.

$$\therefore \lambda = \frac{v_{air}}{\nu_{middle\ C}} = \frac{330 \text{ m/sec}}{262 \text{ Hz}} = 1.26 \text{ m.}$$

$$\nu_{He} = \frac{v_{He}}{\lambda} = \frac{965 \text{ m/sec}}{1.26} = 766 \text{ Hz.}$$

This note is about an octave and a half higher than middle C.

5. Professor Zoom travels down the 401 in his car whistling middle C. A policeman has a defective radar gun but passed PHYS 1410. He hears the pitch change from D immediately above middle C to A# immediately below middle C as the car passes.

a) How fast is Professor Zoom traveling?

$$f_{\text{oncoming car}} = \frac{f_{\text{middle c}}}{1 - \frac{v_{car}}{v_{sound}}}$$

$$f_{\text{Receding car}} = \frac{f_{\text{middle c}}}{1 + \frac{v_{car}}{v_{sound}}}$$

$$\Delta f = f_{\text{mid c}}\left[\frac{1}{1-x} - \frac{1}{1+x}\right] \qquad x \equiv \frac{v_{car}}{v_{sound}}.$$

$$294 - 230 = 262\left[\frac{1}{1-x} - \frac{1}{1+x}\right]$$

$$\frac{64}{262} = \frac{1+x-(1-x)}{1-x^2}$$

$$\frac{32}{131}(1-x^2) = +2x$$

$$x^2 + \frac{262}{32}x - 1 = 0$$

$$\Rightarrow x = 0.12 \quad \Rightarrow v_{car} = 0.12 \times 330 = 40 \text{ m/sec}.$$

b) Should Professor Zoom get a ticket?

$$40\,\frac{\text{m}}{\text{sec}} = 40\,\frac{\cancel{\text{m}}}{\text{sec}} \times 3600\,\frac{\text{sec}}{\text{hr}} \times \frac{1}{1000}\,\frac{\text{km}}{\text{m}}$$

$$= 144 \text{ km/hr}.$$

This clearly exceeds the respect the cop views his old professor meriting a ticket?

Assignment 14
Kinetic Theory of Gases

1. A tank holding a gas initially at one atmosphere pressure at a temperature of 300 K is heated to 400 K.

a) What is the increase in pressure?

$PV = nRT \Rightarrow \frac{P}{T} = $ constant for gas in tank

$\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{400}{300} P_1 = 1.33 P_1$

$\Delta P = P_2 - P_1 = 0.33 P_1 = 0.33 \times 10^5 = 3.3 \times 10^4$ pascals

b) Assuming the tank is a sphere of radius 10 meters, what force does the gas exert on the walls of the tank?

Force $= P_2 \times$ tank surface area

$= 1.33 \times 10^5 \times 4\pi\,(10 \text{ meter})^2$

$= 1.67 \times 10^8$ Nt.

c) How does the force calculated in b) compare to the force exerted by the gas on the tank walls before it was heated?

Original Force $= P_1 \times$ tank surface area

$= 1.26 \times 10^8$ Nt.

d) Why is one normally not concerned about the force in c) but only worried about the difference in the answers to b and c.

Force in c (due to gas in tank) is balanced by air in atmosphere on tank.

2. Five moles of gas expand at a constant temperature of 373 K from a volume of 1 cm^3 to a 100 cm^3. Evaluate the work done by the expanding gas on a piston.

Work $W = nRT \ln(V_f/V_i)$

$= 5 \times 8.314 \times 373 \ln(100/1)$

$= 7.14 \times 10^4$ J

3. Professor X wishes to construct a so called atomic beam which means that the atoms can travel the length of a 5 meter vacuum chamber without colliding with gas molecules. What pressure should the chamber be pumped down to?

Mean Free Path $\lambda = \dfrac{kT}{\pi d^2 P}$ $\quad d \simeq 3$ Å for O_2, N_2

$\therefore P = \dfrac{1.38 \times 10^{-23} \times 300\text{K}}{\pi (3 \times 10^{-10}\text{m})^2 \times 5\text{m}}$

$= 2.93 \times 10^{-3}$ Pascal

$P = 2.93 \times 10^{-8}$ atmosphere

4. Estimate the average time between collisions for a gas at one atmosphere pressure and at room temperature.

Mean Free Path $\lambda = \dfrac{kT}{\pi d^2 P} \simeq 1.5 \times 10^{-7}$ m

Time Between Collisions $\simeq \dfrac{\lambda}{v_{sound}}$

$= \dfrac{1.5 \times 10^{-7}}{330}$

$= 4.4 \times 10^{-10}$ sec.

5. For the Maxwell speed distribution find the following.

a) Show the highest probability occurs at a speed $v_P = (2RT/M)^{1/2}$

$$P_{maxwell} = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}$$

P_{max} occurs at $0 = \frac{dP}{dv}$

$$= 2v e^{-kv^2} + v^2(-2kv) e^{-kv^2}$$

$k \equiv M/2RT$

Soln. is $v = \frac{1}{\sqrt{k}} = \sqrt{\frac{2RT}{M}}$

b) Evaluate v_{ave}, v_{rms} and v_P for a gas of N_2 at room temperature.

$$\sqrt{\frac{RT}{M}} = \left(\frac{8.314 \times 300}{.028}\right)^{1/2} = 298 \text{ m/sec.}$$

$$v_P = \sqrt{\frac{2RT}{M}} \simeq 422 \text{ m/sec}$$

$$\bar{v} = 1.60 \sqrt{\frac{RT}{M}} = 476 \text{ m/sec}$$

$$v_{RMS} = 1.73 \sqrt{\frac{RT}{M}} = 515 \text{ m/sec}$$

$\Rightarrow v_P < \bar{v} < v_{RMS}$

c) Suppose the moon had an atmosphere consisting of hydrogen atoms. (Here, we assume the hydrogen atoms don't combine to form hydrogen molecules.) For a temperature T= 300 Kelvins, estimate the fraction of hydrogen atoms having speed greater than the escape velocity.

$v_{esc\ Moon} = 2374$ m/sec

$$\therefore f = \int_{v_{esc}}^{\infty} P(v)\,dv$$

$$= \frac{4}{\sqrt{\pi}} \int_{x_{esc}=1.063}^{\infty} x^2 e^{-x^2}\,dx \quad \text{where } x \equiv \frac{v}{\sqrt{2RT/M}}$$

$f \simeq 52\%$ using computer pgm. such as EXCEL