**Assignment 11**
**Motion of Rigid Bodies**

1. A person stands at the equator.

a) What is the person's velocity due to the rotation of the Earth?

$$v = \omega R_E = \frac{2\pi R_E}{T} = \frac{2\pi \times 6.4\times10^6}{24\times 3600} = 4.65\times10^2 \text{ m/sec}$$

b) What is the centripetal acceleration and how does this compare to gravity?

$$\frac{v^2}{R} = \frac{(4.65\times10^2)^2}{6.4\times10^6} = 3.4\times10^{-2} \text{ m/sec}^2 = 3.4\times10^{-3} g$$

2. Estimate the angular momentum and energy of the following:

a) the rotating Earth.

$$I = \frac{2}{5}MR^2 = \frac{2\times6\times10^{24}(6.4\times10^6)^2}{5} = 9.8\times10^{37} \text{ kg m}^2$$

$$L = I\omega = 9.8\times10^{37}\frac{2\pi}{24\times3600} = 7.1\times10^{33} \text{ J sec}$$

$$E = \frac{1}{2}I\omega^2 = 2.6\times10^{29} \text{ J}$$

b) the moon orbiting around the Earth

$$L = mvr = m\omega r^2 = 7.4\times10^{22}\frac{2\pi}{29\times24\times3600}(3.8\times10^8)^2 = 2.7\times10^{34} \text{ J sec}$$

$$E = \frac{m}{2}v^2 = \frac{L^2}{2mr^2} = \frac{(2.7\times10^{34})^2}{2\times7.4\times10^{22}(3.8\times10^8)^2} = 3.4\times10^{28} \text{ J}$$

3. Estimate the moment of inertia of a nitrogen molecule.

$$I \sim 2 m_N r^2$$
$$\simeq 2 \times 14 \times 1.67 \times 10^{-27} \times (1 \times 10^{-10})^2$$
$$= 4.7 \times 10^{-46} \text{ kg m}^2$$

4. A car wheel has a mass of 100 kg which can be viewed as contained only on the rim which has a radius of 30 cm. The car travels at 120 km/hr.

a) What is the angular momentum of the wheel?

$$L = m r v$$
$$= 100 \text{kg} \times .30 \text{m} \times 120 \times \frac{1000}{3600} \text{m/sec}$$
$$= 1 \times 10^3 \frac{\text{kg m}^2}{\text{sec}}$$

b) What is the rotational energy of the wheel?

$$E = \frac{m}{2} v^2$$
$$= \frac{100}{2} \left(120 \times \frac{1000}{3600}\right)^2$$
$$= 5.6 \times 10^4 \text{ J}$$

c) What torque must be applied if the wheel is braked to a halt in 10 seconds?

$$N = \frac{\Delta L}{\Delta t}$$
$$= \frac{1 \times 10^3 \text{ kg m}^2/\text{sec}}{10 \text{ sec}}$$
$$= 1 \times 10^2 \text{ Nt m.}$$

5. A cylinder of uniform mass M and radius R rolls down an inclined ramp beginning at a height of 5 meters.

a) What is the final velocity of the cylinder?

$$mgh = K.E._{Rot.} + K.E._{Translation}$$

$$= \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2$$

$$= \frac{1}{2}\left(\frac{1}{2} mR^2\right)\omega^2 + \frac{1}{2} mv^2$$

$$= \frac{3}{4} mv^2 \quad \text{since } v = \omega R$$

$$v = \sqrt{\frac{4gh}{3}} = \left(\frac{4 \times 10 \times 5}{3}\right)^{1/2} = 8.2 \text{ m/sec.}$$

b) Repeat the problem if the cylinder is hollow. (neglect the mass in the ends of the can)

$$mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2$$

$$= \frac{1}{2}(mR^2)\omega^2 + \frac{1}{2} mv^2$$

$$= mv^2$$

$$v = \sqrt{gh}$$

$$= (10 \times 5)^{1/2}$$

$$= 7.1 \text{ m/sec.}$$

6. Explain why a twirling skate rspe3eds up if they bring their arms closer to their body.

Cons. of Angular Momentum

$$m_1 v_1 r_1 = m_2 v_2 r_2 \quad (m_1 = m_2)$$

$$\therefore v_1 r_1 = v_2 r_2$$

If $r_1 = \frac{1}{2} r_2 \Rightarrow v_1 = 2v_2$ i.e. skater twirls faster

**Assignment 12b**
**Elasticity & Fluid Mechanics**

1. A scientist proposes construction of a space elevator by using a steel wire to raise a mass from the Earth's surface to a satellite 1000 km above the Earth's surface. Assume the steel wire has a diameter of 1 mm.
   a) What is the mass of the wire?

Mass of Wire $= \rho_{steel} \times \pi r^2 L$.

$= 8 \times 10^3 \frac{kg}{m^3} \times \pi (5 \times 10^{-4} m)^2 \times 10^6 m$

$= 6.28 \times 10^3$ kg.

   b) Estimate the extension of the wire due to its weight. Assume that all parts of the wire experience the same acceleration of gravity as at the surface of the Earth.

$\Delta L = \frac{L}{Y} \frac{F}{A}$ where $F \simeq \frac{Mg}{2} = \frac{\rho A L g}{2}$

$\simeq \frac{\rho g L^2}{2Y}$

$= \frac{8 \times 10^3 \times 10 \times (10^6)^2}{2 \times 22 \times 10^{10}}$ $\rightarrow \Delta L = 2 \times 10^5$ m $= 200$ km.

   c) Is this realistic?

$\frac{\Delta L}{L}$ is 20% $\Rightarrow$ Unrealistic.

2. Greenland Icecap

a) Estimate the rise in sea level if the Greenland ice cap fully melts.

Volume of Ice $\simeq$ Ice area on Greenland $\times$ Thickness

$\simeq 2 \times 10^{6}\ km^{2} \times 1\ km$

$\simeq 2 \times 10^{6}\ km^{3}$

Ocean Surface Area $\simeq \frac{2}{3} 4\pi R_E^2$

$= \frac{8}{3} \pi (6.4 \times 10^{3})^{2}$

$= 3.4 \times 10^{8}\ km^{2}$.

$\therefore$ sea level rise $= \frac{2 \times 10^{6}\ km^{3}}{3.4 \times 10^{8}\ km^{2}}$

$= 6 \times 10^{-3}\ km$

$= 6\ m$.

b) How much does your answer change if you also consider the melting of ice in the waters surrounding Greenland?

No change since floating ice already displaces its weight.

3. A cube having volume 100 $cm^3$ floats in water.
a) If 70% of the cube is submerged, what is its density?

Floating Criteria: Mass of Cube = Mass $H_2O$ displaced

$= 0.70 \times 100\ cm^3 \times 1\ \frac{gm}{cm^3}$

$= 70\ gm.$

$\therefore$ cube density $= \frac{70\ gm}{100\ cm^3}$

$= 0.7\ gm/cm^3$

b) The cube is next placed in an unknown liquid and 90% of it is submerged. What is the density of the liquid?

Mass Fluid displaced = Cube Mass

$0.90\ \rho_{Fluid}\ 100\ cm^3 = 70\ gm.$

$\rho_{Fluid} = 0.78\ gm/cm^3$

4. A water tank has a small hole near its bottom at a depth h from the top surface. What is the speed of the stream of water emerging from the hole?

See notes.

5. Gas Pressure

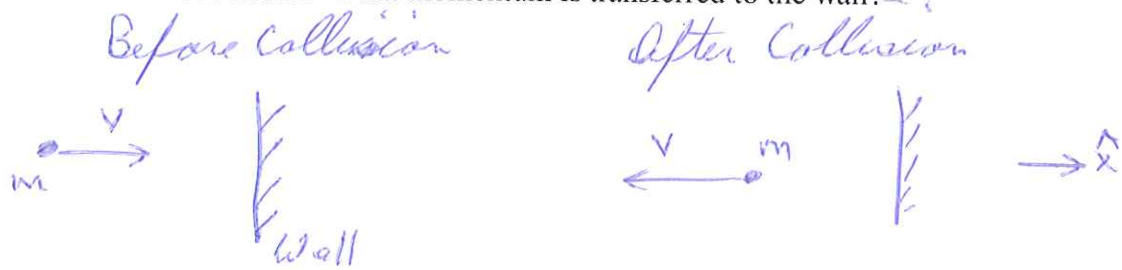
a) Estimate the density of air at sea level and a temperature of 20 °C.

$$\frac{N}{V} = \frac{P}{kT}$$

$$= \frac{1 \times 10^5 \text{ Pascal}}{1.38 \times 10^{-23} \times 293 K}$$

$$= 2.5 \times 10^{25} \text{ particles}/m^3$$

b) Consider a nitrogen molecule travelling at the speed of sound that bounces perpendicularly off the wall of a chamber. Assume its speed is not changed while its direction is reversed. What momentum is transferred to the wall?




$$\Delta P_{wall} = 2mV$$

$$= 2 \times 28 \times 1.67 \times 10^{-27} \times 330$$

$$= 3.1 \times 10^{-23} \text{ kg m/sec.}$$

c) Estimating the number of collisions per $m^2$ per second with the wall by multiplying the half the particle density times the speed. (The factor half takes into account that the molecules move randomly and only half have a velocity component toward the wall.)

$$\text{Collision rate}/m^2 \text{ with wall} = \frac{n}{2} V$$

$$= \frac{2.5 \times 10^{25}}{2} \times 330$$

$$= 4.1 \times 10^{27} /m^2$$

d) Estimate the gas pressure.

$$P = \text{Collision rate}/m^2 \times \Delta P$$

$$= 4.1 \times 10^{27} \times 3.1 \times 10^{-23}$$

$$= 1.2 \times 10^5 \text{ Pascal.}$$