Assignment 9
Work and Kinetic Energy II

1. A 5 kg. mass falls from an initial height of 100 meters.

a) What are the initial potential and kinetic energies of the mass?

$U_i = mgh$ $\qquad T_i = 0$

$= 5\,kg \times 10\,\frac{m}{sec^2} \times 100\,m$

$= 5000\,J$

b) What are the potential and kinetic energies of the mass when it hits the ground?

$U_f = 0$ $\qquad T_f = U_i$

$= 5000\,J$

c) What is the speed of the particle when it hits the ground?

$T_f = \frac{m}{2} v^2$

$v = \sqrt{\frac{2T_f}{m}}$

$= \sqrt{\frac{2 \times 5000}{5}}$

$\therefore v = 44.7\,m/sec$

2. What is the kinetic energy of an electron travelling around the proton in a hydrogen atom? You may assume that it travels 1% of the speed of light.

$$T = \frac{m_{electron}}{2} v^2$$

$$= \frac{9.11 \times 10^{-31}}{2} kg \times (.01 \times 3 \times 10^8 m/sec)^2$$

$$= 4.1 \times 10^{-18} J$$

3. What is the kinetic energy of an air molecule moving at the speed of sound?

$$T = \frac{m_{air}}{2} v^2 \qquad m_{air} \simeq m_{N_2}$$

$$= \frac{28 \times 1.67 \times 10^{-27} kg}{2} (3 \times 10^2 m/sec)^2$$

$$= 2.1 \times 10^{-21} J$$

4. The number of Motor vehicle deaths can be estimated assuming they are linearly related to the kinetic energy of the vehicles in the accident. What effect on the number of fatalities would occur if the average speed on the highway increases by 20% while the car mass increases by 10%? (Note that improvements in road and/or car safety are not considered.)

Accident Deaths $\propto$ Kinetic Energy

$$\propto \frac{m}{2} v^2$$

or $D = k\, m v^2$ k = constant of proportionality

$$\Delta D = k\, \Delta m\, v^2 + k m\, 2v\, \Delta v$$

$$\frac{\Delta D}{D} = \frac{\Delta m}{m} + 2\frac{\Delta v}{v}$$

$$= .10 + 2(.20)$$

$$= .5$$

$$\frac{\Delta D}{D} = 50\%$$

∴ 20% increase in speed + 10% increase in car mass naively causes 50% increase in deaths.

5. A 250 gm block dropped onto a relaxed vertical spring that has a spring constant of 250 Nt/m. The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping.

a) What is the speed of the block just before it hits the spring? Neglect friction.

Kinetic Energy of block hitting spring = Potential Energy of spring when block stops

$$\frac{mv^2}{2} = \frac{k}{2}x^2$$

$$v = \sqrt{\frac{k}{m}}\, x$$

$$= \left(\frac{250}{.25}\right)^{1/2}(.12)$$

$$= 3.8 \text{ m/sec.}$$

b) If the speed of impact is doubled, what is the maximum compression of the spring?

Note above $x \propto v$.

$\therefore$ doubling v, also doubles x.

6. This question considers power usage in Canada.

a) Estimate Canada's total annual electrical energy consumption in kW hours and in joules

1/2 million people use ~1 GW

30 " " " ~60 GW

Annual Energy = 60 GW × 365.25 days × 24 hr.

$= 5 \times 10^{11}$ kWhr.

$= 1.9 \times 10^{18}$ J

b) What fraction of your answer in a) can be supplied by Niagara Falls?

Niagara generates ~ 2 GW which is a few percent of total.

c) Estimate the total solar energy power falling on Canada?

$$P_{Canada} \sim P_{Earth} \frac{\text{Area of Canada}}{\text{Area of Earth}} \times \frac{1}{2}$$

↑ less sun in Canada than at equator

$$\sim 1.7 \times 10^{17} W \frac{1 \times 10^{7} km^2}{4\pi (6.40 \times 10^3 km)^2} \cdot \frac{1}{2}$$

$$\sim 1.7 \times 10^{15} W$$

d) What area would Canada need to cover with solar panels in order to generate all of its electricity? Assume a 10% conversion of sunlight into electricity.

$$\text{Area required} \sim \frac{\text{Power Needed}}{P_{solar}} \cdot \frac{\text{Area of Canada}}{\text{efficiency}}$$

$$\sim \frac{6 \times 10^{10} W}{1.7 \times 10^{15} W} \cdot \frac{1 \times 10^{7} km^2}{0.1}$$

$$\sim 3.5 \times 10^{3} km^2$$

Assignment 10
Gravitation

1. Two 75 kg students are separated by 10 cm. What is the ratio of their gravitational attraction compared to their weight?

$$\frac{F_{Grav}}{mg} = \frac{G\,m^2/r^2}{mg}$$
$$= \frac{6.67\times10^{-11}\times75^2/(.1)^2}{75\times10}$$
$$= 5\times10^{-8}$$

2. A satellite in a circular low earth orbit.

a) What is the velocity of the satellite?

$$v = \sqrt{\frac{G M_E}{R_E}}$$
$$= \left(\frac{6.67\times10^{-11}\times6\times10^{24}}{6.4\times10^{6}}\right)^{1/2}$$
$$= 7.9\times10^{3}\ m/sec$$

b) What is the period of the satellite?

$$T = 2\pi\left(\frac{R_E^3}{G M_E}\right)^{1/2}$$
$$= 2\pi\,\frac{(6.4\times10^{6})^{3/2}}{(6.67\times10^{-11}\times6\times10^{24})^{1/2}}$$
$$= 5100\ sec.$$

c) How do the answers change if the satellite is in low moon orbit?

$$v_{mo} = \sqrt{\frac{G M_m}{R_M}}$$
$$= \left(\frac{6.67\times10^{-11}\times7.4\times10^{22}}{1.7\times10^{6}}\right)^{1/2}$$
$$= 1.7\times10^{3}\ m/sec$$

$$T = 2\pi\left(\frac{R_M^3}{G M_m}\right)^{1/2}$$
$$= 2\pi\,\frac{(1.7\times10^{6})^{3/2}}{(6.67\times10^{-11}\times7.4\times10^{22})^{1/2}}$$

→ $T = 6260\ sec.$

3. Consider a planet in a circular orbit. Equating the gravitational force to the centripetal force, derive Kepler's law showing that the square of the period is proportional to the cube of the radial distance of the orbit.

See notes.

4. A scientist wishes to determine whether neighbouring planets can perturb the moons of Mars. What is the ratio of the gravitational forces of Jupiter and Earth exert on Mars when the planets are closest to Mars?

$$\frac{F_{Jupiter}}{F_{Earth}} = \frac{M_{Jup} / r_{JM}^2}{M_{Earth} / r_{EM}^2}$$

$$= \frac{1.9 \times 10^{27}\ kg}{6 \times 10^{24}\ kg} \cdot \frac{(7.8 \times 10^{7}\ km)^2}{(5.5 \times 10^{8}\ km)^2}$$

$$= 6.4$$

Hence for accurate models, Earth's effect on Mars must be taken into account when studying effect of Jupiter on Mars.