Assignment 5
Newton's Laws I

1. A 500 gm hockey puck experiences two forces $\vec{F}_1 = (5, 2)$ and $\vec{F}_2 = (1, -1)$ Newtons.

a) What is the magnitude and direction of the net force?

$$\vec{F}_{TOT} = \vec{F}_1 + \vec{F}_2$$
$$= (5, 2) + (1, -1)$$
$$= (6, 1)$$

$$|\vec{F}_{TOT}| = \sqrt{6^2 + 1^2} = \sqrt{37} \text{ Nt.}$$

$$\tan\theta = \frac{F_{TOT\,y}}{F_{TOT\,x}}$$

$$\tan\theta = \frac{1}{6} \Rightarrow \theta = 9.5^\circ$$

b) What is the magnitude and direction of the acceleration?

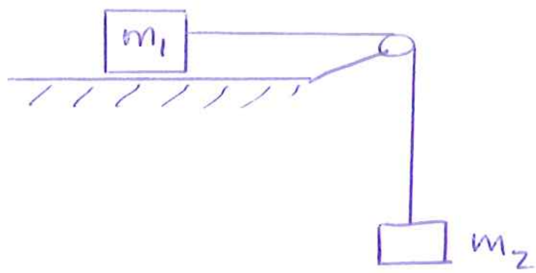
$$\vec{a} = \frac{\vec{F}_{TOT}}{M}$$
$$= \frac{1}{.5}(6, 1)$$
$$= (12, 2)$$

$$\therefore |\vec{a}| = \sqrt{12^2 + 2^2} = 12.2 \text{ m/sec}^2$$

$$\tan\theta = \frac{a_y}{a_x}$$
$$= \frac{2}{12}$$

$\Rightarrow \theta = 9.5^\circ$ (i.e. same direction as $\vec{F}_{TOT}$)

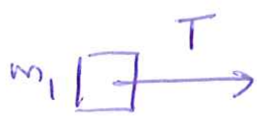
2. A 3 kg mass sits on a table and is connected via a pulley to a 1 kg mass hanging over the table as shown below.



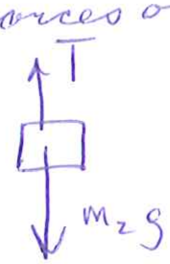
Let T be tension in string.
Let a be acceleration.

a) What is the acceleration of the 3 kg mass?

Forces on m_1



Forces on m_2



$\therefore\ m_1 a = T \quad (1)$ $\qquad$ $\therefore\ m_2 a = m_2 g - T \quad (2)$

$(1)+(2) \Rightarrow \quad (m_1 + m_2) a = m_2 g \Rightarrow a = \frac{g}{4}.$

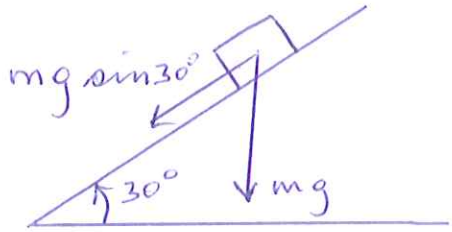
b) What is the tension in the string?

Substitute $a = \frac{g}{4}$ in $(1) \Rightarrow T = m_1 \frac{g}{4}$

$= \frac{3}{4} \times 10$

$\therefore T = 7.5$ Nt

3. A 2 kg block slides down a frictionless plane inclined at angle 30^o. What is the acceleration of the block?



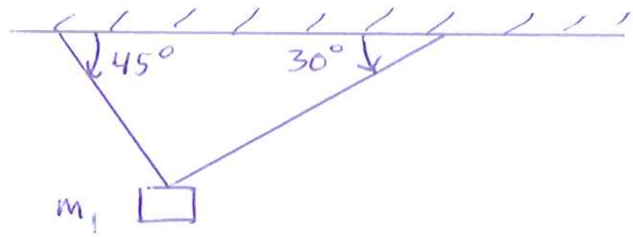
$\therefore \quad m a = m g \sin 30^\circ$

Acceleration $a = g \sin 30^\circ$

$= \frac{g}{2}$

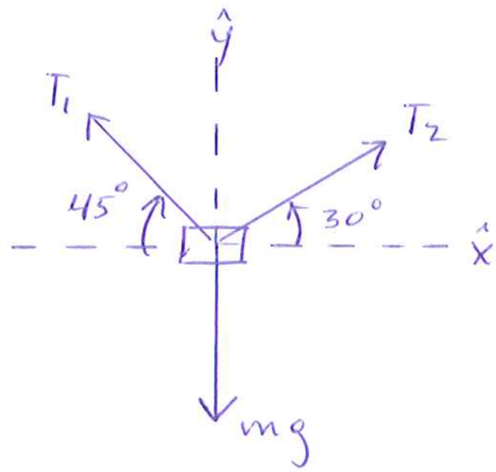
$= 5 \; m/sec^2$

4. A 9 kg mass hangs suspended from the ceiling via two cables as shown below. One cable is at 30° and the other at 45°. What are the tensions in the two cables?



Let tensions in cables be T_1 & T_2.

Forces on m



F_{xTOT} & F_{yTOT} must equal zero for equilibrium.

$F_{xTOT} = 0 \Rightarrow \quad T_1 \cos 45° = T_2 \cos 30°$

$$\frac{T_1}{\sqrt{2}} = \frac{\sqrt{3}}{2} T_2$$

$$T_1 = \frac{\sqrt{6}}{2} T_2 \qquad (1)$$

$F_{yTOT} = 0 \Rightarrow \quad T_1 \sin 45° + T_2 \sin 30° = mg. \quad (2)$

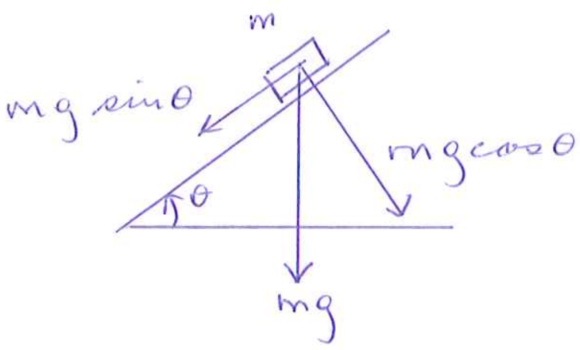
Using (1) $\Rightarrow \quad \frac{\sqrt{6}}{2} \frac{1}{\sqrt{2}} T_2 + \frac{T_2}{2} = mg$

$$T_2 = \frac{mg}{1.37} = \frac{9 \times 10}{1.37} = 66 \text{ Nt.}$$

From (1) $\Rightarrow \quad T_1 = \frac{\sqrt{6}}{2} \times 66 = 81$ Nt.

5. An engineer wishes to design a curve of radius 1 km in the road to help prevent cars from flying off the highway. i.e. She/he wishes to incline the road at an angle so that cars traveling 120 km/hr feel a force toward the center of the curve that is given by the gravitational force component.

a) What are the components of the gravitational force perpendicular and parallel to the inclined road surface?



$$F_{car \perp rd} = mg\cos\theta$$

$$F_{car \parallel rd} = mg\sin\theta$$

b) What angle should the road be inclined in order for the parallel component of the gravitational force found in (a) to equal the centripetal force?

$$mg\sin\theta = \frac{mv^2}{R}$$

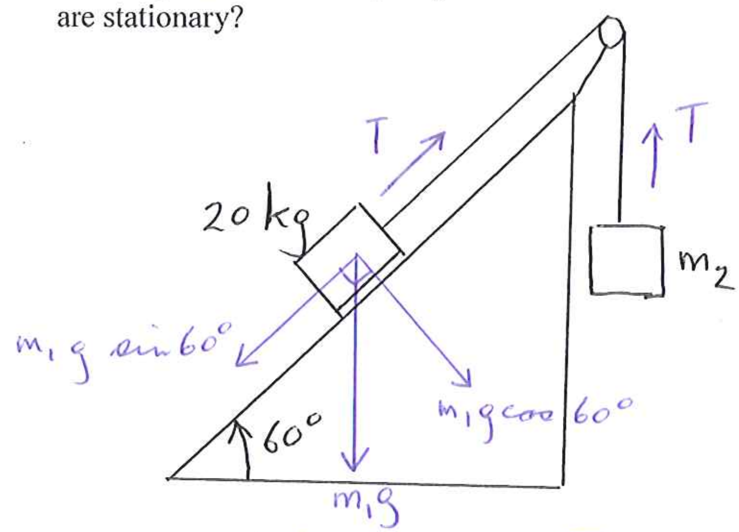
$$\sin\theta = \frac{v^2}{Rg}$$

$$= \frac{\left(120\,\frac{km}{hr} \times 1000\,\frac{m}{km} \times \frac{1}{3600}\,\frac{hr}{sec}\right)^2}{1000\,m \times 10\,m/sec^2}$$

$$= 0.11$$

$$\therefore \theta = 6.4^\circ$$

6. Consider a 20 kg mass located on a plane inclined at 60° from the horizontal. The coefficient of friction is 0.1. The mass is attached to a string that is attached to a mass m_2 via a massless pulley as shown below. Find m_2 so that the two masses are stationary?



For m_2 not to move $T = m_2 g$.

20 kg. mass moves down plane when

$m_1 g \sin 60° > T + \mu m_1 g \cos 60°$

$T < m_1 g(\sin 60° - \mu \cos 60°)$

$m_2 < m_1 (\sin 60° - \mu \cos 60°)$

$< 20\left(\frac{\sqrt{3}}{2} - \frac{0.1}{2}\right)$

< 16.3 kg.

20 kg. mass moves up plane when

$T > m_1 g \sin 60° + \mu m_1 g \cos 60°$

$m_2 > m_1(\sin 60° + \mu \cos 60°)$

$> 20\left(\frac{\sqrt{3}}{2} + \frac{0.1}{2}\right)$

> 18.3 kg.

$\therefore$ masses stationary when $16.3 < m_2 < 18.3$ kg.

Assignment 6
Oscillatory Motion

1. A grandfather clock is moved from one site where g = 9.80 m/sec^2 to a site closer to the equator where g = 9.79 m/sec^2. Assuming the clock owner is not a physicist and does not adjust the length of the clock pendulum.

   a) Will the clock be too slow or too fast?

Period $T = \frac{2\pi}{\omega}$

$= 2\pi \sqrt{\frac{l}{g}}$.

If g decreases, T increases & clock ticks too slow.

   b) By how many minutes is the clock out in one week?

One week $= 24 \times 7 \times 3600$

$= 6.05 \times 10^5$ sec (1 sec = 1 period)

$|\Delta T| = 2\pi \sqrt{l} \ \frac{1}{2} g^{-3/2} \Delta g$

$\frac{\Delta T}{T} = \frac{\Delta g}{2g}$

$= \frac{(\times .01)}{2 \times 9.8}$

$= .0005$

$\therefore \Delta T$ after $T = 6.05 \times 10^5$ sec is 3.0295×10 sec.

or 5.6 minutes.

2. A spring having constant k = 2 Nt/m is connected to a 30 kg mass. The mass is then pulled extending the spring and released.

a) Evaluate the period of the motion.

$$\text{Period} = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\left(\frac{30\text{ kg}}{2\text{ Nt/m}}\right)^{1/2}$$

$$= 24\text{ sec.}$$

b) What happens to the period of the motion if:

i) the mass is doubled

Period increases by $\sqrt{2}$.

ii) the spring constant is doubled

Period decreases by $\sqrt{2}$.

iii) the spring constant and mass are doubled

Period is unchanged.

iv) the initial extension of the spring is doubled

Period is unchanged.