Assignment 6

1a) H ground state has hyperfine levels $F=1,0$.

$$H = H_{hyp} + H_z$$

$$= a\,\vec{I}\cdot\vec{J} + 2\omega_0 S_z \quad \text{where } \omega_0 \equiv \frac{\mu_B B}{\hbar}$$

b) Basis is either $\{|m_s m_I\rangle\} = \{|\tfrac{1}{2}\tfrac{1}{2}\rangle, |\tfrac{1}{2}-\tfrac{1}{2}\rangle, |-\tfrac{1}{2}\tfrac{1}{2}\rangle, |-\tfrac{1}{2}-\tfrac{1}{2}\rangle\}$

OR $\{|F m_F\rangle\} = \{|00\rangle, |11\rangle, |10\rangle, |1-1\rangle\}$

c) Eigenenergies are: $E = \begin{cases} \dfrac{a\hbar^2}{4} \pm \hbar\omega_0 \\ -\dfrac{a\hbar^2}{4} \pm \sqrt{(\hbar\omega_0)^2 + \left(\dfrac{a\hbar^2}{2}\right)^2} \end{cases}$

Eigenstates for: $E = \dfrac{a\hbar^2}{4} + \hbar\omega_0$ is $|m_s m_I\rangle = |\tfrac{1}{2}\tfrac{1}{2}\rangle$.

$\dfrac{a\hbar^2}{4} - \hbar\omega_0$ " " $|-\tfrac{1}{2}-\tfrac{1}{2}\rangle$.

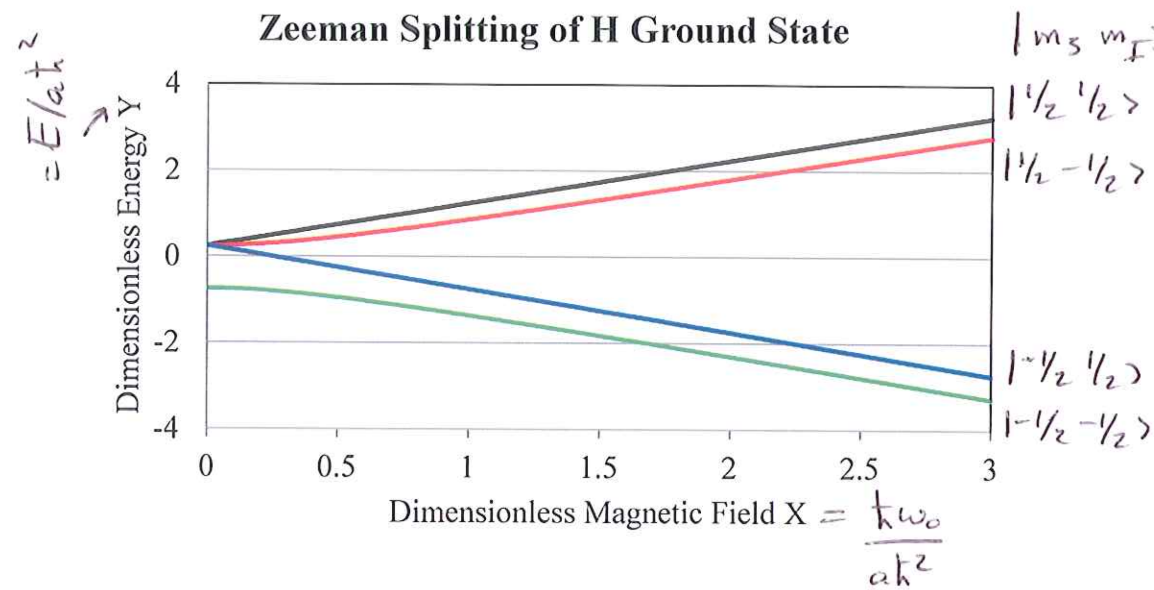
To find eigenstates for $E_\pm = -\dfrac{a\hbar^2}{4} \pm \sqrt{(\hbar\omega_0)^2 + \left(\dfrac{a\hbar^2}{2}\right)^2}$

solve:

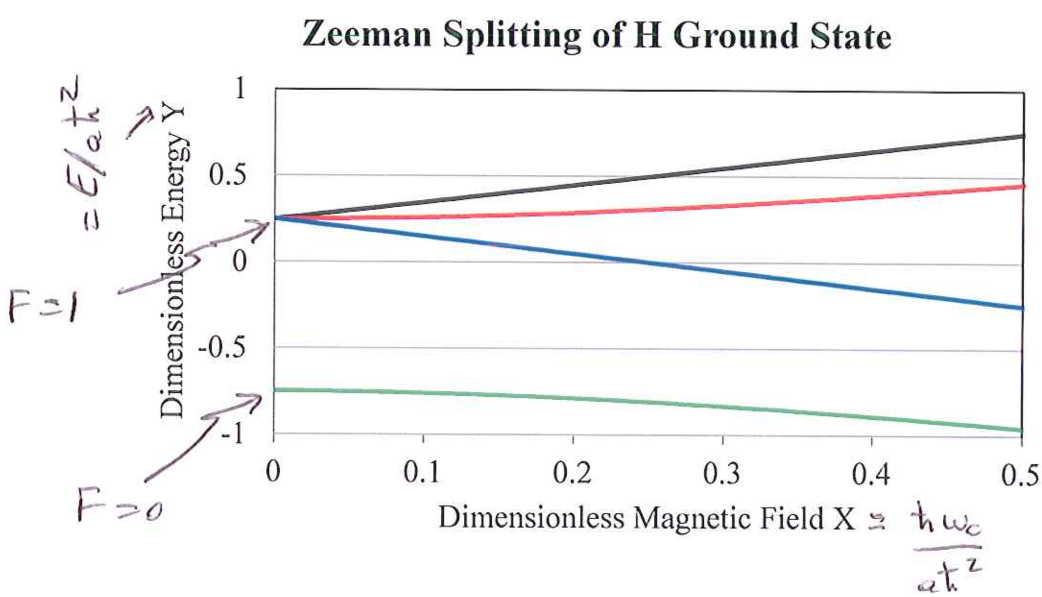
$$\begin{pmatrix} -\dfrac{a\hbar^2}{4} + \hbar\omega_0 - E_\pm & \dfrac{a\hbar^2}{2} \\ \dfrac{a\hbar^2}{2} & -\dfrac{a\hbar^2}{4} - \hbar\omega_0 - E_\pm \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Note: See pg. 80 of notes.

a) High Field



b) Low Field



$$\Rightarrow x_1 = \frac{-a\hbar^2/2}{\frac{-a\hbar^2}{2} + \hbar\omega_0 - E_\pm} x_2$$

$$\Rightarrow \text{eigenstate } \psi_\pm = \frac{\frac{-a\hbar^2}{2}}{\frac{-a\hbar^2}{2} + \hbar\omega_0 - E_\pm} |\tfrac{1}{2} \, {-\tfrac{1}{2}}\rangle + |{-\tfrac{1}{2}} \, \tfrac{1}{2}\rangle.$$

A normalized eigenstate is found by dividing by N where

$$N^2 \equiv \langle \psi_\pm | \psi_\pm \rangle.$$

e) Low Field Limit

Defining $\epsilon \equiv \frac{E}{a\hbar^2}$ & $x \equiv \frac{\hbar\omega_0}{a\hbar^2}$, eigenergies are:

$$\epsilon = \begin{cases} \frac{1}{4} \pm x. \\ -\frac{1}{4} \pm \frac{1}{2}\sqrt{1+4x^2} \end{cases}$$

When $x \ll 1 \Rightarrow$
$$\epsilon = \begin{cases} \left.\begin{array}{l} \frac{1}{4} \pm x \\ \frac{1}{4} + x^2 \end{array}\right\} \text{result for } F=1 \\ \left.-\frac{3}{4} - x^2 \right\} \text{result for } F=0. \end{cases}$$

f) High Field Limit $x \gg 1$.

$\epsilon = \pm x$ corresponding to $|m_s = \tfrac{1}{2}\rangle$ & $|m_s = -\tfrac{1}{2}\rangle$.