Assignment 5

1a) Energy separation between H 1S & 2S states $= \frac{3}{4} E_R = 10.2\,eV$

- mainly due to Coulomb interaction + kinetic energy of electron motion around nucleus.

b) Fine Structure due to:

- spin orbit interaction - results from electron magnetic moment + magnetic field due to moving nuclear charge
- relativistic correction to electron energy
- Darwin term due to Compton wavelength of electron

$$\frac{E_{FS}}{E_n} = \left(\frac{Z\alpha}{n}\right)^2 \left[\frac{3}{4} - \frac{n}{j+1/2}\right] \qquad j = l \pm 1/2$$

There is no obvious shift for ground state since we only have $l = 0 \Rightarrow j = 1/2$.

For H $2P_{1/2}$ + $2P_{3/2}$ separated by 1×10^{10} GHz

c) Lamb Shift

- vacuum is not empty but has e^- e^+ charge pairs popping in + out briefly as long as $\Delta E \, \Delta t \sim \hbar$. $\Rightarrow$ Zitterbewegung

- Difference between H $2S_{1/2}$ + $2P_{1/2}$ states ~ 1 GHz.

d) Hyperfine Interaction

- interaction between electron + nuclear magnetic moments
- finite nuclear size $\Rightarrow$ electric quadrupole interaction

Difference between H $1S_{1/2}$ $F=1+0$ is 1.4×10^{9} Hz
This generates the famous 21 cm line in astronomy.

e) $H_{zeeman} \sim \mu_B B$.

$= 9.27\times10^{-24} \frac{J}{Tesla} \times 0.5\times10^{-4}$ Tesla.

$= 4.6\times10^{-28}$ J

$= 2.9\times10^{-9}$ eV.

This corresponds to a frequency of $\sim 7\times10^{5}$ Hz.

f) $H_{Stark} \sim a_0^3 \; E^2$ ← electric field
↑ polarizability $\sim a_0^3$

$\sim (0.53\times10^{-10}\text{ m})^3 \left(\frac{10^3 \text{ Volt}}{.01\text{ m}}\right)^2$

$\sim 1.5\times10^{-21}$ J

$= 9.3\times10^{-3}$ eV.