Atomic Physics Assignment 3

5 x 2 = 10 marks

1. Using the definition of spherical coordinates show that

   $L_z = -i\hbar \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}\right) = -i\hbar \frac{\partial}{\partial \phi}$

2. Raising and Lowering Operators applied to Spherical Harmonic Functions
   a) Apply L. to Y_{11} to find Y_{10} and Y_{1-1}.
   b) Verify the orthonormalization condition is satisfied for $Y_{2+/-2}$, $Y_{2+/-1}$, Y_{20}

3. Consider the addition of angular momenta $j_1 = 3$ and $j_2 = 1$.
   One can find $|J, M\rangle = \Sigma m_1 m_2 |j_1, j_2, m_1, m_2\rangle \langle j_1 j_2 m_1 m_2 |J, M\rangle$

   Beginning with $|J=3, M=3\rangle = |j_1=2, m_1=2\rangle |j_2=1, m_2=1\rangle$, apply J_ to find expressions for $|3,3\rangle$, $|3,2\rangle$ …. . $|3,-3\rangle$

4. Electron Spin
   Show $[S^2, S_x]=0$ and $[S_x, S_y] = i\hbar S_z$ are satisfied if $S = \hbar/2\ \sigma$ where Pauli spin matrices are:

   $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

   and electron spin wavefunctions are : $\chi_{1/2\,1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\chi_{1/2\,-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

5. Calculation of Landé g_J factor
   Total magnetic Moment $\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S$

   where: $\vec{\mu}_S = -2\frac{\mu_B}{\hbar}\vec{S}$ $\vec{\mu}_L = -\frac{\mu_B}{\hbar}\vec{L}$ $\vec{\mu}_J = -\frac{g_J \mu_B}{\hbar}\vec{J}$

   Show $g_J = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$