Assignment 1

1a) Bohr radius $a_o = \dfrac{\hbar^2}{mkq^2}$

$$= \frac{m_{elect}}{m_{muon}} \frac{\hbar^2}{m_{elect}kq^2}$$

$$= \frac{1}{207} (0.53 \mathring{A})$$

$$= 2.6 \times 10^{-3} \mathring{A}$$

$$a_o = 2.6 \times 10^{-13} m$$

b) Rydberg Energy $E_R = \dfrac{mk^2q^4}{2\hbar^2}$

$$= \frac{m_{muon}}{m_{elect}} \frac{m_{elect}k^2q^4}{2\hbar^2}$$

$$= 207 \times 13.6 \text{ eV}$$

$$= 2.8 \text{ keV.}$$

2) Energy of emitted photon $h\nu = E_{Ryd}\left[\dfrac{1}{1^2} - \dfrac{1}{4^2}\right]$

$$= 13.6 \text{ eV} \left(1 - \frac{1}{16}\right)$$

$$= 12.75 \text{ eV}$$

Photon Momentum $= \dfrac{12.75 \times 1.6 \times 10^{-19} \text{ J}}{3 \times 10^8 \text{ m/sec}}$

$$= 6.8 \times 10^{-27} \text{ kg} \frac{m}{sec}$$

Conservation of Momentum implies atom acquires equal + opposite momentum.

$$\therefore \vec{P}_{atom} = -\vec{P}_{photon}$$

$\therefore$ atom recoil energy $E_{Rec} = \dfrac{P_{atom}^2}{2m_{atom}}$

$$= \frac{(6.8\times10^{-27})^2}{2\times1.67\times10^{-28}} \text{ J}$$

$$= 1.5\times10^{-27} \text{ J}$$

$$= 9.3\times10^{-9} \text{ eV.}$$

3) $E_H = -\dfrac{E_{Ryd}}{n^2}$

One can show $E_{He^+} = -Z^2\dfrac{E_{Ryd}}{n^2}$ where Z = nuclear charge

$\therefore$ if $n_{He^+} = 2n_H \Rightarrow$ same energy level.

eg. H transition $2 \rightarrow 1$ has same energy as

He^+ transition $4 \rightarrow 2$.

4) $E = -\vec{\mu} \cdot \vec{B}$

For $\vec{\mu}$ opposite $\vec{B} \Rightarrow E = \mu B$.

a) $E = \mu_B B$.

$$= \frac{e\hbar}{2m_e} \cdot B_{Earth}$$

$$= \frac{1.6 \times 10^{-19} \times \frac{6.63 \times 10^{-34}}{2\pi}}{2 \times 9.11 \times 10^{-31}} \cdot 0.5 \times 10^{-4} \text{ Tesla.}$$

$$= 4.6 \times 10^{-28} \text{ J}$$
$$= 2.4 \times 10^{-9} \text{ eV}$$

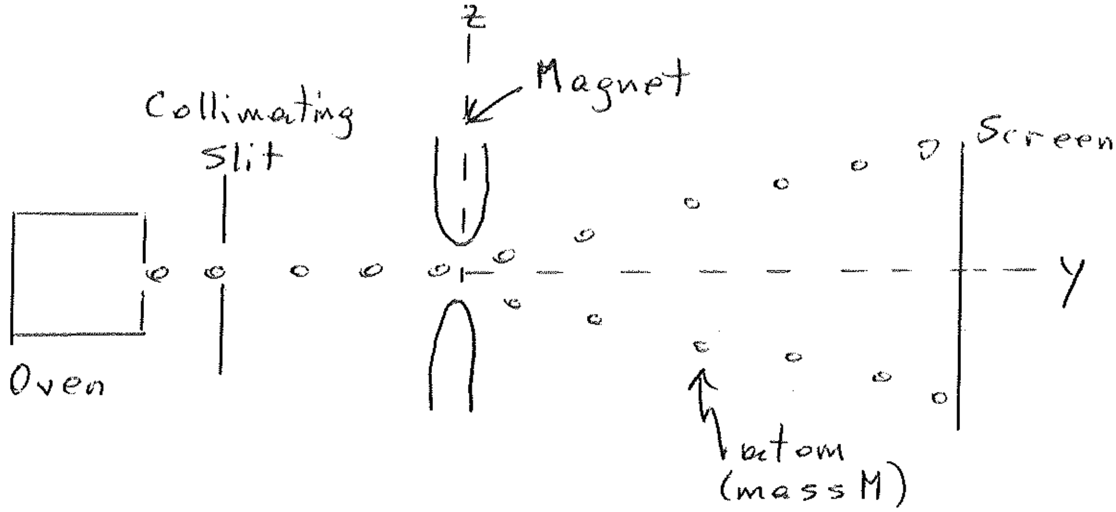
b) $E = \mu_{Nuc} \cdot B$

$$= \frac{e\hbar}{2m_{prot}} \cdot B$$

$$= \frac{m_{elect}}{m_{prot}} \cdot \mu_B B.$$

$$= \frac{1}{1830} \quad 2.4 \times 10^{-9}$$
$$= 1.3 \times 10^{-12} \text{ eV}.$$

5a)



While traversing magnet, atom experiences acceleration:

$$a_z = \frac{1}{M} \mu_B \frac{dB}{dz}$$

Magnet has length l_1 $\Rightarrow$ atom acquires vertical speed

$$v_z = a_z t_1$$

$$= a_z \frac{l_1}{v_y}$$

When atom leaves magnet, one can show deflection on screen is given by

$$z = v_z t_2$$

$$= v_z \frac{l_2}{v_y}$$ where l_2 is distance from magnet to screen.

Note: Here we ignored vertical deflection in magnet which you can show is negligible.

$\therefore$ deflection $z = \frac{1}{M} \mu_B \frac{dB}{dz} \frac{l_1}{v_y} \frac{l_2}{v_y}$.

$$= \mu_B \frac{dB}{dz} \frac{l_1 l_2}{M v_y^2}$$

Stern & Gerlach used Ag atoms.

$$\therefore z = 9.3\times10^{-24} \frac{J}{\text{Tesla}} \cdot \frac{1\ \text{Tesla}}{.01\,m} \frac{(.05\,m)(1\,m)}{108\times1.67\times10^{-27}\,kg\cdot\left(300\,\frac{m}{sec}\right)^2}$$

$$= 2.9\times10^{-3}\ m.$$

$v_{thermal} \simeq v_{sound\ speed}$.

$$\simeq 3\ mm.$$