Atomic Physics Assignment 2

5 x 2 = 10 marks

1. Heisenberg Uncertainty Principle
   Using the definition of the momentum operator show the following.
   a) $[x, p_x] = i\hbar$
   b) $[y, p_y] = i\hbar$
   c) $[z, p_z] = i\hbar$

2. Orbital Angular Momentum
   a) Write down operators defining the orbital angular momentum operators L_x, L_y and L_z.
   b) Show $[L_x, L_y] = i\hbar Lz$
   c) Show $[L_x, L^2] = 0$
   d) Explain which of the operators L_x, L_y, L_z and L^2 may have simultaneous eigenvalues.

3. General Angular Momentum is defined by: $[J_x, J_y] = i\hbar J_z$
   $[J_y, J_z] = i\hbar J_x$
   $[J_z, J_x] = i\hbar J_y$.
   a) Show $[J_z, J_+] = \hbar J_+$ and $[J_z, J_-] = -\hbar J_-$
   b) Show $[J_+, J_-] = 2\hbar J_z$
   c) Show $[J^2, J_+]=0$
   d) Show $J^2 = ½(J_+J_- + J_-J_+) + J_z^2$

4. Prove the following:
   a) $J_- | j, m = -j > = 0$

   b) For $m > -j$, then $J_- |j\ m>$ is an eigenstate of J^2 and J_z with eigenvalues $j(j+1)\hbar^2$ and $(m-1)\hbar$.

5. Derive the matrices for $J = 1$ as given in the notes for
   a) J_x, J_y and J_z
   b) J_+, J_-
   c) J^2.