

Transfer of ultracold ^{87}Rb from a QUIC magnetic trap into a far off resonance optical trap

H. Ming and W.A. van Wijngaarden

Abstract: Ultracold ^{87}Rb atoms were transferred from a QUIC (quadrupole and Ioffe configuration) magnetic trap into a far off resonance optical trap (FORT). FORTs were created by focusing a 150 mW laser beam having a wavelength of 852 nm to a spot having a radius of 20 and 30 μm . A probe laser then passed through the ultracold atom cloud after the magnetic trap was turned off to study the temporal evolution of the optically trapped atoms. Nearly 10^6 atoms could be transferred into the FORT at temperatures as low as 1 μK with an efficiency as high as 50%.

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Résumé : Des atomes ultra-froids de ^{87}Rb sont transférés d'un piège magnétique QUIC dans un piège optique opérant loin de la résonance (FORT). Les pièges de type FORT sont générés en focalisant un faisceau laser de 150 mW de longueur d'onde de 852 nm sur un point de rayon de 20 et 30 μm . Un laser sonde passe alors à travers les atomes ultra-froids après que le piège magnétique ait été éteint, afin d'étudier l'évolution dans le temps des atomes piégés optiquement. Près 10^6 d'atomes ont pu être transférés dans le FORT à des températures aussi basses que 1 μK et avec une efficacité aussi élevée que 50%.

[Traduit par la Rédaction]

1. Introduction

Ultracold atoms have been generated using a variety of magnetic traps allowing the achievement of Bose–Einstein condensation (BEC) [1–5]. These traps generate a magnetic field configuration that traps atoms occupying a single Zeeman sublevel. The currents required to generate the necessary magnetic fields dissipate considerable heat that can create problems for both macroscopic and more recently microscopic traps [6–8]. An alternative is to use a so-called far off resonance trap (FORT) whereby a laser, tuned below the atom's transition frequency, traps atoms at its focus [9, 10]. FORTs trap all Zeeman sublevels and have been used to generate BECs of several elements [11–14]. These traps facilitate the application of magnetic fields to adjust the collision-scattering length, which is important when studying quantum degenerate Fermi gases. The latter have been used to generate a molecular BEC [15] and study the Bardeen, Cooper, and Schrieffer (BCS)–BEC crossover [16, 17]. Optical lattices have

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also been generated using off-resonance laser beams to study the transition between the superfluid and Mott insulator states [18].

The trapping potential of a FORT is given by [19]

$$U = \frac{\hbar I \Gamma^2}{4 I_s \Delta} \quad (1)$$

where \hbar is Planck's constant divided by 2π , I is the laser intensity, and Γ equals 2π times the natural linewidth. The saturation intensity I_s is given by

$$I_s = \frac{2}{3} \pi^2 \frac{\hbar c \Gamma}{\lambda^3} \quad (2)$$

where c is the speed of light and λ is the transition wavelength. For the case of an alkali atom, the detuning Δ is given by

$$\frac{3}{\Delta} = \frac{1}{\delta_{1/2}} + \frac{2}{\delta_{3/2}} \quad (3)$$

where $\delta_{1/2}$ and $\delta_{3/2}$ are the detunings of the laser frequency below the D1 and D2 transition frequencies, respectively.

The largest trap depths are achieved using focused laser beams that are tuned just below the atomic resonance. However, even optical traps created using the most powerful lasers available, have a shallow depth. For example, one of the initial demonstrations of a FORT used a 0.8 W laser beam operating at 814 nm that was focused to a spot having a radius of 9.6 μm to trap Rb atoms [9]. For the Rb 5S–5P transition, Γ is $2\pi \times 6.1$ MHz and the resulting maximum trap depth was 6 mK. In general, atoms must be precooled before they can be loaded into a FORT. The preceding experiment first cooled the atoms in a magneto-optical trap (MOT) and was able to transfer only about 1000 atoms into the FORT.

A larger number of atoms can be loaded into a FORT from a MOT using a larger laser focal spot size. This enabled the Wieman group [19] to transfer several million Rb atoms into a FORT created using 300 mW of laser radiation tuned at 784 nm. The laser beam was focused to a spot having a radius of 26 μm generating a maximum trap depth of 1 mK. Unfortunately, detuning the laser near resonance increases the scatter of photons by the atom. This heats the atoms at a rate given by

$$H = \gamma \frac{\hbar^2 k^2}{M} \quad (4)$$

where γ is the rate at which photons are scattered and $\hbar^2 k^2 / M$ is twice the recoil energy that arises when an atom of mass M either emits or absorbs a photon of momentum $\hbar k$. The scatter rate is given by

$$\gamma = \frac{3 I \Gamma^3}{2 I_s \Delta^2} \quad (5)$$

For the Wieman experiment, the scatter rate was 1300 Hz corresponding to a heating rate of 0.5 mK/s. Hence, the trap lifetime, defined as the ratio of the maximum trap depth to the heating rate,

$$\tau = \frac{\Delta M}{\Gamma \hbar k^2} \quad (6)$$

was only a few seconds.

Heating is critical when studying BECs, which, in the case of alkali vapours, typically have a transition temperature near 100 nK. The absorption and scatter can be reduced if the FORT laser is

detuned far from resonance. Ketterle et al. used a laser operating at 985 nm focused to a radial spot size of $6\ \mu\text{m}$ and having a power of 4 mW, to transfer a Na BEC from a cloverleaf magnetic trap into a FORT [20]. The maximum trap depth was $4\ \mu\text{K}$ and the scatter rate was 0.02 Hz corresponding to a trap lifetime of several hundred seconds. A ^{87}Rb BEC in an optical trap has been demonstrated by Chapman et al. [11] using two intersecting 12 W laser beams generated by a CO_2 laser that were focused to a spot having a radius less than $50\ \mu\text{m}$. About half a million atoms were first transferred from a MOT into the FORT. The laser power was then reduced to 200 mW allowing the hotter atoms to escape yielding a BEC consisting of 3.5×10^4 atoms. More recently, a ^{87}Rb BEC was created in a crossed dipole trap generated using two intersecting 3 W YAG laser beams that were focused to a waist of $300\ \mu\text{m}$ [12].

The alignment of a tiny laser focus with the BEC, both of which have a size on the order of tens of micrometres, is not trivial. A further complication is that a BEC in a magnetic trap is spatially shifted from the center of the MOT that is typically used to load the magnetic trap. This paper shows how atoms were transferred from a so-called quadrupole and Ioffe configuration (QUIC) magnetic trap into a FORT. A QUIC trap can achieve a temperature orders of magnitude lower than is possible using a MOT and furthermore, the atom temperature can be specified much more precisely. The optical trap is created by focusing a ~ 100 mW laser beam operating at 852 nm to a spot having a radius of either 20 or $30\ \mu\text{m}$. This generates a FORT having a maximum trap depth of $75\ \mu\text{K}$. The photon scatter rate is 2.5 Hz and corresponds to a heating rate of less than $1\ \mu\text{K/s}$. Section 2 describes the apparatus emphasizing the details of how the laser focus is superimposed onto the ultracold atoms in the QUIC trap. Section 3 describes how atoms could be optimally loaded into the FORT and also discusses their temporal evolution. Conclusions are given in Sect. 4.

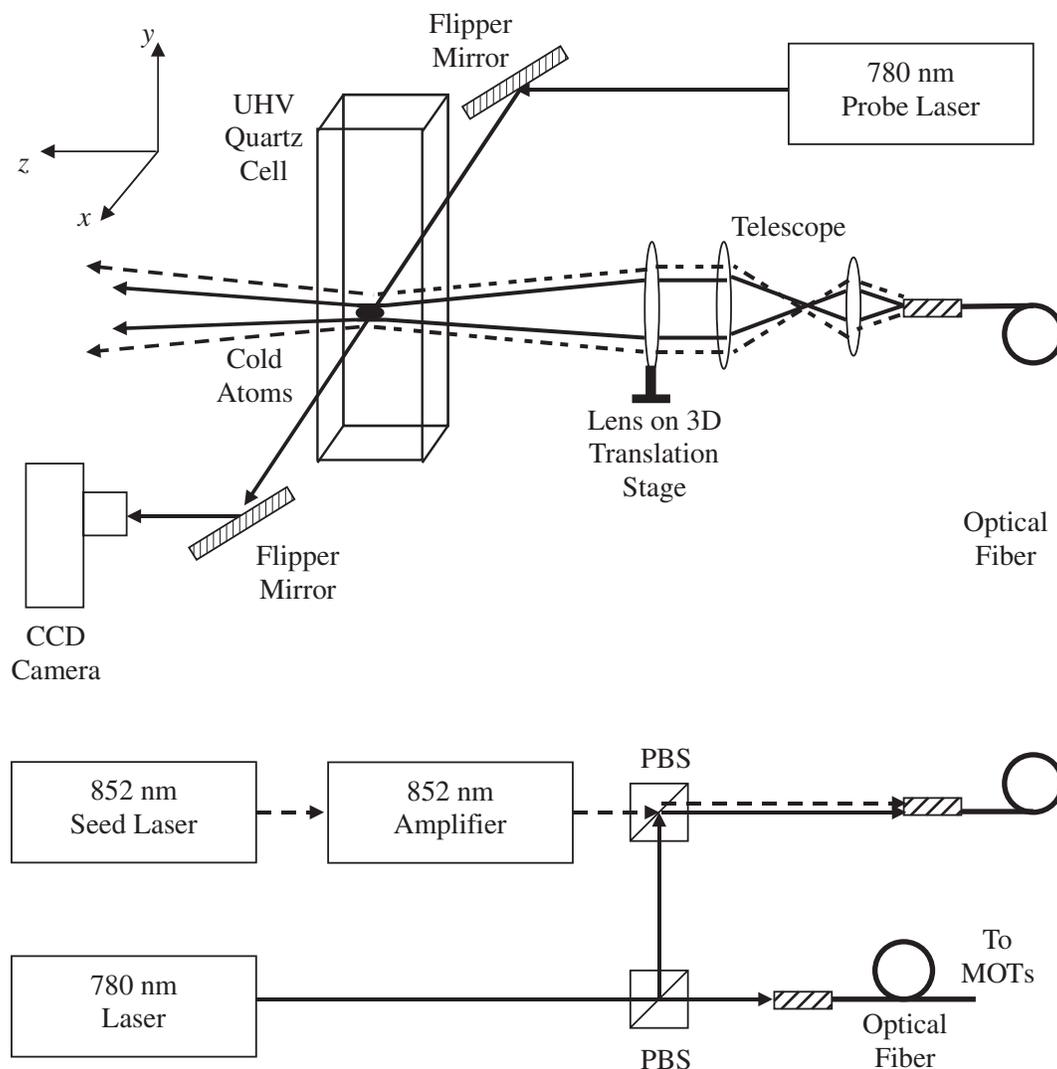
2. Apparatus

The apparatus is illustrated in Fig. 1. The operation of our QUIC trap for generating ultracold atoms including a BEC has been described in detail elsewhere and is, therefore, only briefly discussed [21]. The QUIC trap consisted of a pair of coils positioned on either side of the ultra-high vacuum (UHV) quartz cell along the x -axis carrying oppositely oriented currents plus a third so-called Ioffe coil displaced from the origin along the z -axis. The three coils generate a magnetic field that has a nonzero field minimum where atoms can be trapped if their magnetic moment is parallel with the field.

The QUIC trap was loaded with ^{87}Rb atoms that were cooled from a few hundred degrees Celsius using standard laser-cooling techniques [22]. The atoms were first collected in a MOT operating in a relatively high pressure ($\sim 10^{-9}$ Torr) ($1\ \text{Torr} = 133.322\ \text{Pa}$) vapour cell. A laser beam then pushed the atoms into a second MOT contained in the UHV quartz cell shown in Fig. 1. Each MOT was generated using pairs of counterpropagating laser beams traveling along the x , y , and z directions, which, for simplicity, are not shown in Fig. 1. The pressure in the quartz cell was maintained at about 1×10^{-11} Torr using a combination ion and titanium sublimation pump to minimize collisions with the background gas that can heat the atoms. The atoms were loaded into the magnetic trap by turning on the appropriate currents for the QUIC trap coils.

The final stage of cooling occurred with all the lasers turned off. A radio frequency (rf) signal was applied using a small antenna positioned next to the quartz cell. The atoms absorbed this radiation when the frequency corresponded to a transition between the spin-up and spin-down Zeeman levels. This spin flip caused the atom's magnetic moment to become antialigned with the magnetic field resulting in expulsion from the trap. The rf was tuned from 20 to 3 MHz in 20 s. Initially, only the spin of the hotter atoms farther from the trap minimum was flipped. The remaining trapped atoms underwent thermalizing collisions producing a lower temperature. The result of this so-called evaporative cooling was a cloud of about half a million atoms with a temperature below the BEC threshold of 100 nK. The BEC had an ellipsoidal shape corresponding to the QUIC trap magnetic field with a semimajor axis of approximately $25\ \mu\text{m}$ aligned along the z -axis while the semiminor axis along the radial (x and y) direction was about $5\ \mu\text{m}$.

Fig. 1. Apparatus.



The ultracold atom cloud was probed using a laser beam operating at 780 nm generated by a ν Focus Vortex 6013 laser. The laser frequency was locked to the $^{87}\text{Rb } 5S_{1/2} F = 2 \rightarrow 5P_{3/2} F = 1-3$ crossover peak observed in a vapour cell using saturation spectroscopy. The laser beam passed twice through an acousto-optic modulator (AOM) (IntraAction ATM-1001A2) that shifted approximately 70% of the incoming light by 106 MHz on each pass through the AOM. This frequency-shifted laser beam was in resonance with the $^{87}\text{Rb } 5S_{1/2} F = 2 \rightarrow 5P_{3/2} F = 3$ transition.

The laser beam was coupled into an optical fiber that transported the radiation to the experiment. This ensured that the laser beam exiting the fiber remained aligned from day to day with the subsequent optical components that control the laser polarization, spatial mode etc. The laser beam exiting the fiber was collimated using a telescope not shown in Fig. 1 resulting in a beam having a 4 mm radius. The probe laser beam was then reflected off two computer-controlled mirrors that were flipped into position and directed through the ultracold atom cloud. A CCD camera (Santa Barbara ST-10XME)

that consisted of a 2184×1472 array of pixels, each having a size of $6.8 \mu\text{m}$, detected the transmitted probe laser beam.

Timing with microsecond precision is critical to coordinate the various steps of the experiment. A Labview 6.1 software program was written to control the fast mechanical shutters and AOMs used to adjust the power and length of the pulses of the various laser beams as well as the magnetic fields of the QUIC trap. The probe laser pulse had a duration of $50 \mu\text{s}$ and a power of $100 \mu\text{W}$. The CCD camera was also interfaced to the computer. The absorption of the probe laser beam by the ultracold atoms enabled the determination of the number of trapped atoms. The temperature was found by studying the rate of expansion of the atom cloud after the magnetic trap was switched off [8].

The FORT laser beam was generated using a 852 nm laser diode (SDL 5712). This laser beam passed through an amplifier (Sacher TEC-850-500) yielding a 500 mW laser beam. Two 40 dB optical isolators (ConOptics Model 713) were positioned immediately after the oscillator diode laser and amplifier to prevent optical feedback. The 852 nm laser beam was next combined with part of the 780 nm laser beam used to laser cool the atoms, using a polarization beam splitter (PBS). The two overlapping 780 and 852 nm laser beams were input into a 15 m long single mode polarization maintaining fiber that had a minimum loss at 830 nm (Thorlabs FS-PM-4621). The fiber not only facilitated alignment of the laser beams with subsequent optical components but also ensured that the 780 and 852 nm laser beams were very well collimated and had a Gaussian spatial profile that could be focused to give a diffraction limited spot. The laser beam exiting the fiber was collimated using a telescope as shown in Fig. 1, to generate a laser beam incident on the focusing lens and having an intensity given by

$$I(r, z) = \frac{P}{\pi\omega^2} e^{-(r/\omega)^2} \quad (7)$$

where z is the direction of the laser propagation, r is the radial direction, and P was the laser power. The laser waist ω at the focusing lens was measured to be 2.7 mm. A lens having a focal length f , focused the laser beam to a spot having a waist $\omega^* = \lambda f / \pi \omega$, which was measured by viewing an attenuated laser beam with the CCD camera as shown in Fig. 2. Two lenses, having f equal to 20 and 30 cm, gave focal spots having radii of 20 and 30 μm .

The focus of the 852 nm laser beam was aligned onto the ultracold atoms as follows. First, the 852 nm laser beam was blocked and the focus of the co-propagating 780 nm light was positioned onto the ultracold atoms. The 780 nm radiation could be absorbed by the atoms causing rapid heating. Hence, the alignment of the 780 nm laser focus with the ultracold atom cloud was evident by a large increase in loss of trapped atoms. Initially, a 10 ms pulse of $100 \mu\text{W}$ of 780 nm laser light was directed onto the cold atoms immediately before the QUIC trap was turned off. The number of atoms remaining in the trap was found using the probe laser pulse. The x and y positions of the focusing lens were adjusted to maximize this trap loss. This procedure was repeated gradually lowering the 780 nm laser power to $1 \mu\text{W}$ and reducing the laser pulse duration to 4 ms.

The 852 nm focus was positioned onto the ultracold atoms by adjusting the lens to account for the difference in the focal positions of the 780 and 852 nm laser beams. The focal position of a Gaussian laser beam relative to the lens is given by

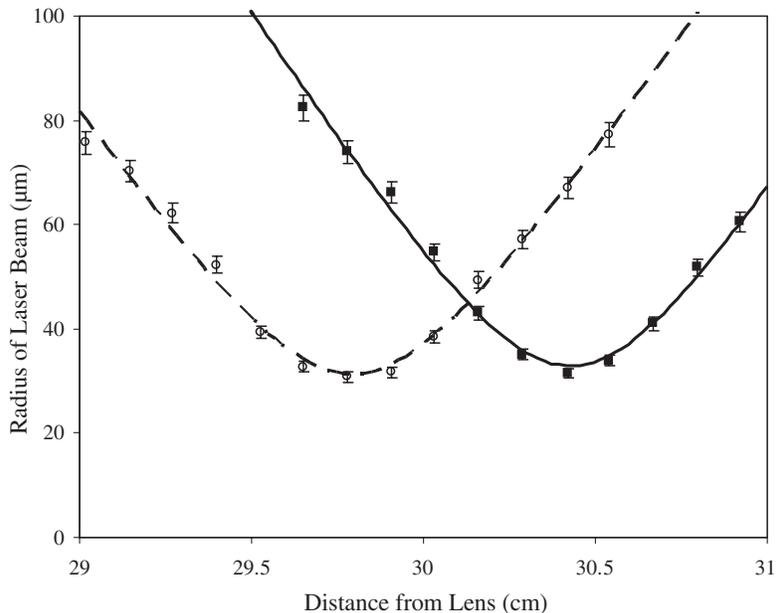
$$z_m = \frac{f}{1 + (f/\omega)^2} \quad \text{where} \quad z_0 = \frac{\pi\omega}{\lambda} \quad (8)$$

Hence, the 852 nm laser beam focus precedes that of the 780 nm laser beam as was observed in Fig. 2 for the case of a lens having f equal to 30 cm. The difference between the focal positions decreased to about 3 mm using the 20 cm focal length lens as predicted by (8).

3. FORT operation

Atoms cooled to a temperature of a few μK were loaded into the FORT as shown in Fig. 3. These images were taken using a 150 mW FORT laser beam that illuminated the atoms for 40 ms immediately

Fig. 2. Radius of 780 (black square) and 852 (open dot) nm laser beams as a function of position from the 30 cm focal length lens. The laser beam was viewed with the CCD camera. The curves are the predicted beam waists for a propagating Gaussian laser beam.



before the QUIC trap was turned off. The data shown in Fig. 3 were taken using a laser focused to a spot having a radius of $30 \mu\text{m}$. The probe laser then passed through the atoms at a time after the QUIC trap was turned off as indicated. Fig. 3a shows the atom cloud falling due to gravity without the FORT laser present. Figure 3b shows that a considerable fraction of the atoms remained trapped when the FORT laser beam was turned on.

The number of atoms in the FORT was proportional to the 852 nm laser power as shown in Fig. 4. A slightly smaller number of atoms could be loaded into the FORT created using a $20 \mu\text{m}$ radial spot as compared to a $30 \mu\text{m}$ spot. Fitting linear functions to the data yielded slopes of 3600 ± 100 and 4400 ± 400 atoms/mW for the 20 and $30 \mu\text{m}$ FORTs, respectively. One advantage of the larger sized FORT trap was that it was easier to align with the ultracold atoms than was the case using the more tightly focused laser. One could expect the $30 \mu\text{m}$ FORT to have more atoms because the laser focus overlaps more of the ultracold atom cloud. However, the maximum trap depth of the $30 \mu\text{m}$ FORT is half that of the $20 \mu\text{m}$ FORT. No significant difference in the number of trapped atoms was observed if the power of the FORT laser was linearly ramped on as compared to when the laser was on at full power, during the 40 ms loading time. The number of trapped atoms was also observed to be independent of the linear polarization axis of the 852 nm laser beam, which could be rotated using a half wave plate.

Figure 5 shows the temperature dependence on trap loading. Each data point is the average found from several measurements with the error bar equal to one standard deviation of the results from their average value. The number of atoms in the FORT increased with temperature because more atoms were then contained in the QUIC trap. A maximum efficiency of about 50% for transferring atoms from the QUIC trap into the FORT occurred at a temperature slightly larger than $1 \mu\text{K}$. The transfer efficiency decreased at higher temperatures because the ultracold atom cloud in the QUIC trap has a larger size that was not entirely overlapped by the FORT laser focus. At temperatures below $1 \mu\text{K}$, the ultracold atom cloud was very small and it was more difficult to maintain stable alignment of the FORT laser focus with the atoms.

Fig. 3. Atom cloud position as a function of time after the QUIC trapped turned off (a) During time t , the atoms fall, when there is no FORT laser beam, a distance $y = 0.5gt^2$ where g is the acceleration due to gravity and (b) some atoms remain trapped in a FORT created by a $150 \mu\text{W}$ laser beam focused to a spot with a $30 \mu\text{m}$ radius.

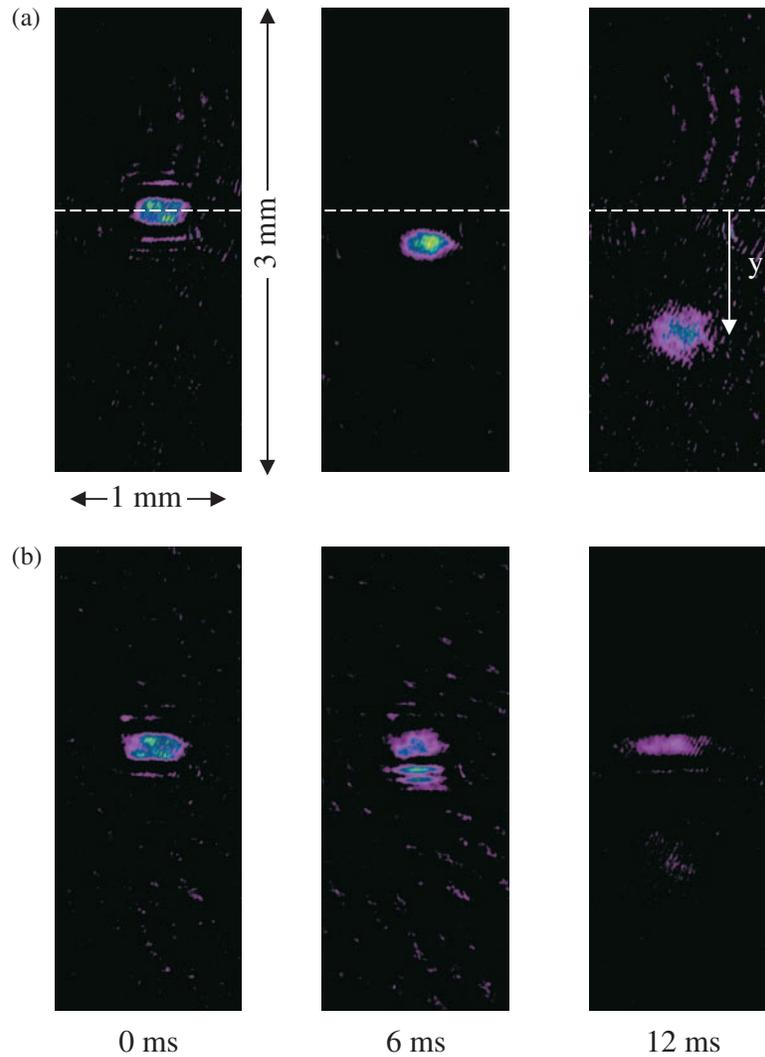


Figure 6 shows the effect of loading time on the number of atoms in a FORT obtained by focusing the 852 nm laser to a spot having a $30 \mu\text{m}$ radius. The FORT laser was turned off 8 ms after the QUIC trap was switched off. This enabled the atoms trapped in the FORT to be distinguished from untrapped atoms as is shown in Fig. 7a The trapped atom number N can be modeled by the following [18].

$$\frac{dN}{dt} = R e^{-t/\tau_L} - \alpha_L N - \beta_L N^2 \quad (9)$$

The first term describes the loading of the atoms into the FORT at a rate R with a time constant τ_L . The second term describes trap loss due to collisions with background gas while the third term takes into account atom loss due to collisions among the ultracold atoms. The continuous curve was found from

Fig. 4. Dependence of number of trapped atoms as a function of FORT laser power. The open (black) dots were taken for a FORT created using a laser focused to a spot with a radius of 20 (30) μm to which a broken (continuous) linear function was fitted.

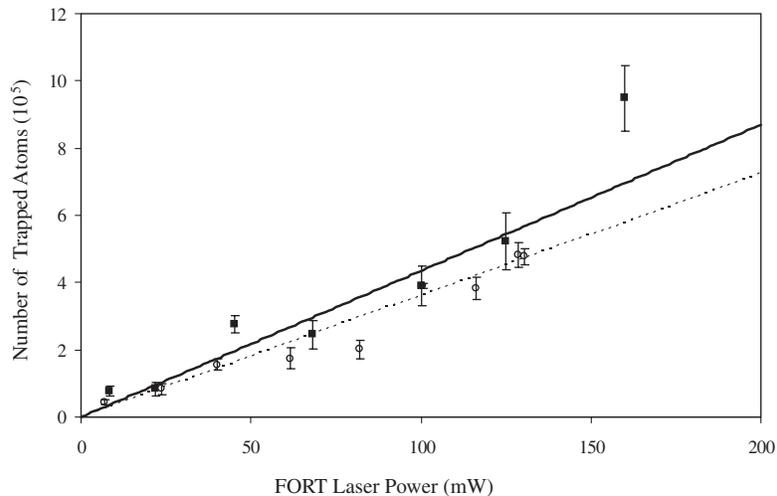
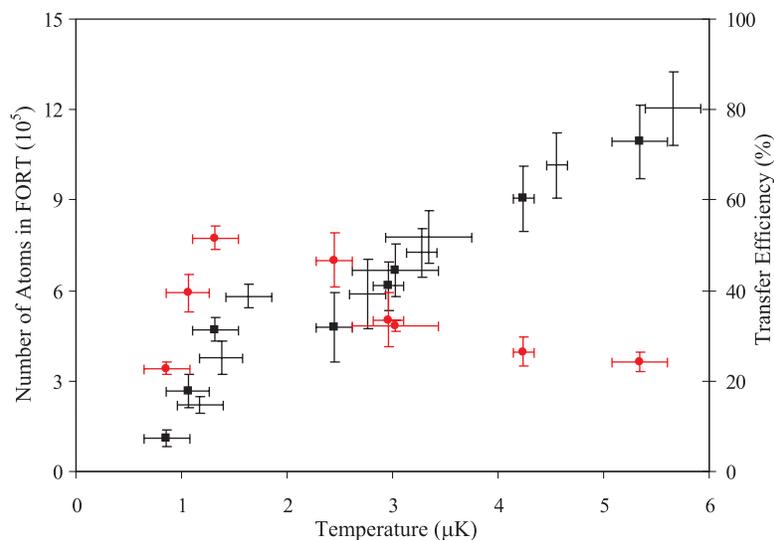


Fig. 5. Temperature dependence of number of atoms in FORT (black) and transfer efficiency (red) of atoms from the QUIC trap to the FORT. This data was taken using a 150 mW 852 nm laser beam focused to a 30 μm spot.



a least-squares fit of (9) to the data yielding values of $R = 1.0 \times 10^7$ atoms/s and $\tau_L = 50$ ms.

It was not possible to precisely estimate α_L and β_L because the fit of (9) to the data using $\alpha_L = 0.324 \text{ s}^{-1}$ and $\beta_L = 0$ was very similar to that found obtained using $\alpha_L = 0$ and $\beta_L = 1.8 \times 10^{-6} (\text{atoms s})^{-1}$. Both of these fits were virtually indistinguishable from that shown in Fig. 6, which used $\alpha_L = 0.087 \text{ s}^{-1}$ and $\beta_L = 0.99 \times 10^{-6} (\text{atoms s})^{-1}$. The exponential decay of atom number predicted by the α_L term in (9) can only be distinguished from the $1/N$ dependence predicted by the β_L term for much longer loading times. Such times were not possible as they are comparable to the QUIC trap

Fig. 6. Number of atoms loaded into a FORT as a function of the loading time before the QUIC trap was turned off. The curve is fitted to the data as is described in the text.

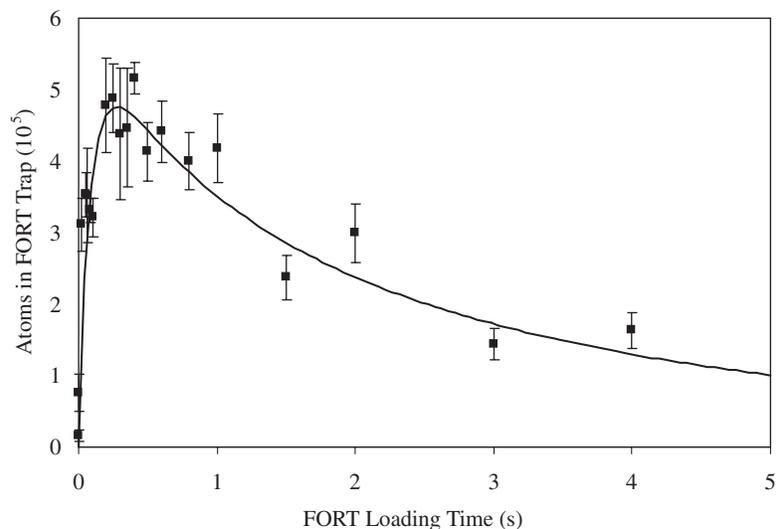
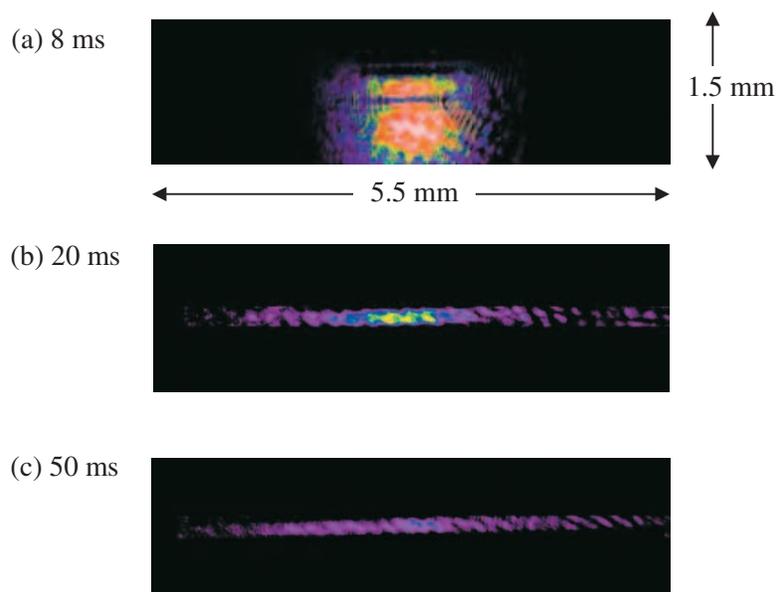


Fig. 7. Time evolution of atoms in FORT after QUIC trap was turned off.



lifetime. A very similar dependence of the atom number on the loading time was found by Wieman et al. who loaded atoms from a MOT into a FORT generated by focusing 305 mW of a 784.5 nm laser to a $26 \mu\text{m}$ radius [19].

The temporal evolution of atoms in the FORT after the QUIC trap is turned off is shown in Fig. 7. These data were taken using the $30 \mu\text{m}$ FORT. Clearly, the atoms were much more tightly trapped in the radial (x and y) directions than along the laser propagation z -axis. Gaussian functions were fitted to the number density in the radial and z directions. Figure 8 shows the dependence of the ultracold cloud

Fig. 8. Dependence of HWHM width of FORT cloud in horizontal (black) and vertical directions (open) as a function of time after QUIC trap shut off.

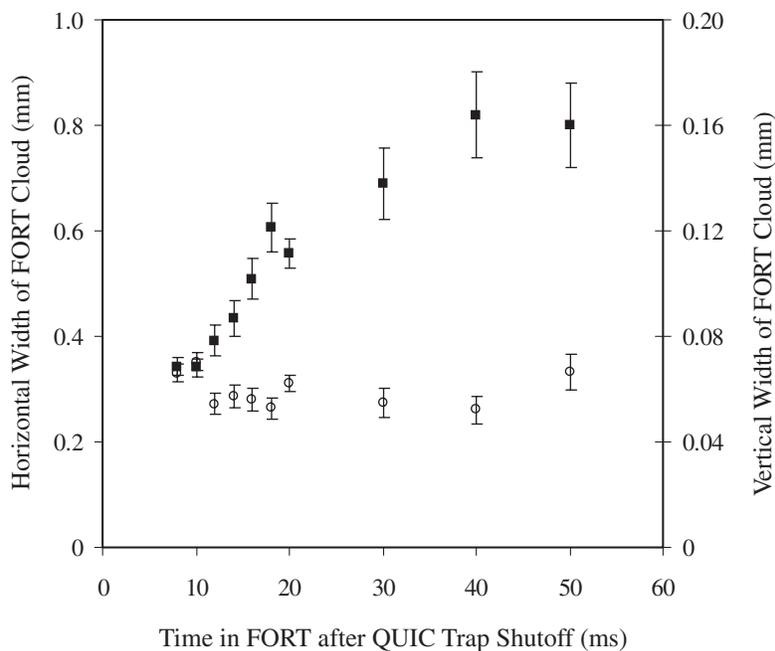
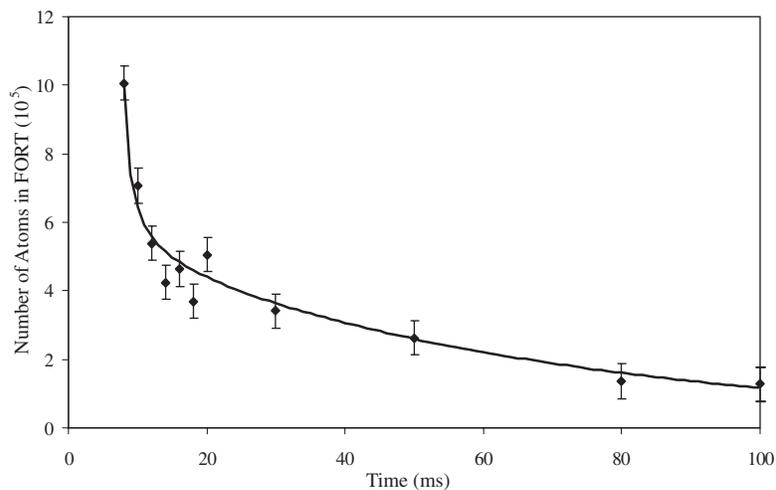


Fig. 9. Temporal dependence of atom number in FORT after the QUIC trap was turned off. The curve is fitted to the data as described in the text.



widths as a function of the time after the QUIC trap was turned off. Similar results were obtained for the case of the $20\ \mu\text{m}$ FORT with the radial size of the atom cloud remaining nearly constant at $60\ \mu\text{m}$ while the size in the z direction increased from $0.2\ \text{mm}$ at $8\ \text{ms}$ to $1.2\ \text{mm}$ at $50\ \text{ms}$ after the QUIC trap was switched off.

The decay of the atoms in the FORT is shown in Fig. 9 as a function of time after the end of the

QUIC trap. The number of trapped atoms is described by

$$\frac{dN}{dt} = -\alpha_D N - \beta_D N^2 \quad (10)$$

Here, α_D and β_D represent trap loss due to collisions with background gas and with other cold atom atoms and differ from α_L and β_L in (9) that describe the trapped atom behaviour in the combined QUIC and FORT traps. The data are well fit by a single exponential function for all but the first few points where the atom density is highest and collisions among the ultracold atoms are most significant² [23]. A least-squares fit of (10) to the data determined $\alpha_D = 15.8 \text{ s}^{-1}$ and $\beta_D = 1.89 \times 10^{-3} (\text{atom s})^{-1}$.

4. Conclusions

This experiment showed how to transfer atoms from a QUIC trap into a FORT. The FORT was generated using a single laser beam having a maximum power of only 150 mW as compared with two CO₂ laser beams of 12 W each by Barrett et al. [11] or two YAG beams of 3 W each by Kinoshita et al. [12]. The maximum trap depth was also more than an order of magnitude smaller than the 1 mK of Kuppens et al. [18]. Nearly 10⁶ atoms could be loaded into the FORT because the atoms were precooled to μK temperatures in a QUIC magnetic trap.

The key experimental challenge to transferring atoms from the QUIC trap into the FORT was to position the 20 or 30 μm laser focus onto the ultracold atom cloud. This alignment was accomplished by superimposing a 780 nm laser beam onto the infrared FORT laser beam using an optical fiber. The 780 nm laser focus could be optimally adjusted by measuring the heating of the atoms that occurred when the laser illuminated the atoms in the QUIC trap. The procedure to align the foci of the 780 and 852 nm laser beams can be simplified using achromatic lenses to collimate the light emerging from the optical fiber and focus it onto the ultracold atom cloud.

Up to 50% of the atoms were successfully transferred into the FORT from the QUIC trap. A larger number of atoms could be loaded into a FORT using a more powerful laser beam. This would also permit a larger laser focus that would facilitate alignment with the ultracold atom cloud and make the trap less susceptible to vibrations that perturb the laser direction and can cause trap instability. The experiment found it was easier to align the larger 30 μm laser focus with the ultracold atoms than was the case with the 20 μm focal spot. A convenient alignment procedure is especially important for the case of a crossed beam FORT where two laser foci must be aligned with the ultracold atom cloud. Such a trap confines the atoms in all three directions, which is necessary to attain BEC. The observed 63 ms ($=1/\alpha_D$) lifetime of atoms in the FORT greatly exceeds the time for currents generating trapping magnetic fields in the QUIC trap to decay. Hence, FORTs provide an environment unperturbed by magnetic fields, which is ideal for studying ultracold atoms.

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