Assignment 5

15 1. Show that $\Phi(\vec{r},t) = \int_V \frac{\rho(\vec{r}';t_r)}{r}\, d^3r'$

$$\vec{A}(\vec{r},t) = \frac{1}{c}\int \frac{\vec{J}(\vec{r}';t_r)}{r}\, d^3r'$$

satisfy the Lorentz condition. Hint: Proceed as follows.

a) Show $\nabla.\left(\frac{\vec{J}}{r}\right) = \frac{1}{r}(\nabla.\vec{J}) + \frac{1}{r}(\nabla'.\vec{J}) - \nabla'.\left(\frac{\vec{J}}{r}\right)$

where ∇ denotes differentiation w.r.t. $\vec{r}$ & ∇' w.r.t. $\vec{r}'$.

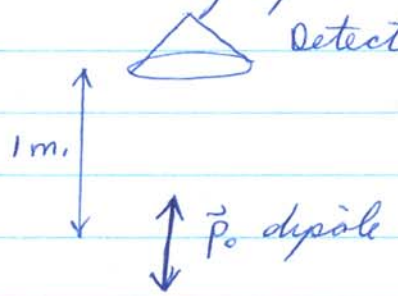
b) Show $\nabla.\vec{J} = -\frac{1}{c}\frac{\partial \vec{J}}{\partial t_r}.(\nabla r)$

c) Show $\nabla'.\vec{J} = -\frac{\partial \rho}{\partial t} - \frac{1}{c}\frac{\partial \vec{J}}{\partial t_r}.(\nabla' r)$

d) Show $\nabla.\vec{A} = -\frac{1}{c}\frac{\partial \Phi}{\partial t}$

10 2a) Discuss the assumptions made when discussing electric dipole radiation.

b) A detector having a radius of 10 cm. is placed 1 meter away from an electric dipole



What fraction of total solid angle does the detector

cover and what fraction of total dipole power does it detect (assuming 100% detection efficiency)?

253. Consider two half wave antennas each having a current

$$\vec{I}(z,t) = \hat{z}\ I_0 \cos\omega t \sin k\left(\frac{d}{2} - |z|\right), \quad k = \frac{\omega}{c}$$

Each antenna has length d and points in direction $\hat{z}$. Antenna one is at position $\left(\frac{\Delta}{2}, 0, 0\right)$ and the other is at $\left(-\frac{\Delta}{2}, 0, 0\right)$

a) Find the vector potential. $\vec{A}(\vec{r}, t)$.

b) Find the electric and magnetic fields.

c) Find $\frac{dP}{d\Omega}$.

d) Evaluate $\frac{dP}{d\Omega}$ in the xy plane when antennae are separated by a distance $\frac{\lambda}{2}$. Along what direction is radiation preferentially emitted?

104. A simple model of H is to picture the electron orbiting the proton.

a) How much energy should the electron emit per second if $P = \frac{2}{3}\frac{e^2 a^2}{c^3}$ holds?

b) What is the kinetic energy of H in the ground state?

c) Crudely estimate how long it takes for the electron to radiate away this kinetic energy.

105. An accelerator has a current of 1 μamp. of electrons at a speed of .9999 c. The accelerator radius is 1 km.

a) How many electrons are in the accelerator beam?

b) What is the power radiated by the e^- beam?

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