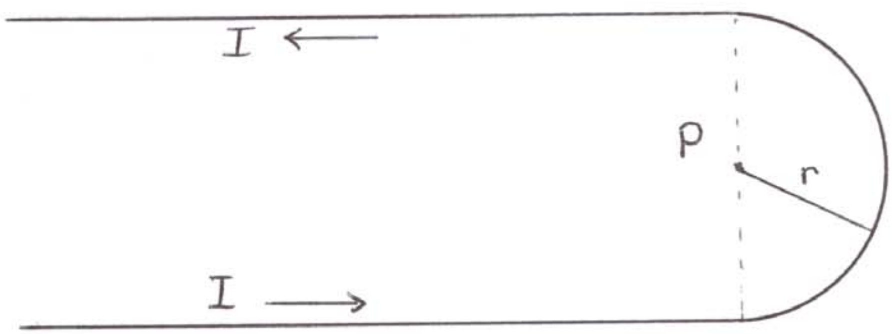
Assignment 7

1. Prove that magnetic fields can't do any any work.

2. A long wire (i.e. extending from infinity) is bent into the hairpinlike shape shown in the figure below. Find an exact expression for the magnetic field at the point P which lies at the center of the half-circle.



3. Consider an infinite solenoid with N turns per unit length, radius R and current I.

   a) Find the vector potential using $\vec{B}$.

   b) Show that $\vec{B} = \nabla \times \vec{A}$.

   c) Show that $\nabla \cdot \vec{A} = 0$.

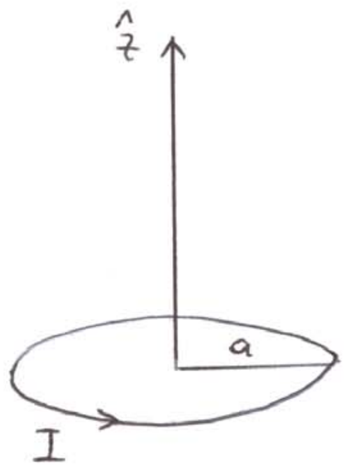
4. If $\vec{B}$ is uniform, $\vec{A} = -\frac{1}{2}(\vec{r} \times \vec{B})$. Verify that this is true by:

   a) Show $\vec{B} = \nabla \times \vec{A}$.

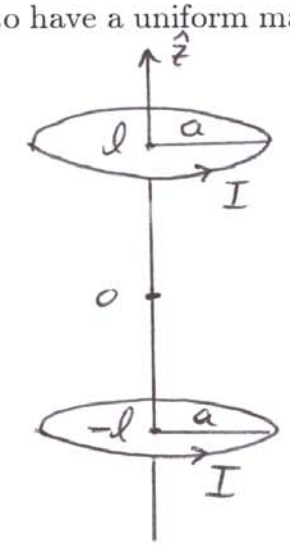
   b) Show $\nabla \cdot \vec{A} = 0$.

5. The magnetic field at height z above a single loop of wire carrying a current I is

$$\vec{B} = \frac{2\pi I a^2}{c(a^2 + z^2)^{3/2}} \hat{z}$$



It is important in many experiments to have a uniform magnetic field. Helmholtz coils are used for his purpose as shown below.



a) What is the field on the z axis due to both coils?

b) Why are odd derivatives of $B(z)$ at the origin equal to 0?

c) Show $\frac{\partial^2 B}{\partial z^2}(z = 0) = 0$ if $2\ell = a$. (i.e. coil separation = coil radius) This is the so called Helmholtz criterion.

d) Verify that $\frac{I(esu/sec)}{c} = \frac{I(amps)}{10}$.

e) The earth's magnetic field is about half a gauss. What current in amps is needed to generate a field at the coil's center that cancels the Earth's field? Assume the number of windings in each coil is 50 and $a = 30$ cm.