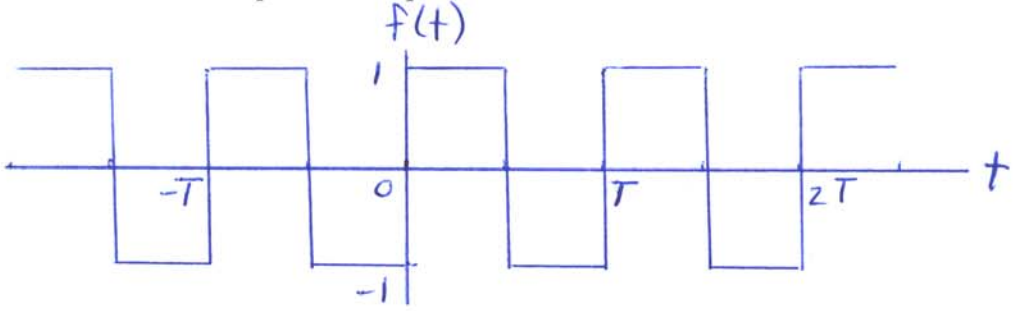
Assignment 4

1. Find the following for the wave $\psi = 5\cos(2x + 3t)$
   a) wave vector
   b) wavelength
   c) frequency
   d) period
   e) phase velocity
   f) amplitude

2. Consider a series of square wave pulses shown below.



Fourier analysis says that this pulse can expanded as:

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t) \qquad \omega \equiv \frac{2\pi}{T}$$

a) Show that $A_n = 4/n\,\pi$ where n is odd, $A_n = 0$ for n is even
$B_n = 0$ and $A_o = 0$

b) Plot the first term, first 2 terms and first 3 terms in the sum.

c) Hence, a pulse of light which can convey information, is composed of many frequencies. Estimate the range of frequencies $\Delta\nu$ required to make a one femtosecond laser pulse using the Heisenberg Uncertainty Principle $\Delta\nu\,\Delta t > 2\pi$.

3. Superposition Principle

a) Show that if ψ_1 and ψ_2 are solutions of the 3 dimensional wave equation that their sum also is a solution.

b) This may seem trivial but show that the superposition principle does not hold for the following differential equation.

$$\frac{d^2\psi}{dx^2} = \psi^2$$

4. Damped harmonic oscillator

$$m\frac{d^2x}{dt^2} = -kx - \gamma\frac{dx}{dt}$$

a) Consider a solution $x = A\,e^{\lambda x}$. Solve for λ. (Result will be complex) This approach is much simpler than using $x = A\cos\omega t + B\sin\omega t$.

b) Write down the general solution for the case of weak damping $km >> \gamma^2$.

c) What is the solution for the case the mass is initially at rest at distance x_o? Plot this solution.

5. Show that the group velocity v_g is related to the phase velocity v by the following equation. Note that for the case of normal dispersion $v_g < c$.

$$v_g = \frac{c}{n + \omega\frac{dn}{d\omega}}$$