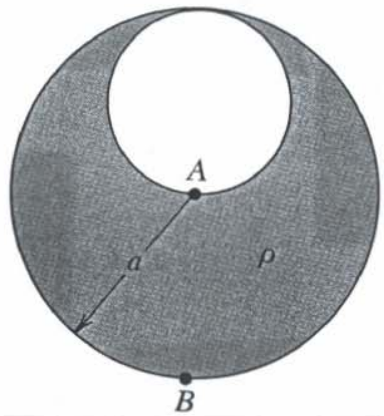
Physics 2020 Assignment 3

1. The sphere of radius a was filled with positive charge at uniform density ρ. Then a smaller sphere of radius a/2 was carved out, as shown in the figure and left empty. What are the direction and magnitude of the electric fields at A? At B?



2. A point charge q is located at the center of a cube of edge of length d.
   a) What is the value of the flux of electric field over one face of the cube?

   b) The charge q is moved to one corner of the cube. What is now the value of the flux of E through each of the faces of the cube?

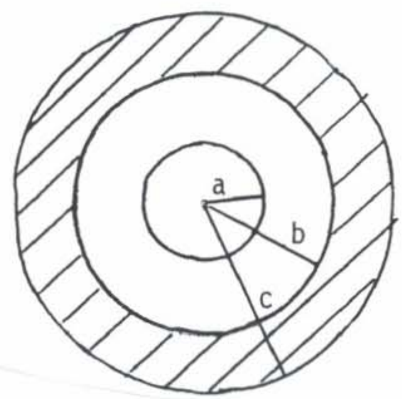
3. Consider two concentric spheres as shown below.

$r < a$ insulator $\rho = \frac{\rho_0 r}{a}$

$a < r < b$ vacuum

$b < r < c$ conductor

$r > c$ vacuum



   a) i. Find the electric field everywhere
      ii. Find surface charge densities on conducting surfaces.

   b) Repeat question 3a but with the conductor grounded. (Note that grounded conductors shield regions from electric fields.)

4. An infinite plane has a uniform surface charge distribution σ on its surface. Adjacent to it is an infinite parallel layer of charge of thickness d and uniform volume charge density ρ. All charges are fixed. Find the electric field everywhere.

5. In this problem you will derive the expression for divergence in cylindrical coordinates.

a) Write expressions relating cylindrical coordinates (ρ, ϕ, z) and Cartesian coordinates (x, y, z).

b) Write expressions for the unit vectors $\hat{\rho}, \hat{\phi}$ & $\hat{z}$ in terms of $\hat{x}, \hat{y}$ & $\hat{z}$ and vice versa.

c) Show that:
$$E_x = E_\rho \cos\phi - E_\phi \sin\phi$$
$$E_y = E_\rho \sin\phi + E_\phi \cos\phi$$

d) $E_x = E_x(\rho, \phi, z)$

Chain rule for partial derivatives is: $\dfrac{\partial E_x}{\partial x} = \dfrac{\partial E_x}{\partial \rho}\dfrac{\partial \rho}{\partial x} + \dfrac{\partial E_x}{\partial \phi}\dfrac{\partial \phi}{\partial x} + \dfrac{\partial E_x}{\partial z}\cdot\dfrac{\partial z}{\partial x}$

Write analogous expression for $\dfrac{\partial E_y}{\partial y}$ and $\dfrac{\partial E_z}{\partial z}$.

e) Using expressions of form
$$\rho = \rho(x, y, z)$$
$$\phi = \phi(x, y, z)$$

Evaluate $\dfrac{\partial \rho}{\partial x}, \dfrac{\partial \phi}{\partial x}$ etc.

f) Show $$\nabla \cdot \vec{E} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\rho) + \frac{1}{\rho}\frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z}$$