Quiz 9

Name: _____

Total = 20 marks

1. (4 marks) Approximately what are the wavelengths of the following.
   a) Red light 650 nm
   b) X ray $\mathring{A} = 10^{-10}$ m
   c) Infrared light 1 μm
   d) UV 100 nm

2. (4 marks) Derive a wave equation for the magnetic field from Maxwell's equations in vacuum.

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \cdot \vec{E} = 0$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\therefore \quad \nabla \times (\nabla \times \vec{B}) = \frac{1}{c}\frac{\partial}{\partial t}(\nabla \times \vec{E})$$

$$\nabla(\underbrace{\nabla \cdot \vec{B}}_{=0}) - \nabla^2 \vec{B} = -\frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial \vec{B}}{\partial t}\right)$$

$$\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$$

3. (6 marks) Neglecting the displacement current term in Maxwell's equations, derive an expression for the skin depth i.e. good conductor limit.

$\nabla \cdot \vec{E} = 0$ $\quad$ $\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ $\quad$ $\nabla \times \vec{H} = \frac{4\pi}{c}\vec{J}$ where $\vec{J} = \sigma \vec{E}$

$$\nabla \times (\nabla \times \vec{E}) = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$\underbrace{\nabla(\nabla \cdot \vec{E})}_{=0} - \nabla^2 \vec{E} = -\frac{4\pi}{c^2}\mu\sigma\frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{E} = \frac{4\pi\mu\sigma}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Let $\vec{E} = \vec{E}_0 e^{i(kz - \omega t)} \Rightarrow -k^2 = \frac{4\pi\mu\sigma}{c^2}(-i\omega)$

$$k^2 = \frac{4\pi\mu\sigma\omega}{c^2} e^{i\pi/2}$$

$$k = \sqrt{\frac{4\pi\mu\sigma\omega}{c}}\, e^{i\pi/4}$$

Let $k = \alpha + i\beta \Rightarrow \beta = \sqrt{\frac{2\pi\mu\sigma\omega}{c}}$

$\therefore |\vec{E}| \propto e^{-\beta z}$ where skin depth

$$\delta \equiv \frac{1}{\beta} = \frac{c}{\sqrt{2\pi\mu\sigma\omega}}$$

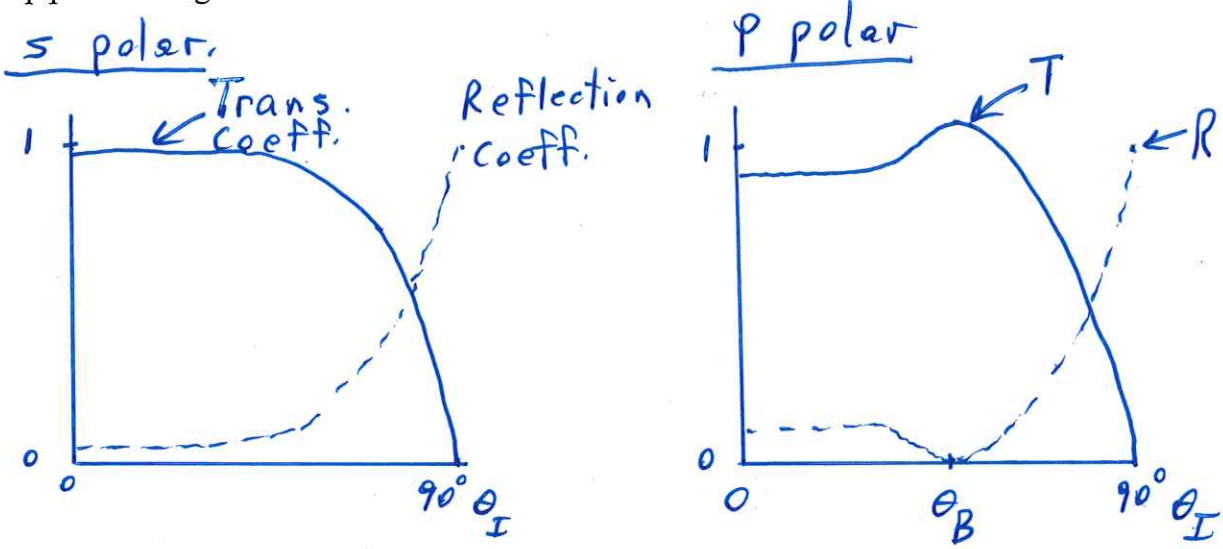
4. (6 marks) Polarization

a) Define what is meant by s and p polarization.

s polarization: $\vec{E} \perp$ incident plane defined by incident, reflected + transmitted wave vectors

p polarization: $\vec{E}$ lies in incident plane

b) Sketch the reflection and transmission coefficients versus angle of incidence for s and p polarized light.



c) Why is Brewster's angle only defined for p polarized light?

There is no θ_I where $R_{s\,pol.} = 0$.