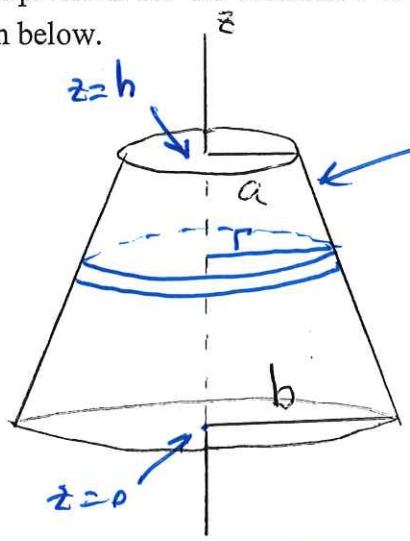
Quiz 8

Name: _____

Total = 20 marks

1. (6 marks) Find an expression for the resistance of a tapered cylindrical object having resistivity ρ as shown below.



$r(z) = b + \frac{a-b}{h} \cdot z$

Consider slice of radius r at height z & thickness dz

Resistance of slice $dR = \frac{\rho\, dz}{\pi r^2}$

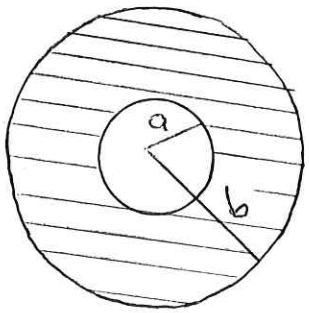
$$\therefore R = \int_0^h \frac{\rho}{\pi\left[b + \frac{a-b}{h} z\right]^2} dz.$$

$$= \frac{\rho}{\pi} \frac{h}{a-b} \left[\frac{-1}{b + \frac{a-b}{h} z} \right]_{z=0}^{z=h}$$

$$= \frac{\rho}{\pi} \frac{h}{a-b} \left[\frac{-1}{a} + \frac{1}{b} \right]$$

$$\therefore R = \frac{\rho h}{\pi a b}$$

2. (6 marks) Consider a conducting sphere of radius a having charge Q surrounded by dielectric material ε up to radius b. Find the total energy stored by the electric field.



Gauss Law gives;

$r < a \quad \vec{D} = \vec{E} = 0.$

$a < r < b \quad \vec{D} = \frac{Q}{r^2}\hat{r} \qquad \vec{E} = \frac{Q}{\epsilon r^2}\hat{r}$

$r > b \quad \vec{D} = \vec{E} = \frac{Q}{r^2}\hat{r}$

$$U_{TOT} = \int \frac{\vec{E}\cdot\vec{D}}{8\pi}\, dV.$$

$$= \frac{1}{8\pi}\int_a^b \frac{Q^2}{\epsilon r^4}\cdot 4\pi r^2 dr + \frac{1}{8\pi}\int_b^\infty \frac{Q^2}{r^4}\cdot 4\pi r^2 dr$$

$$= \frac{Q^2}{2\epsilon}\left(\frac{-1}{b} + \frac{1}{a}\right) + \frac{Q^2}{2}\frac{1}{b}$$

$$= \frac{Q^2}{2}\left[\frac{-1}{\epsilon b} + \frac{1}{\epsilon a} + \frac{1}{b}\right]$$

3. (4 marks) Write down the Lorentz and Coulomb gauge conditions.

Coulomb Gauge $\nabla \cdot \vec{A} = 0$

Lorentz Gauge $\nabla \cdot \vec{A} = -\frac{1}{c}\frac{\partial \Phi}{\partial t}$

4. (4 marks) Write down the differential Maxwell's equations for dielectric and magnetizable materials.

$\nabla \cdot \vec{B} = 0$ $\quad \nabla \cdot \vec{D} = 4\pi \rho_f$

$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ $\quad \nabla \times \vec{H} = \frac{4\pi}{c}\vec{J}_f + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$