Quiz 6

Name: ____________________________

Total = 20 marks

1. (6 marks) An electron travelling initially in the x direction enters a region where a constant and uniform magnetic field points in the z direction.

   a) Write down the equation of motion and solve it for the velocity components.

$$m\frac{d\vec{v}}{dt} = \frac{q}{c}\,\vec{v}\times\vec{B}$$

$$= \frac{q}{c}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix}$$

$$= \frac{q}{c}\left(v_y B, -v_x B, 0\right)$$

$$\frac{dv_x}{dt} = \omega v_y \quad \text{where } \omega = \frac{qB}{mc} \quad (1)$$

$$\frac{dv_y}{dt} = -\omega v_x \quad (2)$$

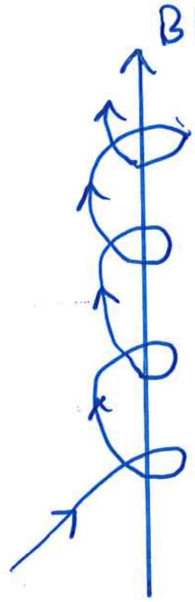
$$\frac{dv_z}{dt} = 0, \quad (3) \Rightarrow v_z = v_{0z} \text{ is constant}$$

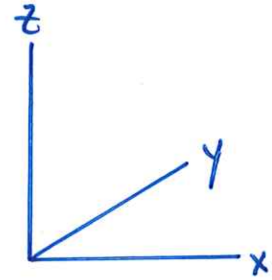
$$(1)+(2) \Rightarrow \frac{d^2 v_x}{dt^2} = -\omega^2 v_x \Rightarrow v_x = A' \cos\omega t + B' \sin\omega t$$

$$v_x = v_{0x}\cos\omega t$$

Subst. v_x into (1) $\Rightarrow v_y = -v_{0x}\sin\omega t$

b) Sketch the path of the electron relative to a magnetic field line.





Looking in $-\hat{z}$ direction, electron spirals clockwise.

i.e. at $t=0$, $v_x = v_{0x}$, $v_y = 0$

$t = \frac{\pi}{2\omega}$ $v_x = 0$, $v_y = -v_{0x}$

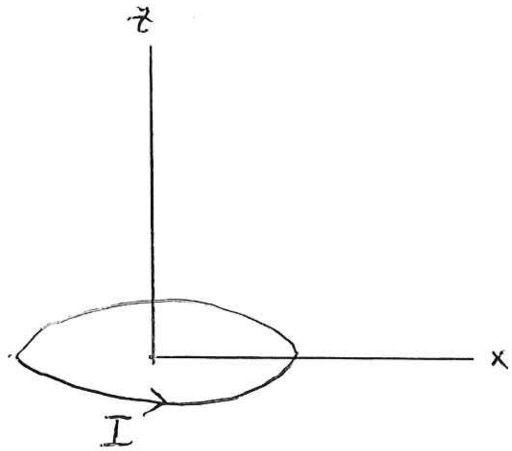
c) Evaluate the Larmor frequency for an electron in a 100 Gauss field.

$$\omega = \frac{qB}{mc}$$

$$= \frac{4.8 \times 10^{-10}\,\text{esu} \times 100\,\text{G}}{9.11 \times 10^{-28}\,\text{gm} \times 3 \times 10^{10}\,\text{cm/s}}$$

$$= 1.76 \times 10^{9}\ \text{rad/sec.}$$

2. (4 marks) Consider a circular loop of radius R lying in the xy plane that carries a current I as shown below.

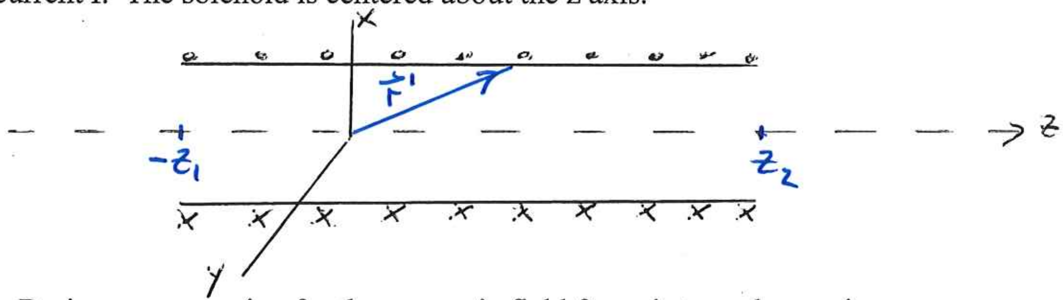


a) Write down an integral expression for the magnetic field at an arbitrary point (x, y, z).

See lecture notes.

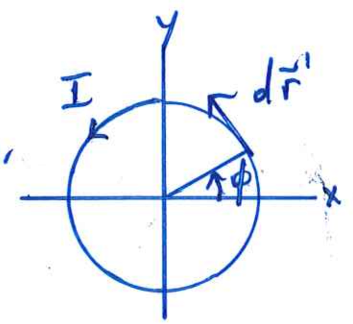
b) Evaluate the field for all points along the z axis.

3. (4 marks) Consider a solenoid of finite length having n turns per unit length and carrying a current I. The solenoid is centered about the z axis.



a) Derive an expression for the magnetic field for points on the z axis.

$$\vec{B}(\vec{r}) = \frac{I}{c}\int d\vec{r}' \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$



Consider cross section view of 1 loop.

$\vec{r}' = (a\cos\phi,\ a\sin\phi,\ z')$

$d\vec{r}' = (-a\sin\phi,\ a\cos\phi,\ 0)\,d\phi$

Let $\vec{r} = (0,0,0)$ i.e. origin

$$\vec{B}(0,0,0) = \frac{nI}{c}\int_{-z_1}^{z_2}\int_0^{2\pi} \frac{(-az'\cos\phi,\ -az'\sin\phi,\ a^2)}{(a^2+z'^2)^{3/2}}\,d\phi\,dz'$$

$$= \frac{2\pi nI}{c}\,a^2\,\hat{z}\int_{-z_1}^{z_2}\frac{dz'}{(a^2+z'^2)^{3/2}}$$

$$= \frac{2\pi nI}{c}\,\hat{z}\,(\sin\theta_2 - \sin\theta_1) \quad \text{where } \tan\theta = \frac{z}{a}$$

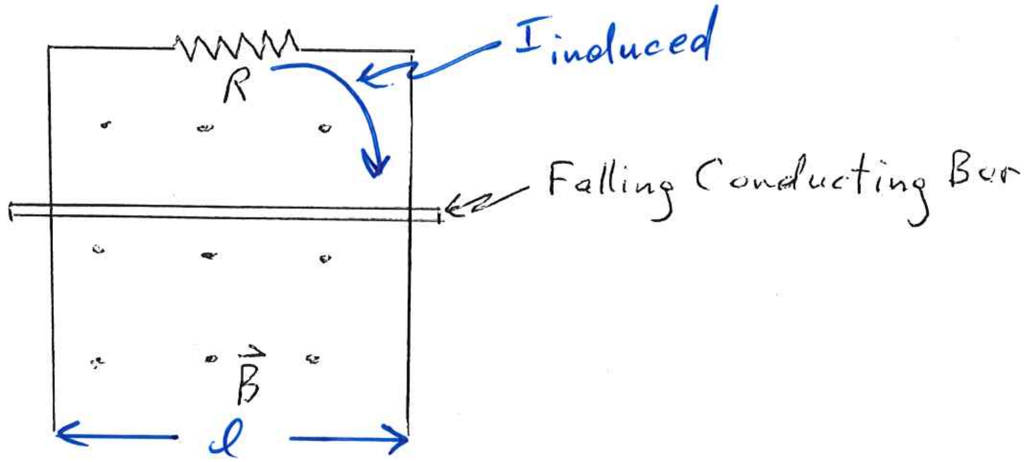
b) What is your answer in the limit of an infinitely long cylinder?

For infinite solenoid $\tan\theta_2 = \infty \Rightarrow \theta_2 = \frac{\pi}{2}$

Similarly $\tan\theta_1 = -\infty \Rightarrow \theta_1 = -\frac{\pi}{2}$.

$$\Rightarrow \vec{B}(0,0,0) = \frac{4\pi nI}{c}\,\hat{z}$$

4. (4 marks) A conducting horizontal bar falls from rest due to gravity. It encloses a circuit having a magnetic field point perpendicular outwards as shown below.



a) Find the induced current in the circuit which has a resistance R.

Induced Voltage $\mathcal{E} = \frac{1}{c}\frac{\partial \Phi}{\partial t}$ ← B field flux through loop.

$$= \frac{1}{c}\frac{\partial}{\partial t}\left(B l \frac{g t^2}{2}\right)$$

$$= \frac{1}{c} B l g t$$

∴ induced current $I = \mathcal{E}/R$

$$= \frac{B l g t}{R c}$$

b) Show the direction of the induced current.

Direction of current generates field that opposes changing flux as shown in diagram,

5. (2 marks) Static Maxwell's equations

a) Write down Maxwell's equations for static fields.

$$\nabla \cdot \vec{E} = 4\pi \rho \qquad \nabla \cdot \vec{B} = 0,$$

$$\nabla \times \vec{E} = 0 \qquad \nabla \times \vec{B} = \frac{4\pi}{\epsilon} \vec{J}$$

b) Show the continuity equation is not satisfied for static Maxwell's equations.

$$\nabla \cdot \vec{J} = \frac{c}{4\pi} \nabla \cdot (\nabla \times \vec{B})$$

$$= 0$$

$$\neq -\frac{\partial \rho}{\partial t}$$