Quiz 4

Name: _____

Total = 10 marks

1. (2 marks) Explain the difference between normal and anomalous dispersion.

Normal: n decreases as λ increases

Anomalous: n increases

2. (4 marks) Wave Equation
   a) Verify that the following wave is a solution of the 1 dimensional wave equation.

$$\psi = A\sin(2x-\omega_1 t) + B\cos(4x-\omega_2 t)$$

$$\frac{\partial^2\psi}{\partial x^2} = -4A\sin(2x-\omega_1 t) - 16B\cos(4x-\omega_2 t)$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega_1^2 A\sin(2x-\omega_1 t) - \omega_2^2 B\cos(4x-\omega_2 t)$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \text{ satisfied if } 4 = \frac{\omega_1^2}{v^2} \text{ and } 16 = \frac{\omega_2^2}{v^2}$$

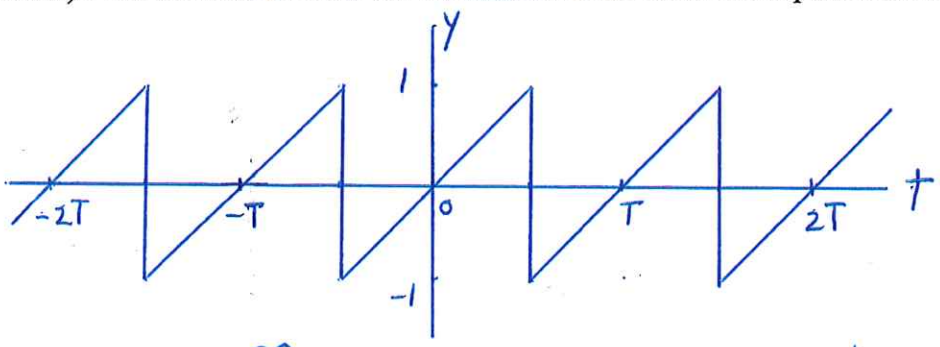
OR $\omega_1 = 2v$, $\omega_2 = 4v$,

   b) If the wave speed is 10 meters/sec, find the values of ω_1 and ω_2. You may assume that the dimensions of all numbers appearing in the wave are in MKS units.

$\omega_1 = 20$ rad/sec

$\omega_2 = 40$ rad/sec

3. (4 marks) Find the fourier series for an infinite series of sawtooth pulses shown below.



$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t) \qquad \omega = \frac{2\pi}{T}$$

$f(t)$ is odd $\Rightarrow A_0 = A_n = 0$.

$$f(t) = \sum_n B_n \sin n\omega t$$

$$\int_{-T/2}^{T/2} \frac{2t}{T} \sin m\omega t \, dt = \sum_n B_n \underbrace{\int_{-T/2}^{T/2} \sin n\omega t \sin m\omega t \, dt}_{= \frac{T}{2}\delta_{nm}}$$

$$B_m = \left(\frac{2}{T}\right)^2 \int_{-T/2}^{T/2} t \sin m\omega t \, dt$$

t	$+$	$\sin m\omega t$
1	$-$	$\frac{-\cos m\omega t}{m\omega}$
0	$+$	$\frac{-\sin m\omega t}{m^2\omega^2}$

$$B_m = \left(\frac{2}{T}\right)^2 \left[-t \frac{\cos m\omega t}{m\omega} + \frac{\sin m\omega t}{m^2\omega^2} \right]_{-T/2}^{T/2}$$

$$= -2 \cdot \left(\frac{2}{T}\right) \frac{\cos\left(m \cdot \frac{2\pi}{T} \cdot \frac{T}{2}\right)}{m\omega}$$

$$= -\frac{4}{m\omega T} \cos m\pi$$

$$B_m = -\frac{2}{\pi m}(-1)^m$$