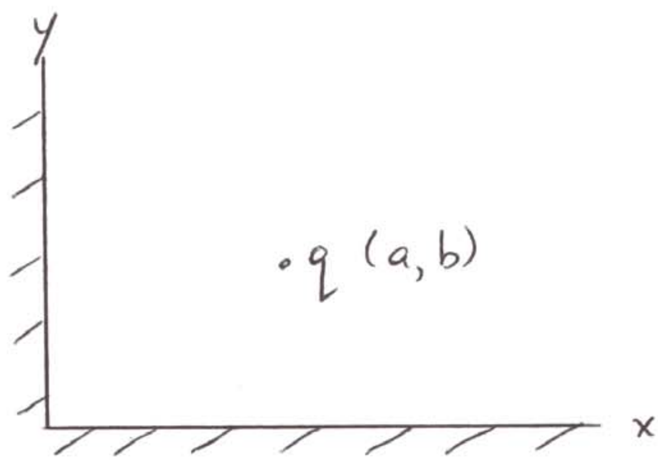
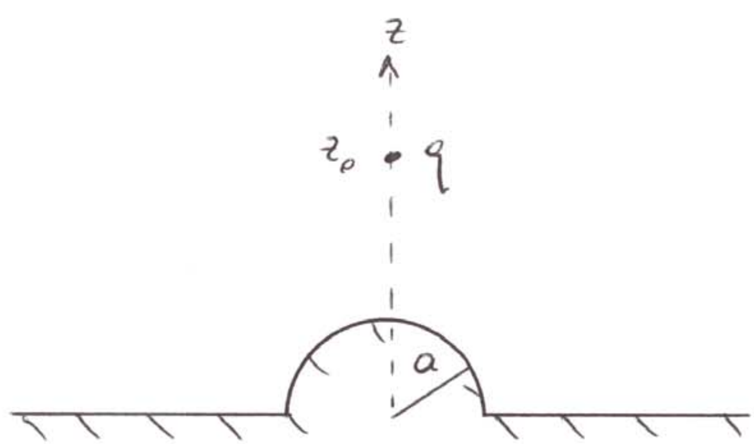
Assignment 4

1. Consider an empty 3 dimensional rectangular cube having all sides at zero potential. What is the potential inside the cube and how do you known this is the only possible answer?

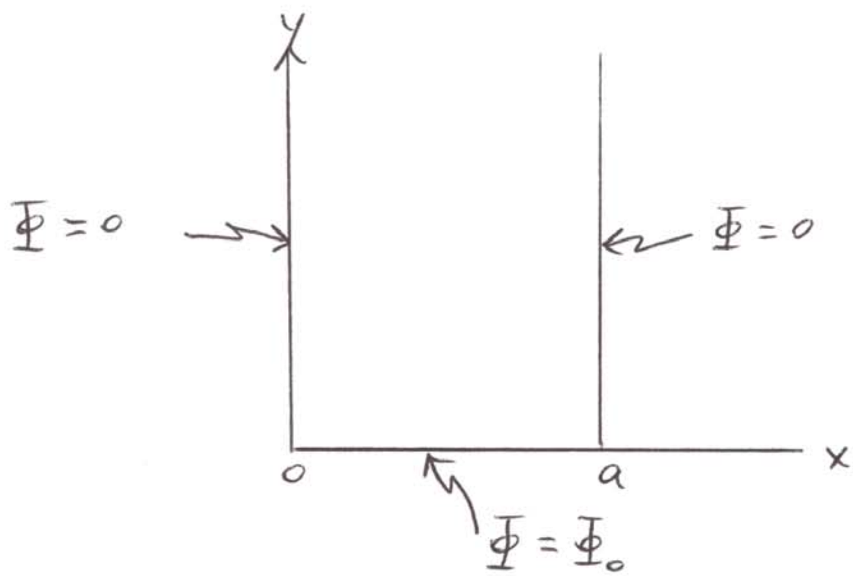
2. An infinite conducting sheet is bent into a 90^o corner. A point charge q is placed near the corner as shown. Find the potential everywhere.



3. An infinite conducting sheet has a hemispherical bubble of radius a. Find the potential everywhere.



4. An infinitely deep trough has its two sides at zero potential and its bottom at potential Φ_o. Find the potential everywhere in the trough.



5. Laplace equation in cylindrical coordinates is

$$0 = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial\Phi}{\partial\rho}) + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Consider the case where Φ is independent of z.

a) Let $\Phi = R(\rho)Q(\phi)$ and find the differential equations for R and Q.

b) i) Set constant=0 and show $R = A\ln\rho + B$, $Q = C\phi + D$.

ii) Set constant $k^2 > 0$ and show $R = A\rho^k + B\rho^{-k}$, $Q = C\cos k\phi + D\sin k\phi$.

iii) Set constant $-k^2 < 0$ and show $Q = Ce^{k\phi} + De^{-k\phi}$.

c) Suppose $\Phi(\rho,\phi) = \Phi(\rho,\phi+2\pi)$ implies $Q(\phi) = Q(\phi+2\pi)$. Show the following.

i) $k = n$ an integer

ii) $\Phi(\rho,\phi) = A\ln\rho + B + \sum_{n=1}^{\infty}(A_n\rho^n + B_n\rho^{-n})(C_n\cos n\phi + D_n\sin n\phi)$