PHYS 3020 Assignment 5

1a) Consider an electron cloud having radius a and uniform charge density $\rho = \dfrac{q}{\frac{4\pi}{3} a^3}$ where

q = electron charge.

$\Rightarrow r < a \quad \vec{E}_e = \frac{4\pi}{3} \rho \vec{r}$.

Since $d \ll a$ the electron cloud

$$\therefore E_e(d) = \frac{4\pi}{3} \rho d$$

$$= \frac{4\pi}{3} \frac{q}{\frac{4\pi}{3} a^3} d$$

$$= \frac{qd}{a^3}$$

b) Dipole moment $p = -qd$ (Note $-q > 0$)

$$\vec{E} = -\vec{E}_e(d)$$

$$E = -\frac{qd}{a^3}$$

$$= \frac{p}{a^3}$$

or $p = a^3 E \Rightarrow$ polarizability $\alpha = a^3$.

For H $\alpha = (.5 \times 10^{-8}\ cm)$

$= 1.25 \times 10^{-25}\ cm^3$

c) Electric field between plates is

$$E = \frac{500 \text{ volts}}{1 \text{ mm.}}$$

$$= 500 \text{ volts} \times \frac{1}{300} \frac{\text{statvolts}}{\text{volts}} \times \frac{1}{.1 \text{ cm}}$$

$$= \frac{50}{3} \text{ esu/cm}^2.$$

Now $E = -\frac{q\,d}{a^3}$ or $d = \frac{E\,a^3}{-q}$

$$= \frac{\left(\frac{50}{3}\ \frac{\text{esu}}{\text{cm}^2}\right)(.5 \times 10^{-8} \text{ cm})^3}{4.8 \times 10^{-10} \text{ esu}}$$

$$= 4.3 \times 10^{-15} \text{ cm.}$$

$$\therefore \frac{d}{a} = \frac{4.3 \times 10^{-15} \text{ cm}}{.5 \times 10^{-8} \text{ cm}}$$

$$= 8.7 \times 10^{-7}$$

2a) Interaction Energy $U = -\vec{p}_1 \cdot \vec{E}$ where $\vec{E}$ is electric field felt by dipole $\vec{p}_1$.

Electric field generated by $\vec{p}_2$ at $\vec{r}_2$ on dipole $\vec{p}_1$ at $\vec{r}_1$ is:

$$\vec{E}(\vec{r}_1) = -\frac{\vec{p}_2}{r_{12}^3} + \frac{3\vec{r}_{12}(\vec{p}_2 \cdot \vec{r}_{12})}{r_{12}^5}$$

where $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$.

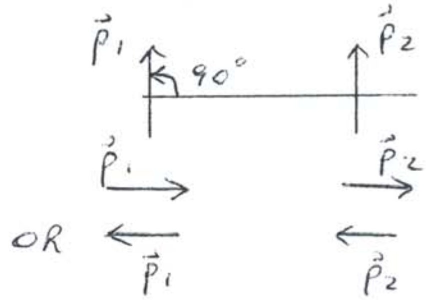
$$\Longrightarrow \quad U = \frac{\vec{p}_1 \cdot \vec{p}_2}{r_{12}^3} - \frac{3(\vec{p}_1 \cdot \vec{r}_{12})(\vec{p}_2 \cdot \vec{r}_{12})}{r_{12}^5}$$

$$\text{or} \quad U = \frac{\vec{p}_1 \cdot \vec{p}_2}{R^3} - 3\frac{(\vec{p}_1 \cdot \vec{r}_{21})(\vec{p}_2 \cdot \vec{r}_{21})}{R^5}$$

b) $\vec{p}_1 \parallel \vec{p}_2 \parallel \hat{z}$.

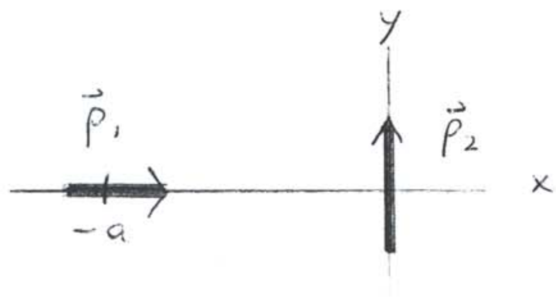
$$U = \frac{p_1 p_2}{R^3} - \frac{3 p_1 p_2}{R^3}\cos^2\theta$$

$$= \frac{p_1 p_2}{R^3}\left[1 - 3\cos^2\theta\right]$$



U_{max} occurs at $\theta = \frac{\pi}{2}$.
U_{min} " " $\theta = 0, \pi$.

3 a).



Electric field due to $\vec{p}_2$ at $\vec{r}$ is:

$$\vec{E} = -\frac{\vec{p}_2}{r^3} + \frac{3\vec{r}(\vec{p}_2 \cdot \vec{r})}{r^5}$$

$$= -\frac{p_2}{r^3}\hat{y} + 3\frac{p_2 y\,\vec{r}}{r^5}$$

b) Force on $\vec{p}_1$ due to $\vec{p}_2$ is:

$$\vec{F}_1 = (\vec{p}_1 \cdot \nabla)\vec{E}$$

$$= p_1 \frac{\partial \vec{E}}{\partial x}.$$

$$= p_1 \frac{\partial}{\partial x}\left[-p_2 \frac{\hat{y}}{r^3} + 3p_2 \frac{y\,\vec{r}}{r^5} \right]$$

$$= p_1 \left\{ -p_2 \hat{y}\,(-3)r^{-4}\frac{\partial r}{\partial x} + 3p_2 y\left(\frac{\partial \vec{r}}{\partial x}\frac{1}{r^5} - \frac{5}{r^6}\frac{\partial r}{\partial x}\vec{r} \right) \right\}$$

$$= p_1 \left\{ \frac{3p_2\,\hat{y}}{r^4}\frac{\partial r}{\partial x} + \frac{3p_2 y}{r^5}\hat{x} - \frac{15\,p_2 y}{r^6}\vec{r}\frac{\partial r}{\partial x} \right\}.$$

$$= p_1 \left\{ \frac{3p_2 x}{r^5}\hat{y} + \frac{3p_2 y}{r^5}\hat{x} - 15\frac{p_2 xy}{r^7}\vec{r} \right\}$$

using $\frac{\partial r}{\partial x} = \frac{x}{r}$.

$$\vec{F}_1 = \frac{3 p_1 p_2}{r^5} \left\{ x \hat{y} + y \hat{x} - 5 \frac{x y \vec{r}}{r^2} \right\}$$

$$\vec{F}_1(-a, 0, 0) = \frac{3 p_1 p_2}{a^5} \left\{ -a \hat{y} + 0 \hat{x} - 0 \vec{r} \right\}$$

$\therefore \vec{F}_1(-a, 0, 0) = -\dfrac{3 p_1 p_2}{a^4} \hat{y}$. is force on $\vec{p}_1$ due to $\vec{p}_2$.