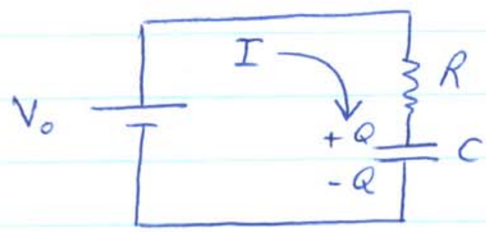
PHYS 2020 Assignment 10

1a)



V_o = voltage across resistor + voltage across cap.

$$V_o = IR + \frac{Q}{C} \quad \text{where } I = \frac{dQ}{dt}$$

b) $$V_o = R\frac{dQ}{dt} + \frac{Q}{C}$$

$$R\frac{dQ}{dt} = V_o - \frac{Q}{C}$$

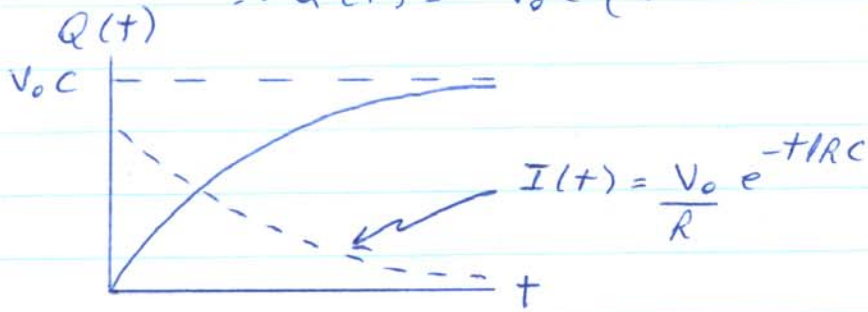
$$\int_0^Q \frac{dQ}{V_o - Q/C} = \int_0^t \frac{dt}{R}$$

$$-C \ln\left(V_o - \frac{Q}{C}\right)\Bigg|_0^Q = \frac{t}{R}.$$

$$\ln\left(\frac{V_o - Q/C}{V_o}\right) = \frac{-t}{RC}$$

$$1 - \frac{Q}{V_o C} = e^{-t/RC}$$

$$\therefore Q(t) = V_o C\left(1 - e^{-t/RC}\right)$$



c) $RC = 500 \text{ ohm} \times 2 \times 10^{-6} \text{ farad}$

$= 10^{-3} \text{ sec.}$

d) $I(t) = \dfrac{dQ}{dt}$

$= \dfrac{V_0}{R} e^{-t/RC}$

7.1) Flux through one loop is $\Phi(t) = A B_E \cos \omega t$

where A = loop area + $B_E = .5$ gauss (earth's field)

$\text{emf} = -\dfrac{1}{c} \dfrac{\partial \Phi}{\partial t}$

$= \dfrac{A B_E \, \omega \sin \omega t}{c}$

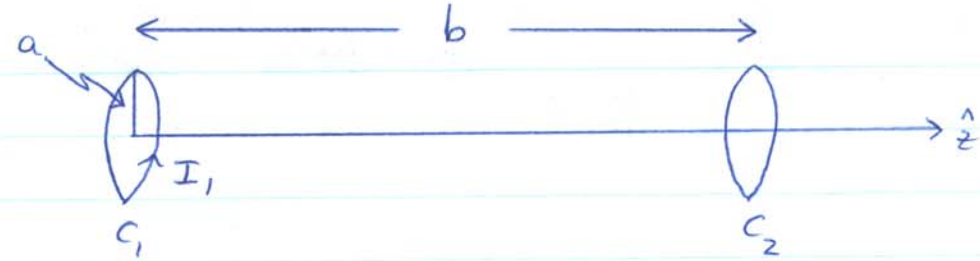
$\therefore$ max. emf is $\dfrac{A B_E \, \omega}{c}$

$\therefore$ max. emf through N loops is

$\dfrac{N A B_E \, \omega}{c} = \dfrac{4000 \times \pi (12 \text{ cm})^2 \times .5 \text{ gauss} \times 2\pi \times 30 \text{ sec}^{-1}}{3 \times 10^{10} \text{ cm/sec.}}$

$= 5.7 \times 10^{-3}$ statvolt

7.9)



Let I_1 be current in C_1.

Field generated by I_1 at C_2 is

$$\vec{B}_1 \simeq \hat{z}\,\frac{2\pi I_1 a^2}{c(a^2+b^2)^{3/2}}$$

$$\simeq \hat{z}\,\frac{2\pi I_1 a^2}{c\,b^3} \quad \text{since } b \gg a.$$

Flux of $\vec{B}_1$ through C_2 is $\Phi_{21} = \pi a^2 B_1$

Voltage or emf induced in C_2 by current I_1 is

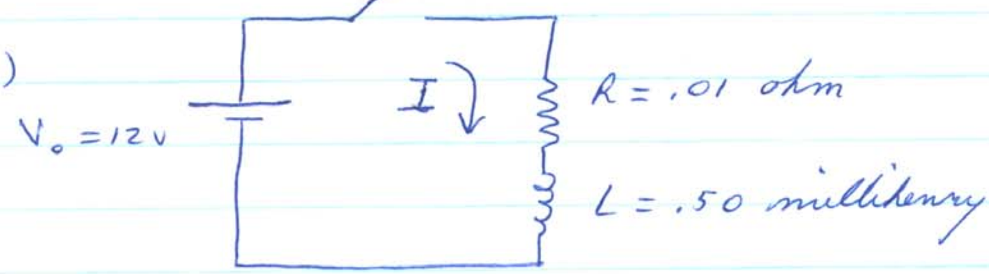
$$\mathcal{E}_{21} = -\frac{1}{c}\frac{\partial \Phi_{21}}{\partial t}$$

$$= -\frac{1}{c}\,\pi a^2\,\frac{2\pi a^2}{c\,b^3}\,\frac{dI_1}{dt}$$

$$\equiv -M_{21}\frac{dI_1}{dt}$$

$\therefore$ mutual inductance $M_{21} = \dfrac{2\pi^2 a^4}{c^2 b^3}$.

7.13)



$$V_0 = IR + L\frac{dI}{dt}$$

$$L\frac{dI}{dt} = V_0 - IR.$$

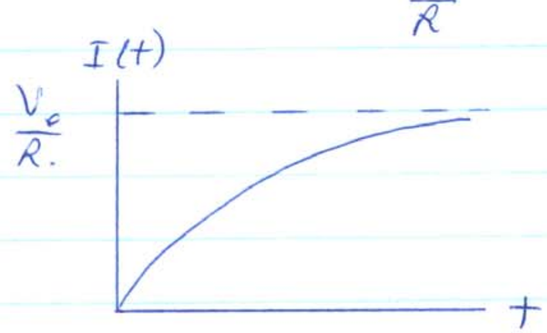
$$\int_0^I \frac{dI}{V_0 - IR} = \int_0^t \frac{dt}{L}$$

$$-\frac{1}{R}\ln(V_0 - IR)\Big|_0^I = \frac{t}{L}$$

$$\ln\left(\frac{V_0 - IR}{V_0}\right) = -\frac{Rt}{L}$$

$$1 - I\frac{R}{V_0} = e^{-Rt/L}$$

$$I = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right).$$



$$I(\infty) = V_0/R.$$

$$.9\frac{V_0}{R} = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right)$$

$$.9 = 1 - e^{-Rt/L}$$

$$e^{-Rt/L} = .1$$

$$-\frac{Rt}{L} = \ln .1$$

$$\therefore t = -\frac{L}{R} \ln .1$$

$$= 2.30 \times \frac{.50 \times 10^{-3} \text{ henry}}{.01 \text{ ohm}}$$

$$= .115 \text{ sec}$$

$\therefore$ current is 90% of its final value after .115 sec.

Energy stored in magnetic field is

$$\frac{1}{2} L I(.115 \text{ sec})^2 = \frac{1}{2} \times .50 \times 10^{-3} \text{ henry} \left(.9 \times \frac{12 \text{ volts}}{.01 \text{ ohm}} \right)^2$$

$$= 292 \text{ joules}$$

Energy supplied by battery up to .115 sec is

$$\int_0^{.115} P \, dt$$

$$= \int_0^{.115} V_0 \, I(t) \, dt$$

$$= \int_0^{.115} \frac{V_0^2}{R} (1 - e^{-Rt/L}) \, dt$$

$$= \frac{V_0^2}{R} \left[t + \frac{L}{R} e^{-Rt/L} \right]_0^{.115}$$

$$= \frac{(12 \text{ v})^2}{.01 \text{ ohm}} \left[.115 + \frac{.50 \times 10^{-3}}{.01} (-.9) \right] = 1008 \text{ joules}$$

7.23) Magnetic field energy density is $\frac{B^2}{8\pi}$.

Total mag. energy in galaxy is

$$\frac{B^2}{8\pi} V_{gal.} = \frac{(3\times10^{-6}\text{ gauss})^2}{8\pi} \times \pi\left(\frac{10^{23}}{2}\text{cm}\right)^2 \times 10^{21}\text{cm}.$$

$$= 2.8\times10^{54}\text{ ergs}.$$

This represents $\frac{2.8\times10^{54}\text{ ergs}}{10^{44}\text{ ergs/sec.}}$

$$= 2.8\times10^{10}\text{ sec}$$

$$= 891\text{ yrs. of starlight energy.}$$

7.25) Mag. energy density $\frac{B^2}{8\pi} = \frac{(10^{12}\text{ gauss})^2}{8\pi}$

$$= 4\times10^{22}\text{ erg/cm}^3.$$

Energy $= mc^2$.

$\therefore$ mag. energy density $= \frac{4\times10^{22}\text{ erg/cm}^3}{(3\times10^{10}\text{ cm/sec})^2}$

$$= 44.2\text{ gm/cm}^3.$$