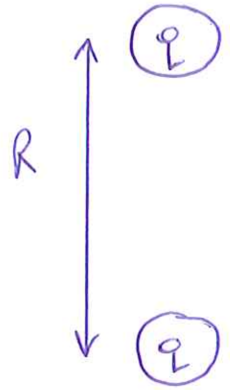
Assignment 15
Electrostatics

1. Two equally charged 1 gram masses repel each other. The lower mass is held fixed.

a) What is the charge on each mass for the Coulomb force to balance the gravitational force of the Earth on the upper mass?



Gravitational Force = Coulomb Force

$$mg = \frac{k q^2}{R^2}$$

$$q = \sqrt{\frac{mg}{k}}\, R$$

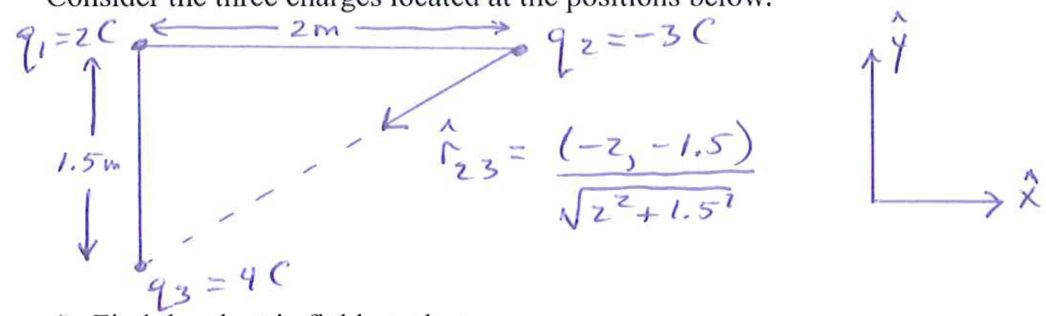
$$\therefore q = \left(\frac{10^{-3} \times 10}{9 \times 10^{9}}\right)^{1/2} \times 1\,\text{m} \text{ (assuming } R = 1\,\text{m})$$

$$= 1 \times 10^{-6} \text{ Coul.}$$

b) How many electrons does this charge represent?

$$\frac{1 \times 10^{-6} \text{ Coul.}}{1.6 \times 10^{-19} \text{ Coul/electron}} = 6.6 \times 10^{12} \text{ electrons}$$

2. Consider the three charges located at the positions below.



a) Find the electric field on charge q_3.

$$\vec{E}_{on\,q_3} = \frac{kq_1}{r_{13}^2}\hat{r}_{13} + \frac{kq_2}{r_{23}^2}\hat{r}_{23}$$

$$= 9\times10^9\left\{\frac{2}{1.5^2}(-\hat{y}) + \frac{(-3)}{2^2+1.5^2}\cdot\frac{(-2,-1.5)}{\sqrt{2^2+1.5^2}}\right\}$$

$$= (3.46, -5.40)\times10^9 \text{ Nt/Coul.}$$

b) Find the force on q_3.

$$\vec{F}_{on\,q_3} = q_3\,\vec{E}_{on\,q_3}$$

$$= 4C \times (3.46, -5.40)\times10^9 \text{ Nt/Coul}$$

$$= (1.38, -2.16)\times10^{10} \text{ Nt.}$$

c) What is the total potential energy of all the charges?

$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$

$$= 9\times10^9\left\{\frac{2\times(-3)}{2} + \frac{2\times4}{1.5} + \frac{(-3)\times4}{\sqrt{2^2+1.5^2}}\right\}$$

$$= -2.22\times10^{10} \text{ J}$$

d) What happens if the charges are free to move?

Charges fly apart because forces are nonzero.

3. Two metal plates have a uniform charge density of 10 Coulombs/meter^2. The plates have dimensions of 0.6 x 0.6 meter^2 and are separated by 2 mm.

a) Ignoring edge effects, the electric field (volts/meter) between the two plates is given by $E = 1.13 \times 10^{11}\ Q / A$ where Q is the charge in Coulombs on one plate and A is the area in meters^2. Evaluate the field.

$$E = 1.13 \times 10^{11} \times 10\ \text{Coul}/\text{m}^2$$

$$= 1.13 \times 10^{12}\ \frac{\text{Nt}}{\text{Coul}}.$$

b) What is the voltage between the two plates?

$$\text{Voltage} = E \times \text{plate separation}$$

$$= 1.13 \times 10^{12} \times .002$$

$$= 2.26 \times 10^{9}\ \text{volts}$$

c) What is the charge stored on one plate?

$$Q = 10\ \frac{\text{Coul}}{\text{m}^2} \times (0.6\ \text{m})^2$$

$$= 3.6\ \text{Coul}.$$

d) Capacitance is defined as the ratio of the charge on one plate to the voltage between the two plates. It has units of Coulomb/volt = farad. Evaluate it.

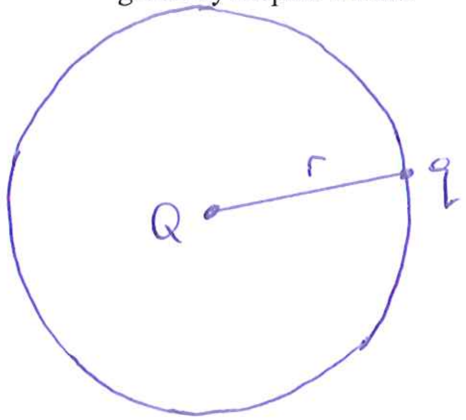
$$C = \frac{Q}{V}$$

$$= \frac{3.6\ \text{Coul}}{2.26 \times 10^{9}\ \text{volts}}$$

$$= 1.59 \times 10^{-9}\ \text{farad}$$

4. Consider two opposite charges. One is very heavy and may be assumed to be fixed while the other one has a mass m and orbits the second at a radius r and velocity v.

a) Show that the relation between the orbital radius and the orbital period is given by Kepler's Law.



Centripetal Force = Coulomb Force

$$\frac{mv^2}{r} = \frac{kqQ}{r^2}$$

$$rv^2 = \frac{kqQ}{m}$$

$$r\left(\frac{2\pi r}{T}\right)^2 = \frac{kqQ}{m}$$

∴ Kepler's Law: $\frac{r^3}{T^2} = \text{constant}$

b) If the radius is doubled what happens to the period?

$$T^2 \propto r^3$$

$$\text{or } T \propto r^{+3/2}$$

If $r \rightarrow 2r$ then $T \rightarrow 2^{3/2} T = 2.83\,T$

Assignment 16
DC Currents

1. 10^{10} electrons ~~travel~~ /sec travel down a wire having a circular cross section. The wire first has a diameter of 1 mm and then narrows to a diameter of 0.1 mm.

a) What is the current in the wire?

$$I = 10^{10} \frac{\text{electrons}}{\text{sec}} \times 1.6 \times 10^{-19} \frac{\text{Coul}}{\text{electron}}$$

$$= 1.6 \times 10^{-9} \text{ amp.}$$

b) What is the current density in the first part of the wire?

$$J = \frac{I}{\pi a^2}$$

$$= \frac{1.6 \times 10^{-9} \text{ amp.}}{\pi (5 \times 10^{-4} \text{ m})^2}$$

$$= 2 \, 1 \times 10^{-3} \frac{A}{m^2}$$

c) What is the current density in the second part of the wire?

$$J = \frac{I}{\pi b^2}$$

$$= \frac{1.6 \times 10^{-9} \text{ amp.}}{\pi (5 \times 10^{-5} \text{ m})^2}$$

$$= 0.2 \frac{A}{m^2}$$

2. A kettle draws 3 A of DC current when it is connected to a 10 V battery.

a) What is the kettle resistance?

Resistance $R = \frac{V}{I}$

$= \frac{10\ V}{3\ A}$

$= 3.33$ ohm.

b) What power is supplied to the kettle?

Power $P = VI$

$= 10$ volts $\times 3$ A

$= 30$ watts

c) If the kettle has 1 liter of water initially at a temperature of 20 C, how long will it take for the water to be heated to 90C?

Energy supplied in time t = Energy heat 1 l H_2O by $(90-20)C$

$P\,t = 1000\ gm \times 70C \times \frac{1\ \text{calorie}}{C\ gm} \times 4.18\ \frac{J}{cal.}$

↑ 30 watts

$\therefore t = 9.75 \times 10^3$ sec.

$= 2.7$ hrs.

This is far from acceptable. One then should go to Tim Horton!

3. a) Find the resistance of a gold cylinder having a diameter of 1 mm and a length of 100 meters.

$$R = \frac{\rho L}{A}$$

$$= \frac{2.35 \times 10^{-8} \text{ ohm meter} \times 100 \text{ m}}{\pi (5 \times 10^{-4} \text{ m})^2}$$

$$= 3 \text{ ohm}$$

b) What is the radius of a copper cylinder having a length of 10 meters if it is to have the same resistance as the object in part a.

$$R = \frac{\rho_{Cu} L}{\pi r^2}$$

$$r = \left(\frac{\rho_{Cu} L}{\pi R} \right)^{1/2}$$

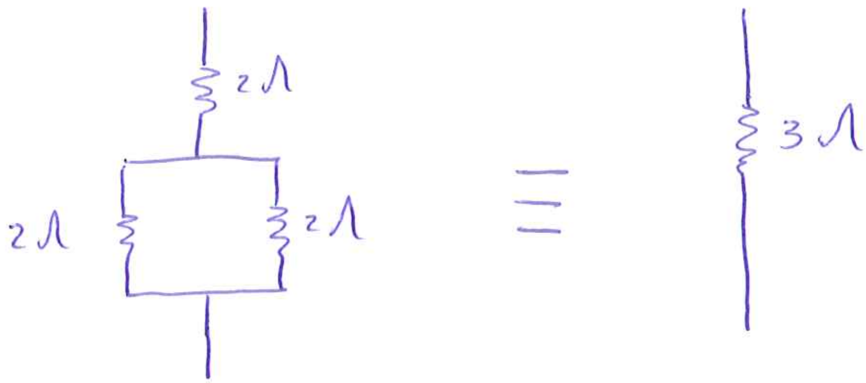
$$= \left(\frac{1.67 \times 10^{-8} \times 10}{\pi \times 3} \right)^{1/2}$$

$$= 1.33 \times 10^{-4} \text{ m}$$

$\therefore r = 0.13$ mm or $133\ \mu m$

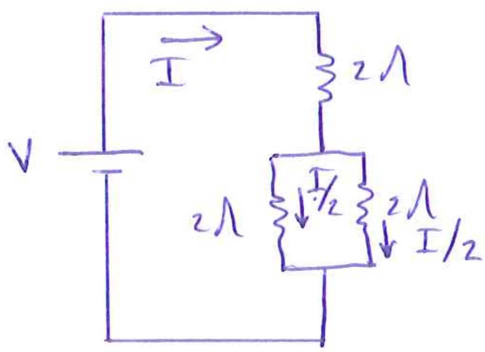
4. A laboratory only as 2 ohm resistors.

a) Draw the circuit required to create a 3 ohm resistance

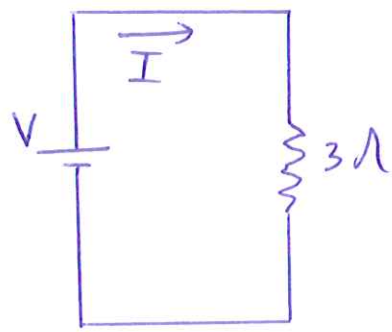


b) Show that the total power dissipated in the above circuit is the same as when a single 3 ohm resistor is available.

Circuit a



Circuit b



In both circuits $I = \frac{V}{3}$.

Circuit a Power dissipation $= I^2 R + \left(\frac{I}{2}\right)^2 R \times 2$

$$= \frac{3}{2} I^2 R$$

$$= \frac{3}{2} \left(\frac{V}{3}\right)^2 \times 2$$

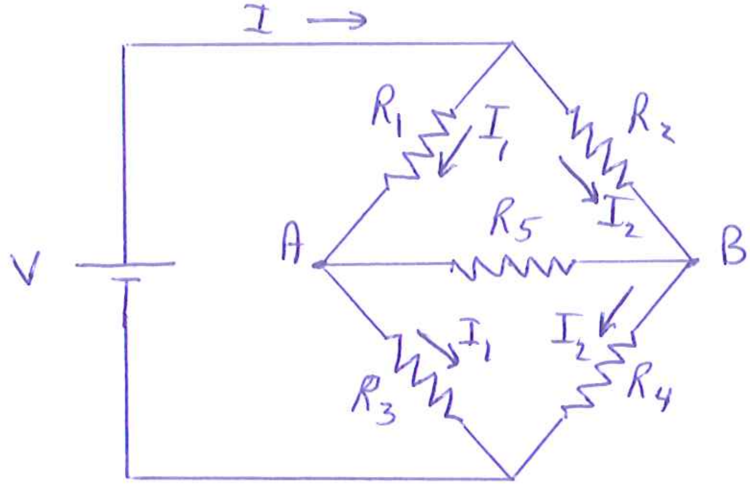
$$P_a = \frac{V^2}{3}$$

Circuit b Power dissipation $= I^2 \times 3$

$$P_b = \frac{V^2}{3}$$

$\therefore P_a = P_b$.

5. Consider the Wheatstone bridge circuit shown below. Show that no current passes through R_5 if $R_1 / R_3 = R_2 / R_4$



No current through $R_5 \Rightarrow V_{AB} = 0$.

$\therefore$ voltage across R_1 = voltage across R_2

$$I_1 R_1 = I_2 R_2 \quad (1)$$

Also voltage across R_3 = voltage across R_4

$$I_1 R_3 = I_2 R_4 \quad (2)$$

$$(1) \div (2) \Rightarrow \frac{R_1}{R_3} = \frac{R_2}{R_4}$$