Assignment 7
Momentum

1. A uranium atom at rest disintegrates into two fragments. An alpha particle (helium nucleus) is measured to have a speed of 3 x 10^5 m/sec. What is the speed of the recoiling atom?

Consider decay of $^{235}U \rightarrow \alpha + {}^{231}X$

recoil speed of X = $\frac{\alpha \text{ momentum}}{\text{mass of } X}$

$= \frac{4}{231} \times 3 \times 10^5$ m/sec

$= 5.2 \times 10^3$ m/sec.

2. A 2 kg mass travelling at 3 m/sec collides and sticks with a 5 kg mass. What is the resultant speed of the combined mass particle?

initial momentum = final momentum

2 kg × 3 m/sec = (2 + 5) kg. × V

$V = \frac{2}{2+5} \times 3$

$\therefore$ v = 0.86 m/sec

3. A rocket is propelled as a result of the very rapid ejection of exhaust gas from the rear of the rocket. Given that the initial mass of the rocket and fuel is 5000 kg and that 4000 kg of fuel is burned in accelerating the rocket to a speed of 600 m/sec, calculate the velocity of the exhaust gases.

|momentum of rocket after burn| = |momentum of fuel after burn|

$1000\ \text{kg} \times 600\ \frac{\text{m}}{\text{sec}} = 4000\ \text{kg}\ v_{exhaust}$

$v_{exhaust} = 150\ \text{m/sec}$.

4. A 1000 kg car travelling at 36 km/hr strikes a tree and comes to rest in 0.1 sec.

a) Calculate the order of magnitude of the force exerted by the tree on the car.

$$\text{Deceleration} = \frac{36 \text{ km/hr.}}{0.1 \text{ sec}}$$

$$= 36 \frac{\text{km}}{\text{hr}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{1}{3600} \frac{\text{hr}}{\text{sec}} \times \frac{1}{0.1 \text{sec}}$$

$$= 100 \text{ m/sec}^2$$

$$\therefore \text{force of tree} = ma$$

$$= 1000 \text{ kg} \times 100 \text{ m/sec}^2 = 10^5 \text{ Nt.}$$

b) What mass has a weight of the same order of magnitude as the force calculated in a).

Equating $mg = 10^5$ Nt.

$m = 10^4$ kg.

$= 10$ tons

c) Would it be comforting if an object of this mass sat on you?

Don't try it.

5. A rubidium atom travelling at the speed of sound absorbs photons from an oncoming laser beam. Each photon can be viewed as a tiny ping pong ball having momentum 7.79×10^{-28} Nt. sec. The atom absorbs a photon which is then reradiated in any direction. Hence, on average each photon absorption/reemission reduces the atom's momentum by a photon momentum.

a) How many photons must an atom absorb and reradiate in order to be stopped?

$$\# \text{ photons} = \frac{\text{atom momentum}}{\text{photon momentum}}$$

$$= \frac{87 \times 1.67 \times 10^{-27} \text{ kg.} \times 3 \times 10^{2} \text{ m/sec}}{7.79 \times 10^{-28} \text{ Nt sec}}$$

$$= 56{,}000$$

b) What is the deceleration experienced by the atom if it can absorb and reradiate a photon every 2.5×10^{-8} sec?

$$\text{Deceleration} = \frac{1}{\text{Rb mass}} \times \frac{\text{photon momentum}}{\text{radiation time}}$$

$$= \frac{1}{87 \times 1.67 \times 10^{-27}} \quad \frac{7.79 \times 10^{-28}}{2.5 \times 10^{-8}}$$

$$= 2.14 \times 10^{5} \text{ m/sec}^2$$

c) What distance does it take to stop an atom?

Time to absorb/radiate 56,000 photons $= 56,000 \times 2.5 \times 10^{-8}$ sec

$= 1.4 \times 10^{-3}$ sec.

Distance travelled $y = v_0 t - \frac{at^2}{2}$

$\therefore y = 3 \times 10^2 \frac{m}{sec} \times 1.4 \times 10^{-3}$ sec $- \frac{2.14 \times 10^5 \, m/sec^2 (1.4 \times 10^{-3} s)^2}{2}$

$= 2.1 \times 10^{-1}$ m

$= 21$ cm

d) How many photons/sec are required to stop a beam of 10^9 atoms/sec?

photons/sec to stop beam = # photons to stop 1 atom × # atoms/sec

$= 56,000 \times 10^9$

$= 5.6 \times 10^{13}$ photons/sec.

Extra: 1 Photon has energy of 2.6×10^{-19} J.

$\therefore$ Power of laser $= 5.6 \times 10^{13} \times 2.6 \times 10^{-19} \frac{J}{sec}$

$= 1.4 \times 10^{-5}$ W

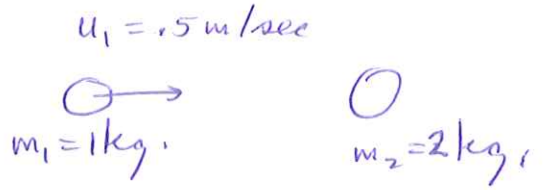
$= 14 \mu W$

$\Rightarrow$ Rather low power.

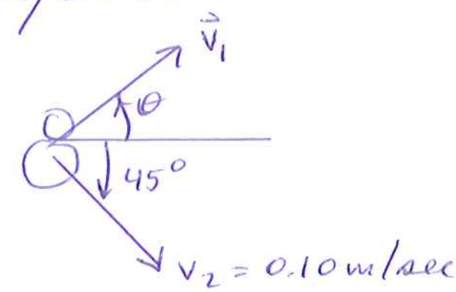
6. A 1 kg hockey puck moving at 0.5 m/sec strikes another puck having a mass at 2 kg such that the 2 kg puck travels at an angle 45° with a speed of 0.10 m/sec with respect to the motion of the original puck.

a) What is the final speed of the first puck?

Before Collision



After Collision



Cons. of p_x:
$$m_1 u_1 = m_1 v_1 \cos\theta + m_2 v_2 \cos 45^\circ$$
$$.5 = v_1 \cos\theta + \frac{0.2}{\sqrt{2}}$$
$$v_1 \cos\theta = .359 \quad (1)$$

Cons. of p_y:
$$m_1 v_1 \sin\theta = m_2 v_2 \sin 45^\circ$$
$$v_1 \sin\theta = \frac{0.2}{\sqrt{2}}$$
$$v_1 \sin\theta = .141 \quad (2)$$

$$(1)^2 + (2)^2 \Rightarrow v_1^2 = .149 \Rightarrow v_1 = .39 \text{ m/sec}$$

b) What is the final direction of the first puck?

$$(2) \div (1) \Rightarrow \tan\theta = \frac{.141}{.359}$$
$$= .393$$
$$\therefore \theta = 21.4^\circ$$

Assignment 8
Work and Kinetic Energy I

1. A weightlifter raises 250 kg from the floor to a height of 2 meters.
   a) How much work does he do if:
      i) the object is raised straight up
      ii) the object is first raised one meter straight up, then sideways 3.5 meters to the left followed by another meter straight up and finally 3.5 meters to the right

i) $W = mgh$

$= 250\ kg \times 10\ m/sec^2 \times 2\ m.$

$= 5000\ J$

ii) Gravitational force is conservative
∴ work done is independent of path
and answer is same as in i.

   b) The mass is then released. What is the velocity of the mass when it hits the weightlifters toe?

Work done = Kinetic Energy.

$5000 J = \frac{m}{2} v^2$

$v = \sqrt{\frac{2 \times 5000}{250}}$

$\therefore v = 6.3\ m/sec$

2. A neutron of mass 1.67×10^{-24} gm travelling at a speed of 10^5 m/sec collides head on with a stationary deuteron with a mass of 3.34×10^{-24} g. The collision is elastic and the particles do not stick together. Calculate the speed of each after the collision.

Before Collision — After Collision

Moment. Cons. $\Rightarrow$ $m_1 u_1 = m_1 v_1 + m_2 v_2$

$$m_1 v_1 = m_1 u_1 - m_2 v_2$$

$$v_1 = u_1 - \frac{m_2 v_2}{m_1} \quad (1)$$

Energy Cons. $\Rightarrow$ $\frac{m_1}{2} u_1^2 = \frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2$

Using (1) $\Rightarrow$ $m_1 u_1^2 = m_1 \left[u_1 - \frac{m_2 v_2}{m_1} \right]^2 + m_2 v_2^2$

$$u_1^2 = u_1^2 - 2\frac{m_2}{m_1} u_1 v_2 + \left(\frac{m_2}{m_1}\right)^2 v_2^2 + \frac{m_2}{m_1} v_2^2$$

$$0 = v_2 \left[-2\frac{m_2}{m_1} u_1 + \frac{m_2}{m_1}\left(\frac{m_2}{m_1} + 1\right) v_2 \right]$$

$$\therefore v_2 = \frac{2\, m_2/m_1}{\frac{m_2}{m_1}\left(\frac{m_2}{m_1} + 1\right)} u_1$$

$$v_2 = \frac{2u_1}{\frac{m_2}{m_1} + 1}$$

But $\frac{m_2}{m_1} = 2 \Rightarrow v_2 = \frac{2}{3} u_1 = \frac{2}{3} \times 10^5$ m/sec

Subst. v_2 into (1) $\Rightarrow$ $v_1 = \frac{-1}{3} \times 10^5$ m/sec.

3. A 2 gm ball travelling 5 cm/sec hits a 3 gm ball and sticks to it.

a) What is the velocity of the combined 5 gm ball?

Mom. Cons. $\Rightarrow$ $2\ gm \times 5\ \frac{cm}{sec} = 5\ gm \times V$

$\therefore\ V = 2\ cm/sec$

b) What is the kinetic energy before the collision?

$$K.E. = \frac{M_1}{2} V_1^2$$
$$= \frac{.002kg}{2}(.05\ m/sec)^2$$
$$= 2.5 \times 10^{-6}\ J$$

c) What is the kinetic energy after the collision?

$$K.E. = \frac{m_1 + m_2}{2} V^2$$
$$= \frac{.002 + .003}{2}(.02)^2$$
$$= 1 \times 10^{-6}\ J$$

d) Can you suggest what happened to the energy?

eg. heat, noise of impact

4. A projectile of mass 20 kg. is projected vertically upward with an initial speed of 50 m/sec

a) What is the original kinetic energy?

$$T_1 = \frac{m}{2} v^2$$

$$= \frac{20\,kg}{2} (50\,m/sec)^2$$

$$= 2.5 \times 10^4\,J$$

b) What is the kinetic energy after 2 sec?

$$v(2\,sec) = v_0 - gt$$

$$= 50 - 10 \times 2$$

$$= 30\,m/sec$$

$$T(2\,sec) = \frac{20\,kg}{2} (30\,m/sec)^2$$

$$= 9 \times 10^3\,J$$

c) What is the change in its gravitational energy during these 2 sec?

$$\Delta U = \Delta T$$

$$= T_1 - T_2$$

$$= 2.5 \times 10^4 - 9 \times 10^3$$

$$= 1.6 \times 10^4\,J$$

5. The Canadian Space Agency wishes to launch a rocket.

a) What is the escape velocity of the rocket if it is launched from the Earth?

$$v_{escape} = \sqrt{\frac{2GM_{Earth}}{R_{Earth}}}$$

$$= \left(\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^{6}}\right)^{1/2}$$

$$= 1.15 \times 10^{4} \text{ m/sec}$$

$$= 11.5 \text{ km/sec}$$

b) Repeat a if the rocket is launched from the surface of the moon.

$$v_{escape} = \sqrt{\frac{2GM_{moon}}{R_{moon}}}$$

$$= \left(\frac{2 \times 6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{1.7 \times 10^{6}}\right)^{1/2}$$

$$= 2.4 \times 10^{3} \text{ m/sec}$$

$$= 2.4 \text{ km/sec}$$

6. The gravitational potential energy is given by $U = -GmM_E/r$. However, for masses close to the Earth's surface $U = mgh$. Are these formulae consistent? Hint: Substitute $r = R_E + h$, where $h <<< R_E$.

$$U = -\frac{GmM_E}{r} \quad \text{where } r = R_E + h \quad h << R_E$$

$$= -\frac{GmM_E}{R_E + h}$$

$$= -\frac{GmM_E}{R_E}\left(1 + \frac{h}{R_E}\right)^{-1}$$

$$\simeq -\frac{GmM_E}{R_E}\left[1 - \frac{h}{R_E}\right]$$

$$\therefore U = \underbrace{-\frac{GmM_E}{R_E}}_{\text{constant } U_0} + \frac{GmM_E}{R_E^2} h$$

But $mg = \frac{GmM_E}{R_E^2}$.

$$\therefore U = U_0 + mgh.$$