Quiz 1

Name: _____ Student Number: _____

NO CALCULATORS ALLOWED.

5 X 2 = 10 MARKS

1. a) sin -75°

$= -\sin(30° + 45°)$

$= -[\sin 30° \cos 45° + \sin 45° \cos 30°]$

$= -\left[\frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}\right]$

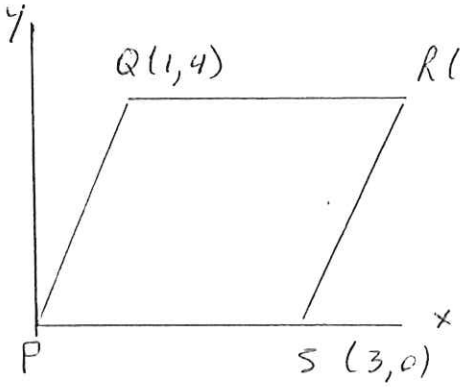
$= -\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

b) What is the period of y = tan 3x?

Period of $\tan x$ is π

$\therefore$ period of $\tan 3x$ is $\frac{\pi}{3}$

2. Find the area of the parallelogram PQRS.



$\vec{PQ} = (1,4,0)$

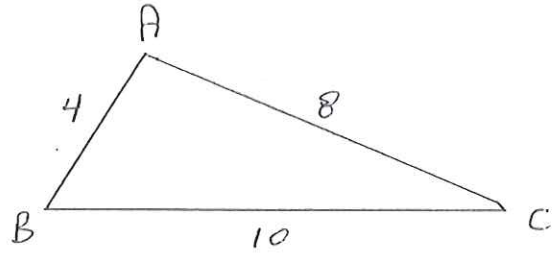
$\vec{QR} = (3,0,0)$

$\vec{PQ} \times \vec{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 0 \\ 3 & 0 & 0 \end{vmatrix}$

$= (0,0,-12)$

$\therefore$ area of PQRS is $|\vec{PQ} \times \vec{QR}| = 12$ square units

3. Find the cosines of all angles in the triangle below given the side lengths indicated.



Cosine Law: $10^2 = 8^2 + 4^2 - 2.4.8 \cos A$

$\cos A = -0.3125$ (Note $\cos A < 0 \Rightarrow A > 90°$)

Also: $8^2 = 10^2 + 4^2 - 2.10.4 \cos B$

$\cos B = 0.65$

Also: $4^2 = 10^2 + 8^2 - 2.10.8 \cos C$

$\cos C = 0.925$

4. Find the unit vector pointing from (1,2,3) to (3,-1,0).

$\vec{r} = (3,-1,0) - (1,2,3)$

$= (2,-3,-3)$

$|\vec{r}| = \sqrt{2^2 + (-3)^2 + (-3)^2} = \sqrt{22}$

$\therefore \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{1}{\sqrt{22}}(2,-3,-3)$

5. Find the first 4 nonzero terms for the Taylor expansion of ln x about the point x=1. Use this to write down an expression for ln 2.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!} (x-a)^{n}$$

$f(x) = \ln x$ $\qquad f(1) = \ln 1 = 0$

$f'(x) = \frac{1}{x}$ $\qquad f'(1) = 1$

$f^{(2)}(x) = \frac{-1}{x^2}$ $\qquad f''(1) = -1$

$f^{(3)}(x) = \frac{2}{x^3}$ $\qquad f^{(3)}(1) = 2$

$f^{(4)}(x) = \frac{-3!}{x^4}$ $\qquad f^{(4)}(1) = -3!$

$\vdots$

$f^{(n)}(x) = (-1)^{n-1} \frac{(n-1)!}{x^n}$ $\qquad f^{(n)}(1) = (-1)^{n-1}(n-1)!$

$$\therefore \ln x = 0 + (x-1)^1 - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots$$

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + - \cdots$$

$$= 0.693$$