Assignment 3
One Dimensional Motion and Vectors

1. An object is shot vertically upward with an initial speed of 40 m/sec. When will it reach a height of 80 meters above the ground?

$$y = v_0 t - \frac{g}{2} t^2$$

$$80 = 40t - 5t^2$$

$$t^2 - 8t + 16 = 0$$

$$(t-4)^2 = 0$$

$$\therefore t = 4$$

$\therefore$ object hits 80 meter height in 4 sec.

2. From a point 70 meters above ground level, an object is sent upwards with an initial velocity upwards of 25 m/sec. How long does it take before it strikes the ground?

$$y = y_0 + v_0 t - \frac{g}{2} t^2$$

$$0 = 70 + 25t - 5t^2$$

$$0 = t^2 - 5t - 14$$

$$t = \frac{5 \pm \sqrt{25 + 56}}{2}$$

$$= \frac{5 \pm 9}{2}$$

$$t = 7 \text{ sec}$$

3. The effect of air resistance is to slow down a moving object. It can be shown that the height of a falling object is given by the following.

$$y = y_o - [t + (e^{-bt} - 1) / b]\, g / b$$

a) Show that for short times this reduces to the expected expression $y = y_o - 1/2\, gt^2$.

For small t, $e^{-bt} \simeq 1 - bt + \frac{b^2t^2}{2!}$

$$\therefore\ y = y_o - \left[t + \left(1 - bt + \frac{b^2t^2}{2} - 1\right)/b\right]\frac{g}{b}$$

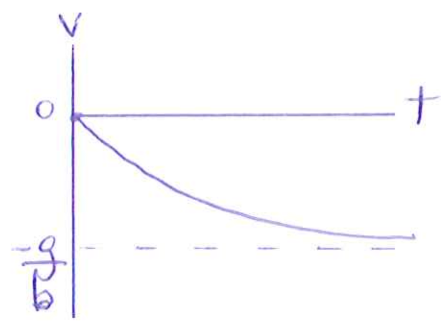
$$= y_o - \left[t - t + \frac{bt^2}{2}\right]\frac{g}{b}$$

$$= y_o - \frac{gt^2}{2}$$

b) Find the velocity and plot it as a function of time. What is the maximum velocity?

$$v = \frac{dy}{dt}$$

$$= -\left[1 - e^{-bt}\right]\frac{g}{b}$$

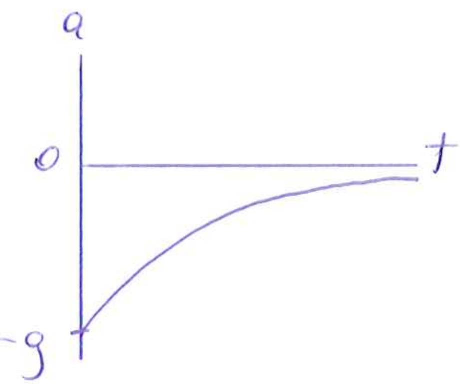


c) Find the acceleration and plot it as a function of time.

$$a = \frac{dv}{dt}$$

$$= -b e^{-bt} \frac{g}{b}$$

$$= -g e^{-bt}$$



4. $\vec{x} = (1, 2, 3)$ $\vec{y} = (-1, 0, 1)$ Evaluate the following.

a) $|\vec{x}|\,|\vec{y}|$

$$|\vec{x}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$|\vec{y}| = \sqrt{(-1)^2 + 0^2 + 1^2} = \sqrt{2}$$

b) $\vec{x} \cdot \vec{y}$ $= (1, 2, 3) \cdot (-1, 0, 1)$

$$= -1 + 0 + 3$$

$$= 2$$

c) $\vec{x} \times \vec{y}$ $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 0 & 1 \end{vmatrix}$

$$= \hat{i} \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix}$$

$$= \hat{i}\,(2 - 0) - \hat{j}\,(1 + 3) + \hat{k}\,(0 + 2)$$

$$= (2, -4, 2)$$

d) Find angle between $\vec{x}$ and $\vec{y}$

$$\cos\theta = \frac{\vec{x} \cdot \vec{y}}{|\vec{x}|\,|\vec{y}|}$$

$$= \frac{2}{\sqrt{14}\,\sqrt{2}}$$ See a & b.

$$= 0.378$$

$$\therefore \theta = 67.8^\circ$$

e) Find vector having unit length perpendicular to both $\vec{x}$ and $\vec{y}$.

$\vec{x} \times \vec{y}$ is $\perp$ to $\vec{x}$ & $\vec{y}$.

$\therefore$ unit $\perp$ vector is $\dfrac{\vec{x} \times \vec{y}}{|\vec{x} \times \vec{y}|} = \dfrac{(2,-4,2)}{\sqrt{4+16+4}}$

$$= \frac{1}{\sqrt{6}}(1,-2,1)$$

f) Explain why or why not the following makes sense $\vec{x} \times (\vec{x} \cdot \vec{y})$

$\vec{x} \cdot \vec{y}$ is a scalar, not a vector

$\therefore \vec{x} \times$ number makes no sense.

↑
should be
a vector for
defined cross product

Assignment 4
Two & Three Dimensional Motion

1. A spaceship initially at rest as measured by an observer experiences a constant acceleration of $\vec{a} = (1, 2, 3)$ m/sec^2.

   a) What is its velocity after 5 seconds?

$$\vec{v} = \vec{a}\, t$$

$$\vec{v}(5\,sec) = (5, 10, 15)$$

   b) What is its speed after 5 seconds?

$$|\vec{v}(5\,sec)| = \sqrt{5^2 + 10^2 + 15^2}$$

$$= 18.7 \text{ m/sec.}$$

   c) What is its position after 5 seconds?

$$\vec{r} = \vec{a}\,\frac{t^2}{2}$$

$$\vec{r}(5\,sec) = (12.5, 25, 37.5)$$

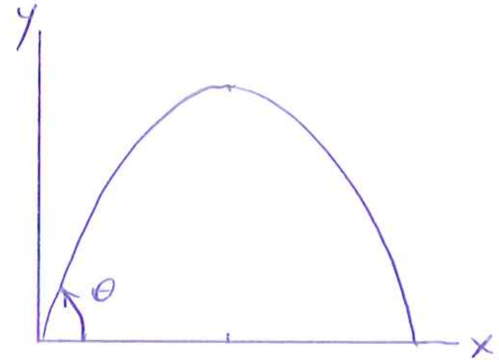
   d) What is the distance it has traveled in 5 seconds?

$$|\vec{r}(5\,sec)| = \sqrt{12.5^2 + 125^2 + 137.5^2}$$

$$= 46.8 \text{ m.}$$

2. A bullet is shot with a speed v_o at an angle θ above the horizontal.

a) What is its maximum height?



$x = v_o \cos\theta \, t$

$y = v_o \sin\theta \, t - \frac{g}{2} t^2$

$v_y = \frac{dy}{dt}$

$= v_o \sin\theta - gt$

at max. height $v_y = 0 \Rightarrow 0 = v_o \sin\theta - gt$

$t = \frac{v_o \sin\theta}{g}$

Max. Height $y\left(\frac{v_o \sin\theta}{g}\right) = \frac{v_o^2 \sin^2\theta}{2g}$

b) How long does it take before it hits the Earth?

Earth $y = 0 \Rightarrow 0 = v_o \sin\theta \, t - \frac{g}{2} t^2$

$= t\left(v_o \sin\theta - \frac{gt}{2}\right)$

$\therefore t = 0, \frac{2 v_o \sin\theta}{g}$

$\therefore$ time of trip is $\frac{2 v_o}{g} \sin\theta$.

c) Where does it strike the Earth?

$$x\left(\frac{2V_0}{g}\sin\theta\right) = \frac{2V_0^2}{g}\cos\theta\sin\theta$$

$$= \frac{V_0^2}{g}\sin 2\theta$$

d) What angle θ gives the maximum range?

Maximize $x \Rightarrow 0 = \frac{dx}{d\theta}$

$$= \frac{V_0^2}{g} 2\cos 2\theta$$

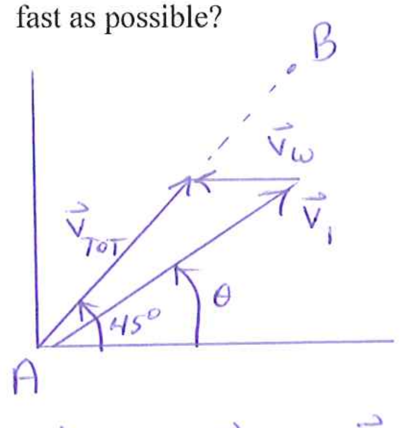
$$= \cos 2\theta$$

$$\therefore 2\theta = 90^\circ$$

$$\theta = 45^\circ$$

3. An airplane wishes to fly from city A to city B which is located 2000 km northeast of A. The maximum speed of the plane in still air is 750 km/hr. There is a wind blowing toward the ~~East~~ West of 50 km/hr.

a) In what direction should the pilot steer the plane to complete the trip as fast as possible?



$\vec{v}_1 = 750(\cos\theta, \sin\theta)$

$\vec{v}_w = (-50, 0)$

$\vec{v}_{TOT} = v_{TOT}(\cos 45°, \sin 45°)$

$$\vec{v}_{TOT} = \vec{v}_1 + \vec{v}_w$$

$$v_{TOT}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 750(\cos\theta, \sin\theta) + (-50, 0)$$

$\hat{x}$ component: $\frac{v_{TOT}}{\sqrt{2}} = 750\cos\theta - 50 \quad (1)$

$\hat{y}$ component: $\frac{v_{TOT}}{\sqrt{2}} = 750\sin\theta \quad (2)$

$(2) \div (1) \Rightarrow \quad 1 = \frac{750\sin\theta}{750\cos\theta - 50}$

Boring algebra $\Rightarrow \theta = 42.3°$

b) How long does the trip take?

Subst. $\theta = 42.3°$ into $(2) \Rightarrow v_{TOT} = 750\sqrt{2}\sin 42.3°$
$= 714$ km/hr.

$\therefore$ trip length $= \frac{2000 \text{ km}}{714 \text{ km/hr.}}$

$= 2.8$ hrs.

4. What is the acceleration of the Earth in m/sec^2 as it orbits the sun?

$r = 1,5 \times 10^{11}$ m.

$T = 1$ yr. $= 3.16 \times 10^{7}$ sec

$$\text{Acceleration} = \frac{v^2}{r}$$

$$= \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r}$$

$$= \frac{4\pi^2 \times 1.5 \times 10^{11}}{(3.16 \times 10^{7})^2}$$

$$= 5,95 \times 10^{-3} \text{ m/sec}^2.$$

5. A circus performer rides a bicycle around a loop. Assuming the loop is a circle with radius 2.7 meters what is the minimum speed for the performer such that at the top of the loop she remains in contact with the loop?

Min. Velocity is such that

$$g = \frac{v^2}{R}.$$

$$v = \sqrt{gR}$$

$$= \sqrt{10 \frac{\text{m}}{\text{sec}^2} \times 2.7 \text{ m}}$$

$$v = 5.2 \text{ m/sec}.$$