Assignment 1
Units

1. How long does it take light to travel from the Sun to the Earth? Give your answer in minutes and in seconds.

$$\text{time} = \frac{\text{Earth Sun distance}}{\text{speed of light}}$$

$$= \frac{1.5 \times 10^{11}\ \text{m}}{3 \times 10^{8}\ \text{m/sec}}$$

$$= 500\ \text{sec}$$

$$= 8.3\ \text{minutes}$$

2. What is the speed in m/sec and km/hr of the Earth as it orbits the sun?

$$\text{speed} = \frac{2\pi \times \text{Earth Sun distance}}{1\ \text{year}}$$

$$= \frac{2\pi \times 1.5 \times 10^{11}\ \text{m}}{365.25 \times 24 \times 3600\ \text{sec}}$$

$$= 2.99 \times 10^{4}\ \text{m/sec}$$

$$= 2.99 \times 10^{4}\ \frac{\text{m}}{\text{sec}} \times \frac{1}{1000}\ \frac{\text{km}}{\text{m}} \times 3600\ \frac{\text{sec}}{\text{hr}}$$

$$= 1.08 \times 10^{5}\ \text{km/hr.}$$

3. How long in seconds and in years would it take an astronaut to travel to the nearest star located 3 light years away if her average velocity is 10 km/sec?

$$\text{time} = \frac{3 \times 365.25 \times 24 \times 3600 \times 3 \times 10^{8}\ \text{m}}{10^{4}\ \text{m/sec}}$$

$$= 2.84 \times 10^{12}\ \text{sec}$$

$$= 2.84 \times 10^{12}\ \text{sec} \times \frac{1}{365.25 \times 24 \times 3600}\ \frac{\text{yr}}{\text{sec}}$$

$$= 9.0 \times 10^{4}\ \text{years}$$

4. Assuming that the sun is mostly made of hydrogen, how many hydrogen atoms are in it?

$$\#\ H\ \text{atoms} = \frac{\text{sun mass}}{H\ \text{mass}}$$

$$= \frac{2 \times 10^{30}\ \text{kg}}{1.67 \times 10^{-27}\ \text{kg}}$$

$$= 1.2 \times 10^{57}$$

5. Estimate the number of atoms contained in the human body.

$$\#\ \text{atoms} = \frac{\text{human mass}}{\text{atom mass}}$$

$$\approx \frac{75\ \text{kg}}{16 \times 1.67 \times 10^{-27}\ \text{kg}}$$

↖ Use O atom as "typical" atom in body

$$= 2.8 \times 10^{27}$$

Assignment 2
Trigonometry

1. Evaluate the following without a calculator.

a) $\sin(-15^\circ)$ $= \sin(30^\circ - 45^\circ)$

$= \sin 30^\circ \cos(-45^\circ) + \sin(-45^\circ)\cos 30^\circ$

$= \frac{1}{2}\,\frac{+1}{\sqrt{2}} + \frac{(-1)}{\sqrt{2}}\,\frac{\sqrt{3}}{2}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

b) $\tan(135^\circ)$ $= -\tan 45^\circ$

$= -1$

c) $\csc(240^\circ)$ $= \frac{1}{\sin 240^\circ}$

$= \frac{-1}{\sin 60^\circ}$

$= -\frac{2}{\sqrt{3}}$

d) $\cos 4\pi/3 = \cos 240^\circ$

$= -\cos 60^\circ$

$= -\frac{1}{2}$

e) $\sec 5\pi/4 = \frac{1}{\cos 225^\circ}$

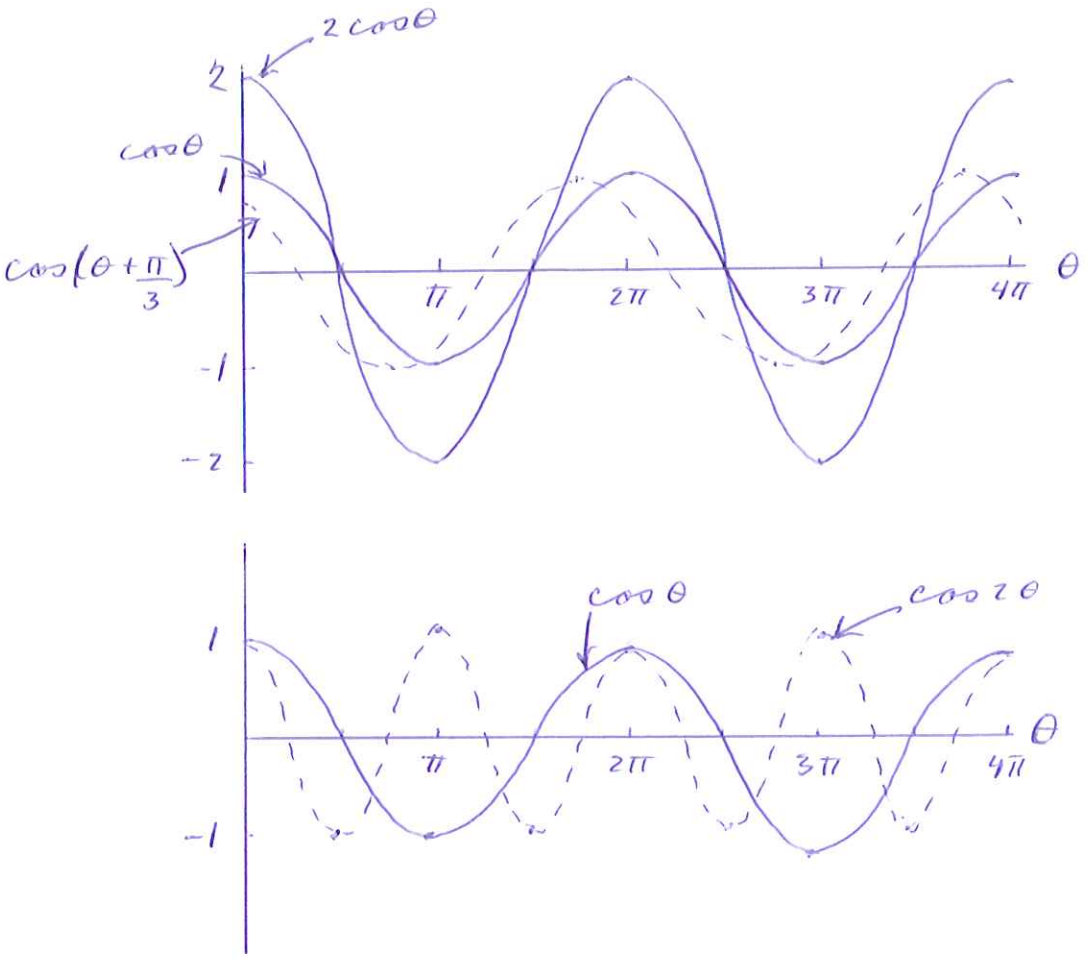
$= \frac{-1}{\cos 45^\circ}$

$= -\sqrt{2}$

f) $\cot 3\pi/2 = \frac{1}{\tan 3\pi/2}$

$= 0$

2. a) Plot the following functions.
   i) $y = \cos\theta$
   ii) $y = 2\cos\theta$
   iii) $y = \cos(\theta + \pi/3)$
   iv) $y = \cos 2\theta$.

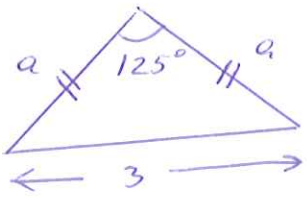


b) What are the period and amplitude of $y = A\cos k\theta$?

Amplitude = A (i.e. y varies from −A to +A)

Period = $\frac{2\pi}{k}$

3. An isosceles triangle has a long side equaling a length of 3 units and one angle equal 125°. Find the length of the two short sides.



Cosine Law: $3^2 = a^2 + a^2 - 2a^2 \cos 125^\circ$

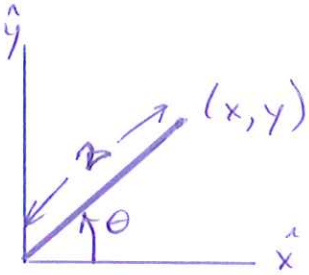
$= 2a^2 + 1.15a^2$

$= 3.15a^2$

$\therefore a = 1.69$ units

4. A 2 meter long bar lies in the xy plane with one end at the origin. Find the position in the xy plane of the other end point of the bar if the angle the bar makes with the x axis is the following.

a) 30°
b) 120°
c) $2\pi/3$
d) $5\pi/6$



$x = 2\cos\theta$

$y = 2\sin\theta$

a) $\theta = 30^\circ \Rightarrow (x,y) = (1.73, 1)$

b) $\theta = 120^\circ \Rightarrow (x,y) = (-1, 1.73)$

c) $\theta = 2\pi/3 \Rightarrow (x,y) = (-1, 1.73)$ (same as b)

d) $\theta = 5\pi/6 \Rightarrow (x,y) = (-1.73, 1)$

5. Find the first 5 terms of the Taylor's expansion for:

a) $y = \cos\theta$

$f(\theta) = \cos\theta \qquad f(0) = 1$

$f'(\theta) = -\sin\theta \qquad f'(0) = 0$

$f^{(2)}(\theta) = -\cos\theta \qquad f^{(2)}(0) = -1$

$f^{(3)}(\theta) = \sin\theta \qquad f^{(3)}(0) = 0$

$f^{(4)}(\theta) = \cos\theta \qquad f^{(4)}(0) = 1$

$$\therefore f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - + \dots$$

b) $y = \tan\theta$.

$f(\theta) = \tan\theta \qquad f(0) = 0$

$f'(\theta) = \sec^2\theta \qquad f'(0) = 1$

$f^{(2)}(\theta) = 2\sec^2\theta\tan\theta \qquad f^{(2)}(\theta) = 0$

$f^{(3)}(\theta) = 4\sec^2\theta\tan^2\theta + 2\sec^4\theta \qquad f^{(3)}(\theta) = 2$

$f^{(4)}(\theta) = 8\sec^2\theta\tan^3\theta + 8\sec^4\theta\tan\theta \qquad f^{(4)}(\theta) = 0.$

$+ 8\sec^4\theta\tan\theta$

$= 8\sec^2\theta\tan^3\theta + 16\sec^4\theta\tan\theta$

$$\therefore \tan\theta = \theta + \frac{2\theta^3}{3!} + \dots$$