York University

Department of Physics and Astronomy

Physics is REALLY Fun!!!?

PHYS 1410 6.0

Laboratory Manual

Summer 2017

Forward

Physics is an experimental science. Everything taught in a classroom is based on observations made over centuries. Experiments are a pain in the neck. Things never work out the first time exactly like we'd like. Everything also has an uncertainty. However, history shows that careful experimental work pays off in wonderful insights. Hopefully, these experiments will encourage you in your studies of physics and show that Physics really is fun.

This manual was originally written by Prof. S. Jerzak of the Physics Department of York University a number of years ago. It was extensively edited this year to clarify and update many items. Undoubtedly, some mistakes remain. I would be grateful to students who point those out.

William A. van Wijngaarden

March, 2017

SUMMER 2017

Dates	Section ODD	Section EVEN
May 2 - 4	-	-
May 9 - 11	-	-
May 16 - 18	Linear Motion	Thrust & Friction
May 23 - 25	Thrust & Friction	Linear Motion
May 30 - Jun 1	Oscillatory Motion	Elasticity
Jun 6- 8	Elasticity	Oscillatory Motion
Jun 13 - 15	Standing Waves	Thermal Physics
Jun 20 - 22	Thermal Physics	Standing Waves
Jun 27 - 29	DC Electricity	Electron Charge/Mass
Jul 4 - 6	Electron Charge/Mass	DC Electricity
Jul 11 - 13	Diffraction of Light & Spectroscopy	Polarization of Light/Lenses
Jul 18 -20	Polarization of Light/Lenses	Diffraction of Light & Spectroscopy

Contents

Chapter 0

Introductory Information

0.1 General Information

0.1.1 Why do Laboratory Work?

Firstly, all science is based on a foundation of experimental data and your experiments will exemplify and illuminate many of the principles studied in the lectures. We try to timetable experiments as near as possible to the related material in the lecture schedule. However, details in the operation of the laboratories prevent us from achieving a perfect match and we ask you to be tolerant in this regard.

Secondly, the laboratory will be a medium for teaching some new material which will not be covered in the lecture course.

Thirdly, it gives you an opportunity to train your brain, eyes and hands in good experimental techniques, while familiarizing yourself with some of the instruments used in experimental science.

Obtaining good results is important, particularly if you intend to go on to more difficult labs. But do not get so involved in the mechanics of ”doing” that you lose sight of the goal of the experiment, the theory behind it, and its wider applications.

You will need:

- this manual (print your name on it)
- the usual writing materials (graph paper is provided)
- a calculator

0.1.2 Prelab Preparation and Reports

You will know from the posted schedule which experiment you will be doing. **Before coming to do the experiment, you are expected to read the appropriate section of this manual.** Be sure you understand the theory involved, consult your textbook, and plan your practical work. Most of the lab outlines contain prelab exercises which must be completed on a separate sheet of paper before you come to the lab. **This preparation is most important.** It is unlikely that you will be able to finish the experiment satisfactorily or learn from them if you do not prepare beforehand. There may be short, unannounced quizzes on the experiment during some labs.

A sample lab report is included in the manual (Appendix D).

We do not require you to write an elaborate report for each experiment. The report should include name, name of partner, title and date. The experimental data, whenever possible, should be summarized in the form of a table, with title, column headings, units and experimental uncertainties. Graphs should have titles, axes labeled and units included. Uncertainties of all measured quantities should be indicated on graphs in the form of error bars. Calculations should be shown and organized in a logical way, with short comments and explanations. Just formulas with substituted data are not acceptable. Calculations of uncertainties is an important part of the lab report (next section in the manual provides more information regarding uncertainty calculations and rounding of final result and its uncertainty).

You are encouraged to record in your report for future reference any comments regarding the theory or method or apparatus which enhance your understanding. Your report should resemble a research scientist's day-to-day experimental log rather than a polished scientific paper.

It is preferred that you write laboratory reports in notebooks, which encourage better organization and neatness. Do not tear pages out of the books, if a mistake is made, simply cross out the mistake neatly. Two books will be required to be used alternately throughout the year. Light weight coil notebooks are suitable. Put your name and lab time clearly on the outside.

The three-hour session should be sufficient for the taking of measurements and for calculations and conclusions, etc. Be punctual - latecomers will find it difficult to complete the assignment. **All lab reports, finished or unfinished, must be handed in to your demonstrator by the end of the three-hour lab session.**

Your report will be marked by the demonstrator whose name appears on the top of the attendance list which you sign. It will be your responsibility to collect your report from this demonstrator during your next laboratory session. At this time you should discuss with your demonstrator any matters concerning the report(s).

0.1.3 Lab Partners

Some students claim that they learn more while working with a lab partner; others prefer to work alone. For certain experiments where basic techniques, etc. are explored, you will be required to work individually - this will be stated in the lab outline for those particular experiments. For the other experiments we will try to provide sufficient apparatus so that you may work with another student who has been assigned the same experiment or alone, as you prefer. For a few of the experiments the mechanical work is so difficult that one person cannot perform the experiment satisfactorily. If two students work together, each should take a turn at reading all the instruments and although both will have the same data, **each student must submit an independent report, with independent calculations. No more than two students working together as lab partners is allowed.**

0.1.4 Cleanliness and Care of Equipment

We do not charge you for accidental breakages, but please report them to the demonstrator or lab technologist immediately, so that equipment can be replaced or repaired.

Students must leave their place of work in the lab neat with all the apparatus complete. Each experimental set-up will be used by approximately forty students before it is retired for the year, so leave it for the next student in the state in which you would like to find it.

When a student hands in a report, the demonstrator will check their place of work to see that it is left in satisfactory condition. When satisfied, the demonstrator will accept the report.

0.1.5 Lab Safety

Scientists very commonly live to a grand old age in spite of their daily encounters with many hazards. The main reason for this is that a scientist doing an experiment is paying very close attention to everything that happens, is expecting the unknown and can react quickly to it. Your best protection against accidents in the lab is a constant thoughtful alertness which never permits your actions to become "mechanical" and "reflex".

Specific hazards which exist in particular experiments will be stressed in the respective lab outline. Please pay very careful attention to these warnings and act accordingly.

Notify the demonstrator or lab technician of any accident or injury no matter how insignificant it may seem.

In the case of a fire, at the sound of the fire alarm in the building, the university stipulates that everyone must leave the building. In the case of a fire in the lab, the demonstrator is responsible for taking the appropriate action to curb it, but the students must leave the building immediately.

A 24-hour Emergency Services Telephone Centre operates on York Campus and can be alerted by calling 33333 on all campus telephones or 736-2100 Ext. 33333 on public or off-campus telephones.

Health services are located in York Lanes.

0.1.6 Academic Honesty

Students will certainly discuss and talk about their studies with their friends and this can be very useful; but any work that you hand in must have been done by yourself. This is the only way to test your own competence and to prepare yourself for positions of responsibility after graduation.

THE UNIVERSITY CONSIDERS ALL FORMS OF COPYING AND CHEATING TO BE SERIOUS OFFENCES.

0.2 Measurement and Uncertainties

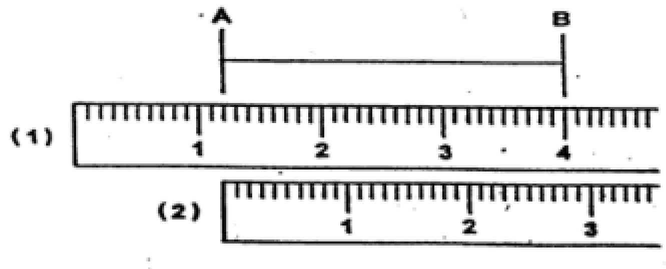
0.2.1 Measurements

There are several requirements that must be met if a measurement is to be useful.

Number of Determinations It is a fundamental law of laboratory work that a single measurement is of little value because of the liability not only to gross mistakes but also to smaller random errors. Accordingly, it is customary to repeat measurements. The laws of statistics lead to the conclusion that the value having the highest probability of being correct is the arithmetic mean or average.

Zero Reading Every measurement is really a difference between two readings, although for convenience, most instruments are calibrated so that one of these readings will be zero. In many instruments, this zero but may shift slightly due to wear or usage. Thus it is essential that the zero be checked before every measurement. In some cases the zero can be reset manually, while in others it is necessary to record the exact zero reading and correct all subsequent readings accordingly. e.g. When measuring the length AB illustrated in Fig. 1, a ruler could be placed (1) with 1.2 cm at A, then length AB = (4.0 - 1.2) cm = 2.8 cm. The more usual ruler position (2) allows the length AB to be read as 2.8 cm directly, but remember this is still the difference between <u>two</u> readings: 2.8 cm and 0.0 cm.

Figure 1: Example of measuring length



Accuracy Quantitative work requires that each measurement be made as accurately as possible. The main units of a scale are usually divided, and the eye can easily subdivide a small a distance of 1 mm into five parts. Thus, if a linear scale is divided into millimeters, e.g. on a high quality ruler, a reading could be expressed to 0.2 of a millimeter; e.g. 4.6 mm, 27.42 cm, where 3/5 and 1/5 of a mm are estimated by eye. In cases where the reading falls exactly on a scale division, the estimated figure would be 0; e.g. 48.50 cm, indicating that you know the reading more accurately than 48.5 cm. But it would not be possible to take a reading with greater accuracy then 0.2 mm with this equipment. If the scale is not finely engraved, the lab meter sticks for example, it could probably only be read as 0.5 mm.

Significant Figures A significant figure is a digit which is reasonably trustworthy. One and only one estimated figure can be retained and regarded as significant in any measurement, or in any calculation involving physical measurements. In the examples in (3) above, 4.6 mm has two significant figures, 27.42 cm has four significant figures.

The location of the decimal point has no relation to the number of significant figures. The reading 6.54 cm could be written as 65.4 mm or as 0.0654 m without changing the number of significant figures - three in each case. The presence of a zero is sometimes troublesome. If it is used merely to indicate the location of the decimal point, it is not called a significant figure, as in 0.0654 m; if it is between two significant digits, as in a temperature reading of 20.5°C , it is always significant. A zero digit at the end of a number tends to be ambiguous. In the absence of specific information we cannot tell whether it is there because it is the best estimate or merely to locate the decimal point. In such cases the true situation should be expressed by writing the correct number of significant figures multiplied by the power of 10.

Thus a student measuring the speed of light, 186,000 mi/s is best written as $1.86x10^5 mi/s$ to indicate that there are only three significant figures. The latter form is called standard notation, and involves a number between 1 and 10 multiplied by the appropriate power of 10. It is equally important to include the zero at the end of a number if it is significant. If reading a meter-stick, one estimates to a fraction of a millimeter, then a reading of 20.00 cm is written correctly. In such a case, valuable information would be thrown away if the reading were recorded as 20 cm. The recorded number should always express the degree of accuracy of the reading. Most of your calculations will probably be limited to three significant figures. A calculator produces about 8 digits, but, in your final answer, only record physically significant figures.

0.2.2 Uncertainties (Errors)

A quantity measured or calculated if its uncertainty is stated. An uncertainty of 50% or even 100% is a vast improvement over no knowledge at all: an accuracy of ±10% is a great improvement over ±50% and so on. The uncertainty in a reading or calculated value is called on error. This word does not imply a mistake or sin. This manual uses the words **error** and **uncertainty** interchangeably.

Systematic Errors A **systematic** error always produces an error of the same sign. Systematic errors may be sub-divided into three groups: instrumental, personal and external. Corrections should be made for systematic errors when they are known to be present.

An **Instrumental Error** is caused by faulty or inaccurate apparatus; for example an undetected zero error in a scale, an incorrectly adjusted watch. If 0.2 mm has been worn off the end of this ruler, all readings will be 0.02 cm too high.

Personal Errors are due to some peculiarity or bias of the observer. Probably the most common source of personal error is the tendency to assume that the first reading taken is correct. A scientist must constantly be on guard against any bias and make each measurement as if it were completely isolated from all previous experience. Other personal errors may be due to fatigue, the position of the eye relative to a scale, etc

External Errors are caused by external conditions (wind, temperature, humidity, vibration); examples are the expansion of a scale as the temperature rises or the swelling of a meter stick as humidity increases.

Random Errors Random errors occur as variations which are due to a large number of factors. Each factor adds its own contribution to the total error. Resulting error is a **matter of chance** and therefore positive and negative errors are equally probable. Because random errors are subject to the laws of chance, their effect in the experiment may be lessened by taking a large number of observations. A simplified statistical treatment of random errors is described in Appendix A.

The Error Interval A reading of 6.540 cm might imply that it lay between 6.538 and 6.542 cm. The reading would then be recorded as (6.540 ± 0.002 cm). The scales on most instruments are as finely divided by the manufacturer as it is practical to read. Hence, the error interval will probably be some fraction of the smallest readable division on the instrument; typically 0.5 of a division. **Note that it is essential to quote an error with every set of measurements.**

Absolute Uncertainty The estimate of an error interval gives what is called an "absolute" uncertainty. It has the same units as the measurement itself; e.g. (6.540 ± 0.002) cm.

Relative and Percentage Uncertainties Frequently a statement of the absolute uncertainty δx, is not as meaningful as a comparison of the size of the uncertainty with the size of the measurement itself, x. This comparison is expressed by a relative uncertainty:

$$\frac{\delta x}{x} = \frac{0.03}{2.56} \simeq 0.01 = 1\%$$

An uncertainty $\delta x = \pm 0.2cm$, for example, is much more important in a measurement of $2cm$ ($\delta x/x = 0.1$ or 10%) than in a measurement of $2m$ ($\delta x/x = 0.2/200 = 0.001$ or 0.1%).

Uncertainties in Calculated Quantities Usually the measurements are used to calculate something. How are the uncertainties in the measurement compounded when these measurements are used in calculations? The rules illustrated below are derived in Appendix B.

RULE 1: For Addition and Subtraction

> **Whenever addition and/or subtraction occur in a calculation the resultant <u>absolute</u> uncertainty in the answer is the <u>sum of the absolute</u> uncertainties of all the measured quantities occurring.**

Example: Let $x = 2.66 \pm 0.02$ and $y = 1.79 \pm 0.02$. Find the magnitudes of the uncertainties of $(x + y)$ and $(x - y)$.

Solution:

$$\begin{array}{r} (\mathbf{x}+\mathbf{y}) \\ 2.66 \pm 0.02 \\ +1.79 \pm 0.02 \\ \hline 4.45 \pm 0.04 \end{array}$$

$$\begin{array}{r} (\mathbf{x}-\mathbf{y}) \\ 2.66 \pm 0.02 \\ -1.79 \pm 0.02 \\ \hline 0.87 \pm 0.04 \end{array}$$

This is reasonable since

$$\begin{array}{lcl} x = 2.66 \pm 0.02 & \rightarrow & 2.64 \leq x \leq 2.68 \\ y = 1.79 \pm 0.02 & \rightarrow & 1.77 \leq y \leq 1.81 \end{array}$$

Adding and subtracting in the most unfavourable ways to obtain the maximum possible uncertainty gives

$$4.41 \leq (x+y) \leq 4.49$$
$$0.83 \leq (x-y) \leq 0.91$$

which can be expressed as 4.45 ± 0.04 and 0.87 ± 0.04 as above.

RULE 2: For Multiplication and Division

> **Whenever multiplication and/or division occur, the <u>relative uncertainty</u> of the product or quotient is equal to <u>the sum</u> <u>of the relative uncertainties</u> of each factor in the function.**

Example: Let $x = 2.66 \pm 0.02$ and $y = 1.79 \pm 0.02$. Find the magnitudes and the uncertainties of $z = (x \times y)$ and $w = (x/y)$.

Solution:

$$\mathbf{z = (x \times y) = 2.66 \times 1.79 = 4.7614}$$
$$\frac{\delta z}{z} = \frac{\delta x}{x} + \frac{\delta y}{y}$$
$$\frac{\delta z}{z} = \frac{0.02}{2.66} + \frac{0.02}{1.79}$$
$$\frac{\delta z}{z} = 0.0075 + 0.011$$
$$\frac{\delta z}{z} = 0.0185$$

where $z = 0.0185$ which gives $\delta z = 0.09$. Hence, $z = 4.76 \pm 0.09$. Please observe that the uncertainty $\delta z = 0.09$ was rounded to one significant figure.

Similarly,

$$\mathbf{w = (x/y) = 2.66/1.79 = 1.486}$$
$$\frac{\delta w}{w} = \frac{\delta x}{x} + \frac{\delta y}{y}$$
$$\frac{\delta w}{w} = \frac{0.02}{2.66} + \frac{0.02}{1.79}$$
$$\frac{\delta w}{w} = 0.0185$$

where $w = 1.486$ which gives $\delta w = 0.03$. Hence, $w = 1.49 \pm 0.03$.

Where addition and/or subtraction and multiplication and/or divisions are all involved in one formula, the calculation is more complicated. The rules, above, should then be applied as illustrated below.

Example: Find the uncertainty of the quantity given by $Z = AB^2 + CD^2$ where A, B, C, D are measured quantities, and $\delta A, \delta B, \delta C, \delta D$ are the corresponding absolute uncertainties.

Solution:

$$\text{Let } I_1 = AB^2 = A \times B \times B \text{ and } I_2 = CD^2 = C \times D \times D$$

$$\frac{\delta I_1}{I_1} = \frac{\delta A}{A} + \frac{2\delta B}{B} \quad , \quad \frac{\delta I_2}{I_2} = \frac{\delta C}{C} + \frac{2\delta D}{D}$$

$$\delta I_1 = (\frac{\delta A}{A} + \frac{2\delta B}{B})I_1 \quad , \quad \delta I_2 = (\frac{\delta C}{C} + \frac{2\delta D}{D})I_2$$

$$\delta I = \delta I_1 + \delta I_2 = (\frac{\delta A}{A} + \frac{2\delta B}{B})I_1 + (\frac{\delta C}{C} + \frac{2\delta D}{D})I_2$$

Rules for Stating Uncertainties Uncertainties should be rounded to one significant figure. The only exception is when the leading uncertainty digit is 1. In this case two significant digits should be given. For example, the uncertainty 0.14 rounded to one significant digit would be reduced very significantly. The final answer should have the last significant digit in the same decimal position as the uncertainty. Examples include: 26.3 ± 0.5 s, 48 ± 2 m, 36.82 ± 0.06 N and $(15.34 \pm 0.14) \times 10^2$ kg.

Comparison Occasionally in the lab you will be asked to compare:

1. several values for the same quantity which you measured using different methods.
2. a value which you measured or calculated with a standard value in a table.

In scientific terms the word "compare" means a mathematical comparison.

In case (1) the best way to compare the value is to calculate what percentage the average deviation is of the mean.

In case (2), you should ask the question does:

$$|\text{your value} - \text{standard value}| \leq \text{your uncertainty}$$

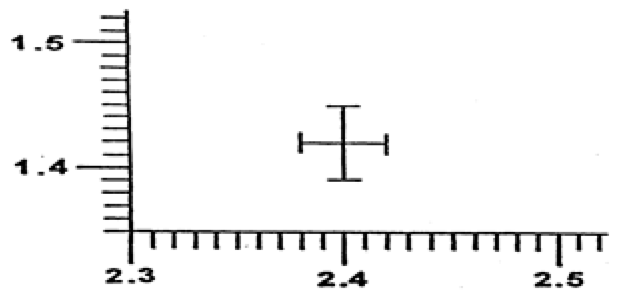
If the above is true, then the measurement agrees with the standard value.

0.3 Graphs

In this course it will frequently be necessary to plot a series of results on graph paper. A graph is often the most concise and meaningful way to display data. Plotting experimental data and deriving significant information from the resulting graph is rather different from the process of plotting the graph of a known analytic function. An experimental measurement is not exact, but rather is represented by a small range of possible values; e.g. $2.38 \leq x \leq 2.42$ which we usually express as $x = 2.40 \pm 0.02$; or $y = 1.42 \pm 0.03$.

On a graph we would represent the uncertainty by plotting the point as in Fig.2 where the two "error bars" cross at $(2.40, 1.42)$ and their lengths are 2×0.02 and 2×0.03.

Figure 2: Plotted data point with Error Bar



Once the points are plotted, the task is to draw the best smooth curve (usually this will be a straight line) through the field of points. Due to the uncertainties the experimental points will never fall exactly on the analytic curve which you try to fit to them. Draw the curve which comes closest to the most points and in general lies within the error bars of all points.

Generally, one variable is under your control; and is known as the **independent variable**, by convention this is plotted on the horizontal, x axis. The second measured quantity is called the **dependent variable**.

The following guidelines should be used when preparing graphs:

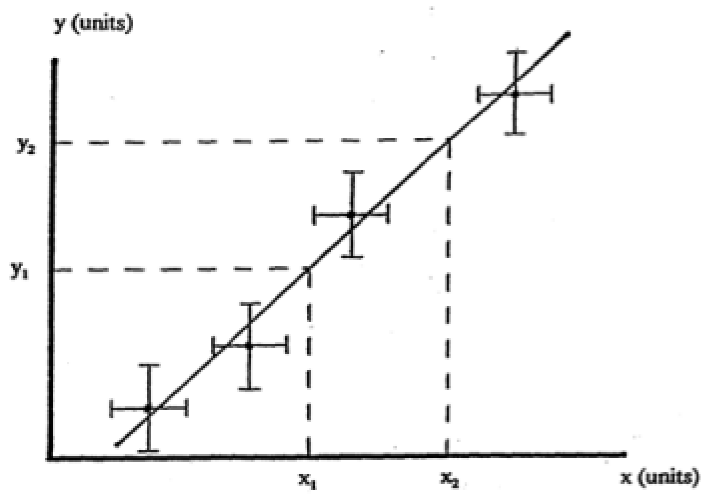
0.3.1 Labeling

Graphs should have a title. Axes should always be labelled with the name of the quantity being displayed and the units in which it is measured. e.g. Spring extension (mm).

0.3.2 Scales

Aim to spread your data out across as much of the graph paper as possible. If the value for one variable ranges from 90 to 220, it would not be necessary to fit in a scale from 0 to 220 but only from 90 to 220.

Figure 3: Determining the Slope from Plotted Data



Avoid the tendency to force the extrapolation of a graph through the origin even if intuition tells you it should go there. Limitations of apparatus and other side effects can sometimes cause unusual results close to the origin. Thus, by forcing your graph through the origin, you may be distorting it in the region of interest and thereby distorting the information which you wish to obtain.

0.3.3 Plotting

Draw graphs with a sharpened pencil so that mistakes can be erased. A transparent ruler is particularly useful for deciding on the "best" straight line through experimentally determined points. Coordinates will be in an appropriately labelled table in your report and not beside the points so as not to clutter up the graph.

0.3.4 Straight Line Graphs

If the relationship between two variables is linear, i.e. $y = mx + b$, a straight line graph of y vs x can be plotted to determine the slope m and y-intercept b.

0.3.5 To Determine the Slope

Once the line has been drawn (Fig.3) you will probably need to calculate its slope to determine some information pertinent to the experiment.

From analytical geometry

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Where (x_1, y_1) and (x_2, y_2) are points which lie **on** the line. Since your experimental points, in most cases, do not lie exactly on the line, do not use them for determining the slope. Instead, choose two arbitrary points lying (exactly) on the line **as far apart as possible** and determine the slope from them (draw in the rise and run or in some way indicate the points you have used). Since the variables you plot will be physical quantities with units, the slope will also have units. Another method frequently used to calculate slope is called the Method of Least Squares. For details see Appendix C.

0.3.6 Logarithmic Graphs and Exponential Data

In some experiments, you will collect data which does not fit to straight line, rather to an exponential. Suppose variable p and q are related by the expression:

$$q = A_0 e^{Tp}, \text{ where } A_0, T \text{ are constants, and } e \text{ is Euler's number}$$

Taking the natural logarithm of both sides gives

$$ln(q) = ln(A_0) + ln(e^{Tp})$$
$$ln(q) = ln(A_0) + Tp$$

Suppose now we collected data x, y which had this expected exponential trend. If we were to plot x vs $ln(y)$ then the results would be a straight line whose slope m would be equal to the constant T, and whose y-intercept b would be equal to the $ln(A_0)$. This is a convenient a powerful method of determining parameters of an exponential trend using normal graph paper. An alternative method would be to use Semi-Logarithmic graph paper, which has the y-axis as on a logarithmic scale and the x-axis on a linear scale.

Chapter 1

Uniformly Accelerated Linear Motion

Motion and changes in motion lie at the heart of physics.

Objective

1. To learn how to analyze experimental data and how to prepare the lab report.
2. To determine the acceleration of glider on an air track.

Apparatus Air track, computer, data acquisition system, meter stick, graph paper.

1.1 Introduction

The experience of several centuries shows three quantities: displacement, velocity and acceleration, along with time, are sufficient to describe the motion of an object along a straight line.

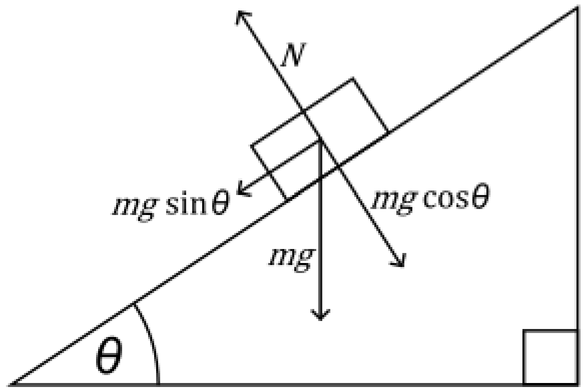
The displacement x is defined as the change in the position of the object: $\Delta\vec{x_1} = \vec{x_2} - \vec{x_1}$,where $\vec{x_2}$ is the final position and $\vec{x_1}$ is the initial position of the object.

$$\vec{v} = \frac{d\vec{x}}{dt}$$

The velocity v is the rate at which the position changes. The acceleration a is defined as the rate of change of velocity:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Figure 1.1: Forces on an object on an inclined plane.



In this experiment you will consider a motion with constant acceleration. This is the motion when velocity increases or decreases at the same rate during the motion. It occurs whenever a constant net force acts. For this kind of motion the acceleration is

$$\vec{a} = \frac{\vec{v} - \vec{v_0}}{t}$$

or equivalently

$$\vec{v} = \vec{v_0} + \vec{a}t \tag{1.1}$$

where $\vec{v}$ is the final velocity, and $\vec{v_0}$ is the initial velocity.

Since we want to examine uniformly accelerated motion we must control the design of the experimental procedure to ensure that we are indeed studying such motion. The basic requirement is a CONSTANT NET FORCE. In this experiment you will study the motion of a glider on the inclined air track. The air track permits a nearly constant net force from gravity because friction forces are so small they can be ignored. The glider, released from rest, starts to move with constant acceleration (Fig. 1).

Figure 1 shows the decomposition of weight into forces parallel and perpendicular to an inclined plane. Only the force $mg\sin\theta$ is responsible for the motion along the inclined plane. Thus according to Newton's second law

$$\begin{aligned} ma &= mg\sin\theta \\ a &= g\sin\theta \end{aligned}$$

where $g = 9.81$ m/s^2 is the acceleration due to gravity. Experimentally the acceleration of the glider can be determined using the equation $\vec{v} = \vec{v_0} + \vec{a}t$.

1.1.1 Prelab Exercise 1

Prelab exercises should be done before coming to the lab. Prelabs will be collected at the start of each lab.

The results of measurements of velocity v of a particle versus time, for constant acceleration motion, are given in the table below:

t (s)	0	2	7	14	18	24	26	36
v (m/s)	0	8	16	33	40	57	64	90

Uncertainties $\delta t = 0.2s$ and $\delta v = 2.0m/s$ are the same for all experimental points.

Draw the graph and determine acceleration and its uncertainty. Follow the procedure explained in Section 0.3.5.

1.1.2 Prelab Exercise 2

Determine the density, and its uncertainty, of a rectangular prism made of wood. Dimensions of the prism (length L, width W, height H) and mass M were measured. Results of measurements are summarized below:

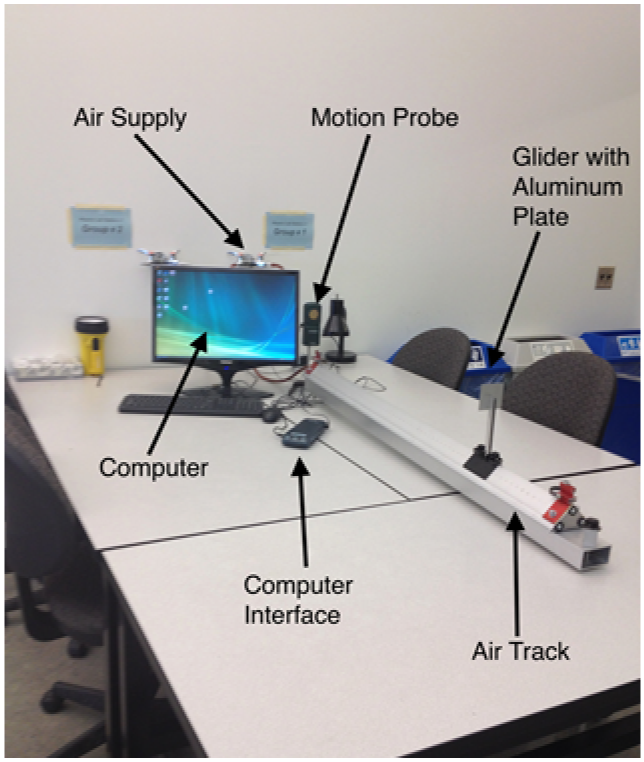
$$L = (34 \pm 1)mm$$
$$W = (91 \pm 1)mm$$
$$H = (248 \pm 1)mm$$
$$M = (745 \pm 5)g$$

1.2 Experimental and Measurements:

The setup is shown below. The computer is interfaced with a sound probe to measure the position of the glider, which is capable of nearly frictionless motion because it is supported above the track by a thin cushion of air. The computer serves to collect, analyze and graphically display the experimental data.

A probe emits short pulses of sound which are reflected from the aluminum plate mounted on the glider. The reflected pulses are detected by the same probe that emits them. The electrical signal from the probe is amplified and digitized by the electronic unit and then transmitted to the computer. The time t that elapses between the emission and detection of pulses is measured precisely by the computer, and is used to compute the distance of the glider from the probe using the formula $x = vt/2$, where $v = 343m/s$ is the speed of sound in air. The data from the experiment is graphically displayed on the computer's screen while it is collected.

Figure 1.2: Experimental Setup.



Follow these steps.

1. If there is experimental data on the computer monitor left by the previous students, close this file <u>without</u> saving it (click on **File**, then click on **Exit**.)

2. To begin the experiment, double-click on the **Linear Motion** icon on the desktop.

3. Turn the air supply full on (both handles must be parallel to the air nozzles.) Place the glider (fitted with a sound reflecting plate) at the top end of the inclined air track.

4. While holding the glider motionless, click **Experiment** on the menu bar and then click on **Zero** so that the sound probe sets the distance between it and the glider plate to zero.

5. When you are ready to begin the experiment, click on the 

button located in the menu bar releasing the glider shortly <u>after</u> the button turns red so that the computer starts collecting data at precisely the same time as the glider starts moving.

6. Your experimental data will be plotted on the monitor in near real time. Once your data collection is completed, print two copies for you and your lab partner.

7. The data table should be in the form of columns of t and x. Close the file (click on **File**, select **Exit** and click on **No** so that the data is not save in the document). content...

1.3 Calculations

1. Use your experimental data of position x vs time t to find the dependence of velocity $\vec{v}$ vs time t. The instantaneous velocity is equal to the slope of the graph of x vs t at a particular instant of time. It can be approximated by the average velocity $\vec{v} = \vec{x}/t$, where $x = x_{i+1} - x_i$ is the change of the position in time $t = t_{i+1} - t_i$.

Time (s)	Position (m)	Velocity (slope) = $\frac{\text{Rise}}{\text{Run}}\left(\frac{\text{m}}{\text{s}}\right)$	Median Time (s)
0.0	0.000	$\frac{\text{Rise}}{\text{Run}} \frac{(x_{i+1} - x_i)}{(t_{i+1} - t_i)} = 0.075$	$\frac{0.0+0.2}{2} = 0.1$
0.2	0.015		
0.4			
0.6			

Take the adjacent experimental points when there is a significant curvature of the graph of x vs t. Use a larger part of the curve when it is nearly linear.

Assume that the absolute uncertainty for position is $\pm 1.0mm$ and for time $\pm 0.005s$. As $x = x_{i+1} - x_i$ (difference between two positions), then $\delta x = \delta x_{i+1} + \delta x_i$, where δx_{i+1} and δx_i are uncertainties of positions x_{i+1} and x_i, respectively. Thus, $\delta x = \delta x_{i+1} + \delta x_i = 1.0 + 1.0 = 2.0mm$.

The uncertainty $\delta t = \delta t_{i+1} + \delta t_i = 0.005 + 0.005 = 0.01s$ and is determined by the electronic unit controlling the Logger Pro acquisition system.

2. Plot the dependence of velocity vs median time. To find the uncertainty of velocity δv use the procedure described in section "Measurements and Uncertainties" of the lab manual (steps shown below). Calculate the uncertainty δv for the first and last experimental points only.

$$v = \frac{x}{t}$$

where $x = x_{i+1} - x_i$ and $t = t_{i+1} - t_i$

$$\frac{\delta v}{v} = \frac{\delta x}{x} + \frac{\delta t}{t}$$

$$\delta v = \left(\frac{\delta x}{x} + \frac{\delta t}{t}\right)v$$

3. Calculate the slope of the straight line representing the dependence of velocity vs time. According to equation $\vec{v} = \vec{v_0} + \vec{a}t$, the slope is equal to the acceleration. Use the procedure described in section “Graphs” of the manual to find the uncertainty δa of the acceleration.

4. Use the equation $a = g \sin \theta$ to determine the value of the gravitational acceleration g. Measure the angle θ using a protractor. Compare your result and see if it is consistent with the standard value. See section **Compare** in 0.2.2.

1.4 Questions

Answer all questions in your own words. Write legibly and be brief. Do not copy from your partner.

1. Is there any experimental evidence that indicates friction may have affected your results?

2. Using the expression $a = g \sin \theta$ and experimental value of acceleration a of the glider, Susan computed the gravitational acceleration $g = 10.2 m/s^2$. Help Susan to account for the value of g greater than $9.81 m/s^2$.

Table 1.1: Uniformly accelerated linear motion- data table

Time (s)	Position (m)	Velocity = $\frac{x_{i+1}-x_i}{t_{i+1}-t_i}$ (m/s)	Median Time (s)

Chapter 2

Thrust and Friction

In the application of Newton's Laws, the concepts of Thrust and Friction appear time and time again. Thrust is a force which acts on an object (for example an airplane or a boat) in reaction to a force being applied on something else (like the air or water). It is a consequence of Newton's Third Law. For example, in a motor boat, the propeller forces water backwards, thus forcing the boat forwards - the force on the boat forwards is in reaction to the force the propeller exerts on the water. Friction is the force which results from the interaction of the surface of a moving object with another surface. The force of friction is always directly opposite to the velocity, and in some systems is a source of unwanted resistance to motion. However, in many cases, friction is highly desirable (imagine walking to class without friction).

Apparatus Cart track, Cart with fan, blocks or computer, data acquisition system.

2.1 Prelab

1. A rocket is launched vertically from rest, it reaches in altitude of $4.7km$ after $15s$ of constant acceleration. What was the magnitude of the acceleration of the rocket? How many times larger than the acceleration due to gravity is this? Is this rocket safe for human passengers?

2. Give one example of static friction.

3. Give one example of kinetic friction.

2.2 Thrust

2.2.1 Introduction

In this lab activity you will study the acceleration of a cart when acted upon by a thrust force generated by a fan attached to the cart. Note that in contrast to the Linear Motion experiment, the track on which the cart moves will be horizontal, so the force of gravity will always be perfectly balanced by the Normal force. You will investigate the acceleration of the cart for forces of varying magnitude, carts of varying weights, and forces applied at various angles with respect to the direction of motion. Note that when the fan is turned on at a particular setting, it very quickly spins up to full speed and achieves a condition of providing a constant thrust (force).

2.2.2 Setup and Explore

In this exercise, you will have a chance to make qualitative observations regarding the motion of the cart, but first the track must be setup.

1. It is critical that the track is perfectly level so that the force of gravity can be perfectly balanced by the Normal force. To do this, use the digital level, and adjust the four support feet of the track.
2. Gently place the cart on the track, and ensure the wheels of the cart are properly in the grooves of the track. Notice the removable weights which can be placed along the sides of the cart.
3. **Draw** a free-body diagram of the cart with the fan turned off.
4. Turn on the fan to the lowest setting and observe the motion of the cart.
5. **Draw** a free-body diagram of the cart with the fan turned on. **Question:** Is this constant velocity? **Question:** Is this constant acceleration?
6. Observe and **comment** on the motion of the cart as a function of fan speed.
7. Observe and **comment** on the motion of the cart as a function of fan angle.
8. Observe and **comment** on the motion of the cart with and without the weights.

2.2.3 Quantify

In this exercise, we will investigate more closely the effect of the direction of the thrust (fan angle). To do this, we want to determine the acceleration of the cart. We could use a computer-based data-collection system like was used in the Linear Motion lab. Here, however, we will apply our physics knowledge to determine the acceleration.

$$\Delta\vec{x} = \vec{v_0}t + \frac{1}{2}\vec{a}t^2 \tag{2.1}$$

To determine the acceleration $\vec{a}$, you should measure the time t it takes the cart, starting from rest ($\vec{v_0} = 0$), to travel a given distance ($\Delta\vec{x}$). When collecting data, hold the cart fixed at some location, turn on the fan, then start the timer as you release the cart. For this exercise, you should use the slowest acceleration to get accurate timing. Namely, Fan Setting 1, and using the weights. (Note that if it takes longer than about 3 seconds to travel down the track for a fan angle of 0°, perhaps the batteries in the fan need changing. Consult the TA.)

Tasks:

1. Measure the time it takes for the cart to travel a given distance down the track for the various fan angles given in Table 2.1. Record your values in that table.
2. Calculate the acceleration of the cart using the formula above, and record the result in the table.
3. **Question:** From geometry, what is the equation for the component of Thrust in the direction of motion as a function of fan angle?
4. **Question:** What force balances the component of Thrust perpendicular to the track?
5. Plot your data points for Acceleration vs Fan Angle on the graph below the table.
6. **Question:** Does your data follow the expected trend?
7. **Question:** What assumption was made in order to use the formula?

2.3 Friction

2.3.1 Introduction

In this lab you will be investigating sliding and static, dry forces of friction, as opposed to fluid friction (viscous drag). The static force of friction f_s is given by the following inequality:

$$f_s \leq \mu_s N \tag{2.2}$$

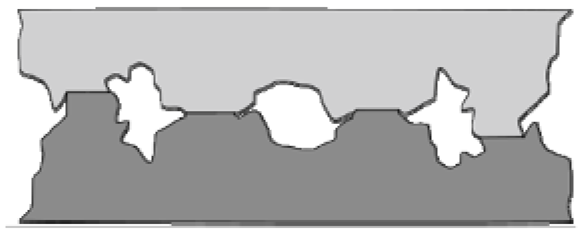
where μ_s is the coefficient of static friction and N is the normal force. (Here "normal" means that the force is perpendicular to the direction of f_s.)

The kinetic force of sliding friction f_k is given by the following equation:

$$f_k = \mu_k N \tag{2.3}$$

where μ_k is the coefficient of kinetic friction and N is the normal force (directed perpendicular to the direction of motion).

The kinetic force of sliding friction does not depend on the speed of the object, although at very low and very high speeds this may not be true. Static and kinetic forces of friction do not depend on the apparent surface area although they depend on the actual or real contact area between the two surfaces. The difference between the apparent and actual contact areas is explained in the figure below.



Surfaces, at a microscopic level, are very rough. Therefore, two surfaces are never in total contact. The object that slides is supported by the surface at the top of roughness irregularities, called asperities. These asperities deform until the weight of the object is fully supported. Approximately only 1 part in 10,000 of the apparent area is usually in contact, which is the reason why friction is effectively independent of the apparent area of contact. For very smooth surfaces the actual contact area may be a significant fraction of the apparent contact area and in this case friction is dependent on the size of the surface.

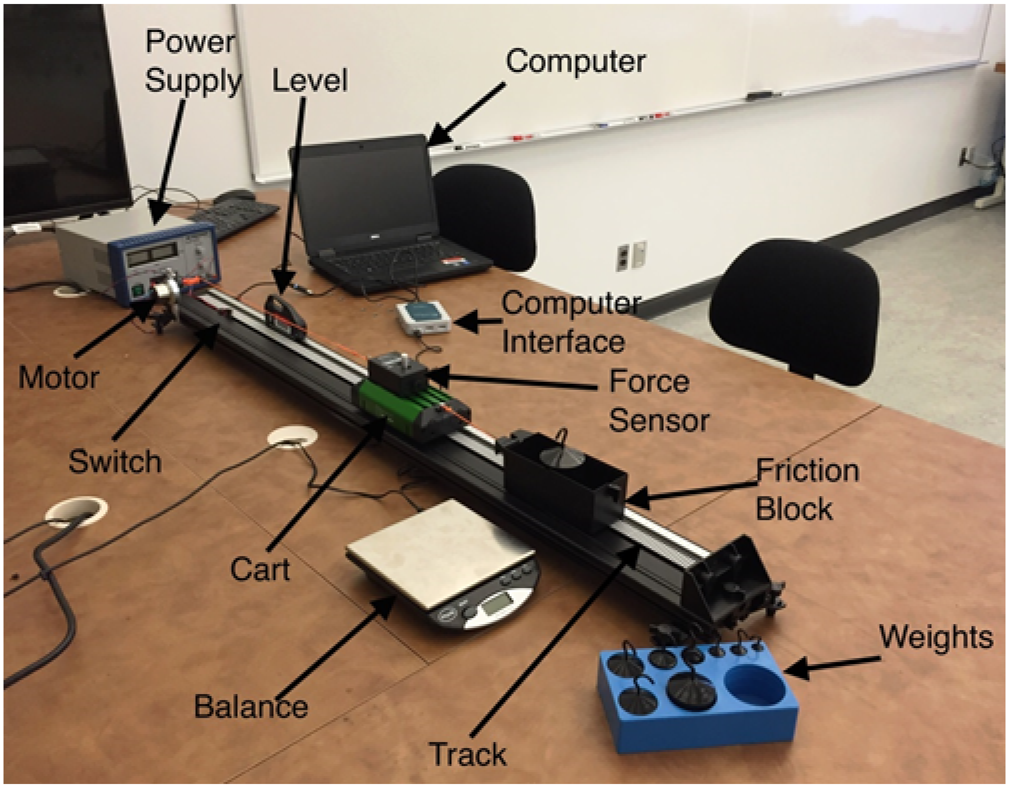
The actual contact area depends on the magnitude of the normal force. The larger the normal force the more the two surfaces push against each other, increasing the actual contact area and the force of friction. Because friction, from the microscopic point of view, occurs due to interactions between electrical charges, the larger the actual contact area between two surfaces, the larger the force of friction.

2.3.2 Measuring Static and Kinetic Coefficients of Friction

You've likely had the experience of pushing or pulling a heavy object on the floor- it is harder to get the object moving initially than it is to keep it moving. That is because the coefficient of static friction is greater than the coefficient of kinetic friction, meaning the static friction for is greater. In this experiment, we will apply a slowly increasing force to a stationary object, and measure the forces which oppose the motion.

2.3.3 Setup

The force sensor is connected by a string to a friction block. Additional weights can be mounted on the block in order to increase the magnitude of the normal force. A force sensor can be pulled either by hand or by a DC motor. You should try both ways but it is very likely that you will conclude that pulling the block by hand does not give as good



results as using the motor. A variable power supply is used to control the speed of the motor. For voltages of about 5V, the speed of the motor is very low, which allows collecting many experimental points during the transition of the block from being stationary to being in motion. The string connecting the force sensor to the motor should be wrapped around the small pulley mounted on the shaft of the motor. A separate string should connect the force sensor to the block being pulled. The force sensor is connected to the computer interface.

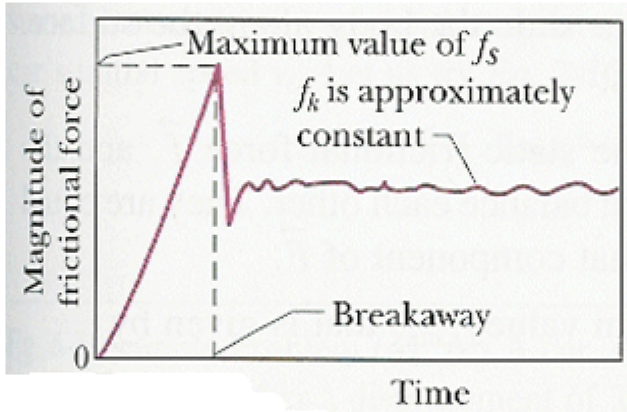
2.3.4 Procedure

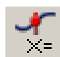
- Start Logger Pro and then zero the force detector by clicking on the Zero icon in the toolbar. Both strings should not be under tension at this point.
- Change the duration of the data collection to 10 s. You can do it by clicking on the maximum value of time on the horizontal axis and typing new value or by clicking on the Data Collection icon in the toolbar and changing Duration to10 s.
- Click on 

to begin to acquire data just before you turn the motor on.

A typical dependence of the force as a function of time is shown below.

The maximum magnitude of the recorded force is equal to the maximum value of the static force of friction f_s, max.



Click on the Examine tool  in the tool bar and determine $f_{s,max}$ and then use the equation $f_s \leq \mu_s N$ to determine μ_s. The normal force is $N = (M_{block} + M_{added})g$, where M_{block} is the mass of the block and M_{added} is the additional mass placed on the block, and g is the acceleration of gravity. $N = (M_{block} + M_{added})g$ only if the string connecting the force sensor to the block is horizontal. Measure both masses using the electronic balance.

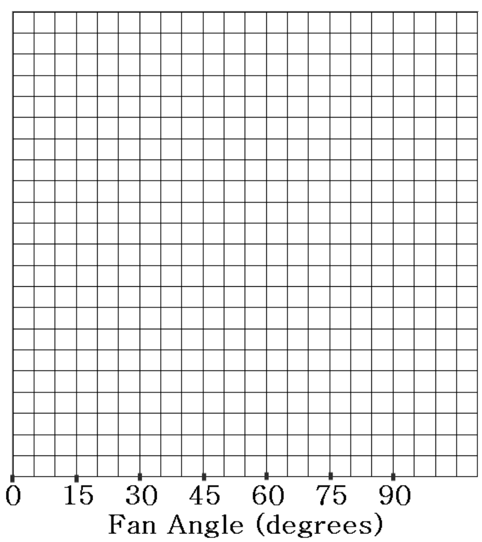
Determine the kinetic force of friction f_k using the approximately horizontal part of the graph. Small fluctuation of the force should be averaged out by using the Statistics tool (STAT icon in the toolbar ) and finding the mean value of f_k. Use the equation$f_k = \mu_k N$ to determine μ_k. As for the static friction, $N = M_{block} + M_{added})g$.

- Repeat for the same material, with 3 different masses.
- Repeat for a block which has a different material on its base.
- **Question:** Is the force of kinetic friction independent of velocity? What evidence do you have?

Distance used throughout :

Table 2.1: Acceleration vs Fan Angle

Fan Angle	Time (s)				Acceleration (m/s^2)
0°				avg:	
15°				avg:	
30°				avg:	
45°				avg:	
60°				avg:	
75°				avg:	
90°				avg:	



Chapter 3

Oscillatory Motion & Conservation of Energy

Christian Huygens and Robert Hooke gave us a strong foundation for analyzing oscillatory motion. This motion is fundamental to many areas of physics, (electricity, magnetism, waves).

Objective

1. To study oscillations of a spring-mass system.
2. To measure the decay constant of damped oscillations.
3. To study Conservation of Energy in a spring-mass system.

Apparatus masses, springs, stopwatch, data acquisition system, linear graph paper

3.0.5 Prelab Exercises

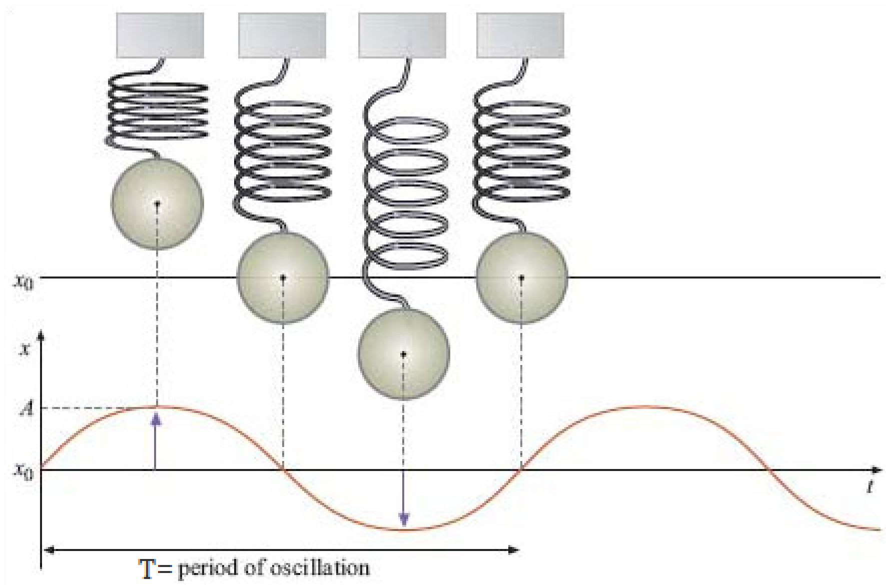
A mass $m = 50g$ is suspended from a spring with spring constant $k = 8.0N/m$.

1. Determine the force acting on the mass when it is displaced $8cm$ from its equilibrium position. If the mass is then released, what is the period of oscillation? What is the frequency?
2. What is expression for the uncertainty in the total energy E_{tot} of equation 3.5? Assume an uncertainty in k of δk, uncertainty in m of δm, uncertainty in x of δx and an uncertainty in v of δv. Assume g is perfectly known ($\delta g = 0$). Refer to the section on the Uncertainty Calculations in the introduction, Section 0.2.2, for more details. (Hint: first compute the uncertainties of the three terms in the sum, then add the uncertainties in the sum together.)

3.1 Spring Mass System

3.1.1 Introduction

In this part of the lab you will analyze a special kind of motion, called oscillatory motion. The a good illustration of that kind of motion is the motion of the mass attached to the spring. If the spring is displaced from the equilibrium position and then released, the mass



oscillates up and down. The pen attached to the mass traces a sinusoidal curve on the moving paper.

If the frictional forces, which are always present, could be neglected the spring would oscillate for an indefinite period of time. That idealized, frictionless oscillatory motion of the mass on the spring is called simple harmonic motion (abbreviated "SHM"). There is a long list of systems to which the ideal of SHM applies. The list includes: pendula, vibration of parts of musical instruments, vibration of atoms and molecules in solids, oscillation of an electric and a magnetic field in electromagnetic wave, time variation of voltage and current in alternating-current circuits. Simple harmonic motion can be represented by

$$\vec{x} = \vec{A}\cos\left(\frac{2\pi t}{T} + \theta_0\right) \tag{3.1}$$

where

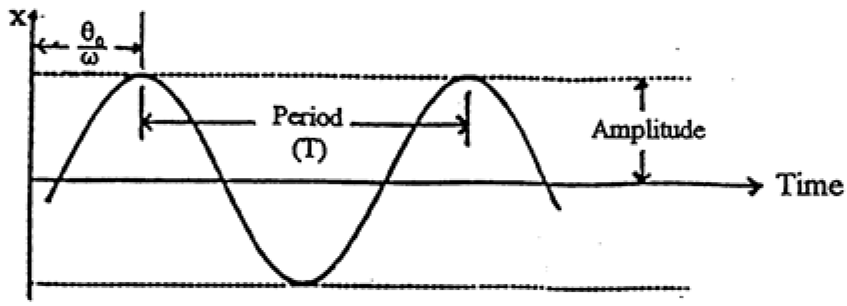
$\vec{x}$ displacement from the equilibrium position ($x = 0$ is the equilibrium position)

$\vec{A}$ Amplitude (maximum displacement from the equilibrium position)

T period (time of one complete oscillation).

θ_0 phase constant (defines the displacement at $t = 0$).

Figure 3.1: Illustration of meaning of parameters



The frequency f of the oscillatory motion is defined as the inverse of the period T

$$f = \frac{1}{T}$$

It is equal to the number of oscillations in 1 second. The unit of frequency is hertz [Hz]. Generally an angular frequency $\omega = 2\pi f$ is used (unit is radian/second). Thus, the equation 3.1 can be rewritten in the more commonly used form

$$\vec{x} = \vec{A}\cos(\omega t + \theta_0) \tag{3.2}$$

So far we have described what SHM is but we have not explained what causes that kind of motion. When the mass connected to the spring is displaced, then the spring exerts a force on the mass in the opposite direction. The bigger the displacement, the bigger the force.

$$\vec{F} = -k\vec{x}$$
$$k = \text{ spring constant}$$

The negative sign expresses the fact that the force $\vec{F}$ has a direction opposite to the displacement. Using Newton's second law

$$k\vec{x} = m\vec{a}$$
$$\text{rearranging gives} \quad \vec{a} = -\frac{k}{m}\vec{x}$$
$$\text{but} \quad \vec{a} = \frac{d^2\vec{x}}{dt^2}$$
$$\text{so that} \quad \frac{d^2\vec{x}}{dt^2} = -\frac{k}{m}\vec{x}$$

The solution of the above differential equation is

$$\vec{x} = \vec{A}\cos\left(\sqrt{\frac{k}{m}}t + \theta_0\right) \tag{3.3}$$

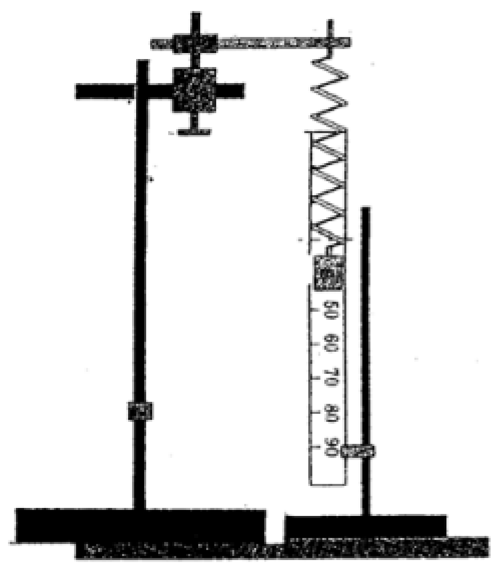
Comparing equations 3.1 and 3.3 leads us to conclude

$$\frac{1}{T} = f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{3.4}$$

The frequency of oscillations depends on mass m and spring constant k, and not on the amplitude.

The experimental setup is shown below:

Figure 3.2: A mass is suspended at the end of a spring.



Measurements and Calculations

1. Displace the mass $m = 200g$ suspended at the end of the spring about 10 to 15 cm from the equilibrium position and measure the time t for $N = 10$ oscillations. Repeat the measurement of the time t at least three times ($n = 3$). The uncertainty of your measurement is not given by the accuracy of the stopwatch but rather by your reaction time. The best estimate of the time t is the average of your three, or more, measurements and the uncertainty δt is given by the average deviation (Appendix A). Record your results and calculations in the following table.

i	Time for N=10 oscillations $t_i(s)$	Period $T_i = t_i/N(s)$	Average Period $\overline{T}(s)$	Average Deviation $\lvert T_i - \overline{T}\rvert(s)$
1				
2				
3				
avg		ΣT_i	$\Sigma T_i/n$	$\delta T = \frac{\Sigma\lvert T_i - \overline{T}\rvert}{n}$

Write your results in the form

$$(\overline{T} \pm \delta T)s$$

2. The spring constant k can be determined using the following formula (equation 3.4 , in which $T = 1/f$)

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$\text{or} \quad k = 4\pi^2\frac{m}{T^2}$$

Determine the spring constant k and its uncertainty δk. Based on the two rules described in Section 0.2.2 the uncertainty is calculated as:

$$\frac{\delta k}{k} = \frac{\delta m}{m} + \frac{2\delta T}{T}$$

The uncertainty δm is specified as the last significant digit written on the mass. For example, $10g$ indicates $\delta m = 0.5g$. If you use two masses $10g + 5g$, then the total uncertainty is $\delta m = 1.0g$.

Record the result in the form: $(k \pm \delta k)N/m$

3. A different way to determine the spring constant is to measure the elongation of the spring under a load mg. If the elongation of the spring under the load mg is d, then

$$mg = kd$$
$$\text{or,} \quad k = mg/d$$
$$\text{its uncertainty} \quad \frac{\delta k}{k} = \frac{\delta m}{m} + \frac{\delta d}{d}$$

Measure the spring constant using this technique and record the result in the form: $(k \pm \delta k)N/m$.

4. Verify if the spring constants determined using two different methods are equal within the experimental uncertainty, that is, if the uncertainty intervals overlap. The uncertainty intervals overlap if the difference between the two spring constants, $|k_1 - k_2|$, is smaller than the sum of their two uncertainties $\delta k_1 + \delta k_2$.

3.1.2 Questions

1. Does the period T depend on the amplitude of the oscillation? Verify your answer experimentally.
2. Mass of the spring should be taken into account when analyzing oscillations of a spring-mass system. Using calculus, it can be shown that one third of the mass of the spring must be considered as a portion of the total mass. Designating the mass of the spring by m_s and the suspended mass by m

$$k = 4\pi^2 \frac{(m + m_s/3)}{T^2}$$

   Measure the mass of the spring using a balance available in the lab. Was it justified to ignore mass of the spring? Explain.
3. Researchers try to design their experiments in such a way that the contribution of various measured quantities to the overall uncertainty are comparable. Which of the quantities: mass m, period T or elongation of the spring d, contributed most to the overall uncertainty δk?

3.2 Conservation of Energy

3.2.1 Introduction

The Law of Conservation of Energy states that Energy can be converted into different forms, but it cannot be created or destroyed. In an oscillating spring/mass system, energy can be transfered between: spring potential energy, gravitational potential energy, kinetic energy, energy lost as heat in the spring, and energy lost due to drag (viscous friction). In this experiment you will investigate this energy transfer.

The spring potential energy PE_{spring} is given by the expression:

$$PE_{spring} = \frac{1}{2}kx^2$$

where k is the spring constant of the spring, and x is the displacement from the equilibrium position.

Gravitational potential energy PE_g is given by:

$$PE_g = mgh = mgx$$

where m is the mass of the oscillating object, g is acceleration due to Earth's gravity, and h is the height with respect to some reference level, and since we have a vertically mounted spring, and we already called the displacement from the springs equilibrium x, this h is equal to x.

Kinetic Energy KE is given by

$$KE = \frac{1}{2}mv^2$$

where m is the mass of of the oscillating object, and v is the speed of the object.

The energy lost as heat in the spring and energy lost due to drag (viscous friction) occur in all systems, however, in the system under investigation are expected to be small, and for now, will be ignored (we have no easy way to measure them) . Therefore, the total energy of the system is:

$$E_{tot} = PE_{spring} + PE_g + KE = \frac{1}{2}kx^2 + mgx + \frac{1}{2}mv^2 \tag{3.5}$$

3.2.2 Setup and Data Collection

To collect data in this experiment, a sound probe (as was used in the Linear Motion Experiment) is used to measure the distance from the probe to the hanging mass and graphically display the experimental data. The software which interfaces with this probe is called LoggerPro. Run LoggerPro if it is not already running, and clear any old data (the easiest way is the brute-force method of closing LoggerPro, not saving the data when asked, and reopening.)

Ideally, we would have a spring which hangs without its coils compressed together. To achieve this situation, we will hang an initial mass onto spring using the mass holder before we start. The mass holder weighs $50g$, and you should place a disk of $200g$ onto the holder. This is the initial mass, m_i. This mass is needed simply to allow the spring to act as more like an ideal spring. The level of the bottom of the mass hook will be the reference level for the gravitational potential energy and spring potential energy.

- We need to assign the present position of the spring as Zero. With the mass holder steady, select Experiment ⇒ Zero from the top menu.

- Now we will apply our oscillating mass to the spring. Chose a value which doesn't overly extend the spring, and place it on the mass holder. This is the added mass, m_{added}. The spring will now hang a lower position (strongly resist the urge to re-zero the probe).

- To get the system to oscillate, pull the mass holder down by a few centimeters and release. Click on 

to begin data collection. You will see a graph being plotted of the position vs time. Notice that it is a nice sinusoidal function (simple harmonic motion). Also notice there is a table on the left hand side of the screen of time, position, and velocity. These will be use for us to determine the energy.

- Print off two copies of the results, one for you, one for your partner.

3.2.3 Analysis

Based on the Law of Conservation of Energy, the total energy of the system should remain constant over time (for all rows in the table). What energies do we need to consider? From the list above, spring potential energy, gravitational potential energy and kinetic energy are certainly key for this experiment, and can be computed from the Position and Velocity values in the table. Energy lost as heat in the spring, and energy lost due to drag (viscous friction) are variables we cannot measure, but are likely quite small in this experiment, so we'll ignore them for now. So was energy conserved? Pick 5 times during the oscillations, preferably when the mass is at a variety of different positions and compute the spring potential energy, gravitational potential energy and kinetic energy for each.

Special Note When computing the Spring Potential Energy and the Gravitational Potential Energy, we only need to consider m_{added}, not the initial mass m_i required to extend the spring. (At all times, the extra Spring potential energy of m_i cancels the extra Gravitational Potential Energy of m_i) However, when computing the Kinetic Energy, we need to consider the total mass of $m_i + m_{added}$, since there is actually a larger mass moving than just m_{added}.

To get an idea of the Uncertainty in your measurement, calculate uncertainty for one line of your table. Assume the uncertainty in the Position reading is $1mm$, and the uncertainty in the velocity is $1mm/s$. Use a value of $k = 18.0 \pm 0.5N/m$.

Question: Was energy conserved in this experiment?

Question: Did we need to include the heat lost in spring and work done by viscous drag in our analysis? Explain.

Table 3.1: Mass on a spring data and analysis

i	Time for N=10 oscillations $t_i(s)$	Period $T_i = t_i/N(s)$	Average Period $\overline{T}(s)$	Average Deviation $\lvert T_i - \overline{T}\rvert(s)$
1			-	
2			-	
3			-	
avg	-	$\Sigma T_i =$	$\Sigma T_i/n =$	$\delta T = \frac{\Sigma\lvert T_i - \overline{T}\rvert}{n} =$

Table 3.2: Mass on a spring data and analysis

i	Time for N=10 oscillations $t_i(s)$	Period $T_i = t_i/N(s)$	Average Period $\overline{T}(s)$	Average Deviation $\lvert T_i - \overline{T}\rvert(s)$
1			-	
2			-	
3			-	
avg	-	$\Sigma T_i =$	$\Sigma T_i/n =$	$\delta T = \frac{\Sigma\lvert T_i - \overline{T}\rvert}{n} =$

$m_i =$	
$m_{added} =$	
$m_{Total} =$	

Table 3.3: Conservation of Energy of a Spring-Mass System

Time(s)	Position (m)	Velocity (m/s)	Spring Potential Energy (J)	Gravitational Potential Energy (J)	Kinetic Energy (J)	Total Energy (J)

Show sample calculation here:

Chapter 4

Elasticity

Many situations which we examine in our study of Physics involve individual, isolated particles. However, much of the natural world consists of collections of particles, i.e. of gases, of liquids, of solids, and of plasmas. The particles within these collections can interact with one another which results in phenomena that could NOT occur were the particles to be isolated. One important such phenomenon is ELASTICITY. This is of particular importance for solids.

We shall assume that the size of any distortion produced in the "normal" solid due to the action of a force upon it is LINEAR, i.e. $F = k_1 x$ and NOT $F = k_1 x + k_2 x^2 + ...$(only $k_1 \neq 0$). Also, we shall assume that once the distorting force is removed the solid returns to its "normal" shape, i.e. NO permanent change occurs as a result of the distorting force (should such a permanent change occur we will have exceeded the elastic limit of the material).

Objective

1. To measure the shear modulus and the Young's modulus of different materials.
2. To use the least squares method to analyze the experimental results.
3. To use the computer to prepare the lab report.

Apparatus Young's modulus apparatus, torsion apparatus, Vernier and micrometer callipers, millimeter ruler, graph paper.

4.1 Introduction

Solids have a well defined shape and size. When a force is applied to the solid, the size and shape of the object are changed. The ability of the object to return to the original shape when the force is removed is called elasticity. Two terms are important to describe the elastic properties of solids: stress and strain. Stress σ is defined as a force F over the area A to which the force is applied

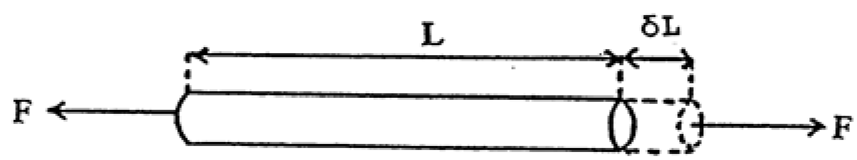
$$\sigma = F/A$$

The SI unit of stress is a Pascal, $Pa = N/m^2$. Strain is a measure of the deformation caused by a stress. In Fig. 1 a long solid cylinder of length L is elongated by δL under the stress $\sigma = F/A$, where A is the cross-sectional area of the cylinder. The strain ϵ in that case is

$$\epsilon = \frac{\delta L}{L}$$

According to Hooke's Law, strain is proportional to stress

Figure 4.1: Illustration of an object under Strain

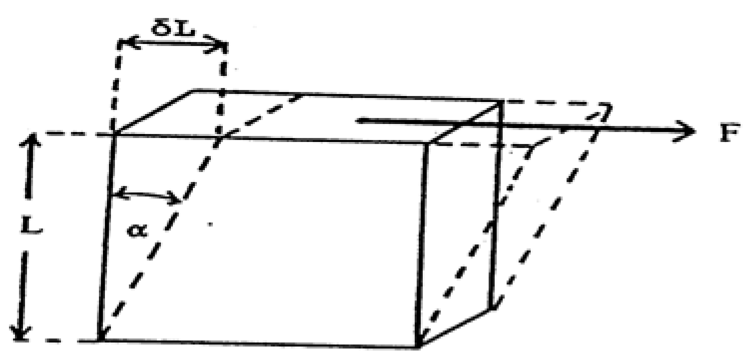


$$\frac{F}{A} = Y\frac{\delta L}{L} \tag{4.1}$$

The proportionality coefficient Y is called **Young's modulus**. It is a measure of the difficulty to stretch the material. On the microscopic level, the elongation of the solid cylinder corresponds to the shift of atoms along the cylinder under the action of the stress. When the stress is removed the atoms return to their original positions.

Another kind of deformation of the material is shown below. Under the action of force F

Figure 4.2: Illustration of an object under Shear



applied to the surface of area A (so called shear stress), the cube is deformed. The measure of deformation is strain ϵ.

$$\epsilon = \frac{\delta L}{L}$$

For $\delta L \ll L, \delta L/L = \alpha$, where angle α is in radians. For small values of stress, Hooke's Law can be applied

$$\frac{F}{A} = S\alpha \tag{4.2}$$

where the proportionality coefficient S is called the shear modulus. On a microscopic level the shear modulus corresponds to the shift of one layer of atoms with respect to adjacent layers. Values of Young's modulus for different materials are tabulated, see HRW p.317. It is important to remember that strain is linearly related to stress only for small values of strain (Hooke's Law). Materials strained beyond certain values do not return to their original dimensions when the stress is removed.

4.1.1 Prelab Exercise

Two quantities x and y are linearly related. A set of data points collected in the lab by a student is tabulated below:

x (kg)	0	5.2	8.8	12.6	14.6	22.5
y (cm)	0	13.0	22.3	28.9	37.2	52.3

Use the Method of Least Squares described in Appendix C to determine the slope and the intercept of the best straight line through the set of data points.

Show your calculations (compute $\Sigma x_i, \Sigma y_i, \Sigma(x_i)^2$ and $\Sigma x_i y_i$). Use graph paper to plot the best straight line and all experimental points.

4.2 Shear Modulus

4.2.1 Experimental

A torsion apparatus shown in Fig.4.3 is used to determine the shear modulus of a steel wire. The twisting of the wire leads to a shear as shown in Fig. 4.4.

A wire twists under the action of the torque $\tau = F_1 R = F_2 r$, where $F_1 = mg$ is the applied force, R is the radius of disc, F_2 is the force applied tangentially to the wire and r is the radius of the wire. According to equation 4.2

$$\frac{F_2}{A} = S\alpha \tag{4.3}$$

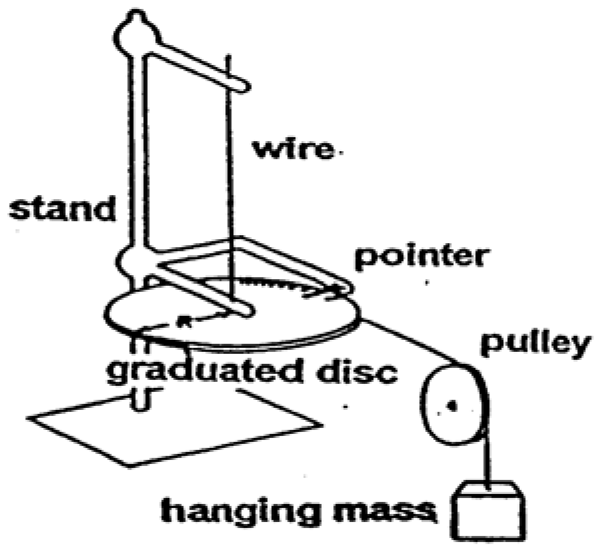
For with the cross-sectional area $A = \pi r^2$, and stress $\sigma = F_2/(\pi r^2)$. At the outside of the wire we can use some approximations and simple geometry Fig. 4.4 to obtain

$$\tan\alpha = \frac{\text{chord } \overline{AB}}{L} \approx \frac{P}{L}$$

Since $\tan\alpha \ll 1$, then $\alpha = P/L$. Also $P = \Psi r$ from the relation, arc length = angle x radius. Thus,

$$\alpha = \frac{\Psi r}{L} \tag{4.4}$$

Figure 4.3: Apparatus for studying torsion



The deformation of the wire is not identical along the radius of the wire. The above relation is correct only for the outer part of the wire. In the center of the wire the deformation is equal to zero. The average deformation can be taken as

$$\alpha_{av} = \frac{\Psi r}{2L} \tag{4.5}$$

Taking into account equations 4.3, 4.4 and 4.5 we obtain:

$$\frac{F_2}{\pi r^2} = \frac{S\Psi r}{2L} \quad \text{or}$$
$$F_2 r = S\frac{\pi r^4}{2L}\Psi$$

Taking into account the fact that the torque $\tau = F_1 R = F_2 r$, the above equation can be rewritten in the form

$$\tau = D\Psi \tag{4.6}$$

where

$$D = S\frac{\pi r^4}{2L} \tag{4.7}$$

Thus the shear modulus S can be determined from the slope D of the graph of the torque τ vs twist angle Ψ.

Taking into account equations 4.6, 4.7 the fact that the size of torque $\tau = mgR$, and converting angle from degrees to radians (angle in radians = $\pi/180$ x angle in degrees), the

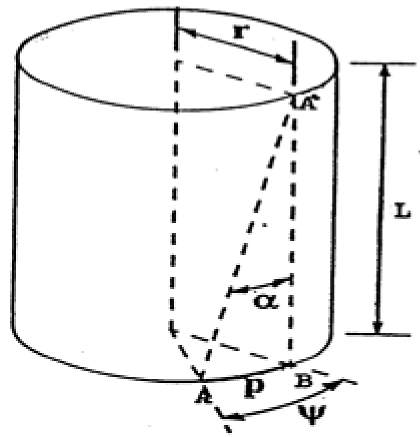


Figure 4.4: Detail of a twisting wire.A' used to be vertically above A. Twisting changes this. It is now above B. The twist angle is Ψ (angle subtended at the wire's centre by arc AB.) The causes a small shear angle α (α and Ψ measured in radians.)

following relation between the twist angle in degrees and the mass can be derived

$$\Psi = \frac{360LgR}{\pi^2 r^4 S} m$$

The slope W of this dependence is

$$W = \frac{360LgR}{\pi^2 r^4 S}$$

The shear modulus S can thus be calculated as

$$S = \frac{360LgR}{\pi^2 r^4 W}$$

where W is the slope determined from the graph.

4.2.2 Measurements and Calculations

1. Assemble the torsion apparatus as shown in Fig. 4.4.
2. Measure:
   - radius of the wire $(r \pm \delta r)$ m
   - length of the wire $(L \pm \delta L)$ m
   - radius of the disc $(R \pm \delta R)$ m

   Precise measurement of the radius r of the wire is very crucial as the shear modulus depends on the fourth power of the radius r (a 3% uncertainty in r becomes a 12.6% uncertainty in the shear modulus). Remember to close the micrometer's jaws to make a zero reading. The zero reading should be added or subtracted from the measurement of the radius r.

3. With no load attached to the wire, read the angle indicated by the pin marker. That is the zero reading.

4. Apply torque to the disc by hanging masses in increments of about $20g$ at the end of the rope passing through the pulley. For each mass measure the twist angle Ψ. Record your measurements in the table.

Hanging Mass m (kg)	Angle Ψ (deg)

5. Plot the graph of angle Ψ in degrees vs hanging mass m. Include the error bars. Remember that uncertainty δm is determined by the last significant digit written on the mass (see Section on Uncertainties 0.2.2). The shear modulus S can thus be calculated as

$$S = \frac{360LgR}{\pi^2 r^4 W}$$

where L is the length of the wire, r is the radius of the wire, R is the radius of the disc and W is the slope determined from the graph. Determine the shear modulus S and its uncertainty using the procedure described in Section 0.2.2. The procedure to determine the uncertainty δW of the slope W is described in section 0.3.5.

$$\frac{\delta S}{S} = \frac{\delta L}{L} + \frac{\delta R}{R} + \frac{4\delta r}{r} + \frac{\delta W}{W}$$

6. Verify if the measured value of the shear modulus S is consistent with the standard value $S_{standard}$, which will be provided in the lab. That is, verify if $|S_{standard} - S| \leq \delta S$.

4.2.3 Questions

Suggest modifications to the experimental set up used in the lab which would allow you to determine the shear modulus more precisely.

4.3 Young's Modulus

4.3.1 Experimental

Two wires support a sensitive leveling instrument. When the bar connecting the two wires is horizontal, a bubble should be seen in a window located at the center of the level. The micrometer screw serves to restore the horizontal position of the level when the wire is elongated under the stress. The difference between the initial and the final reading of the micrometer is equal to the elongation of the wire.

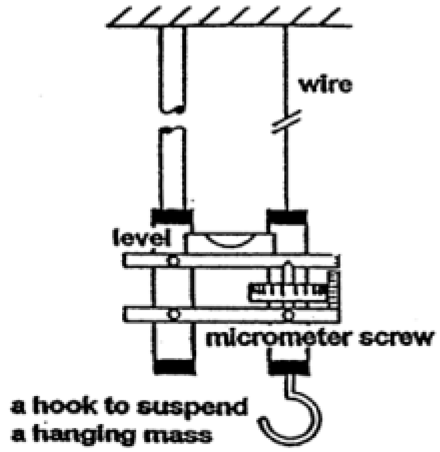


Figure 4.5: Instrument used to measure Young's modulus

4.3.2 Measurements and Calculations

In this part of the experiment you will use a computer to store your data and to do the desired calculations with the data. You, as in all experiments, are responsible for examining those calculations and drawing conclusions from them.

1. Use the micrometer caliper to measure the radius ($r \pm \delta r$) of the wire. This measurement is very crucial to determine Young's modulus precisely. Before using the micrometer screw, close its two jaws and make a zero reading. The zero reading should be added or subtracted from your measurement of the radius of the wire.

2. Measure the length ($L \pm \delta L$) of the wire between the two supports (two screws).

3. Adjust the leveling screw to make the level horizontal. Make a zero reading using the micrometer screw.

4. Add masses in $200g$ increments to the hook at the end of the wire. Each time adjust the levelling screw to make the level horizontal. The elongation δL of the wire is the difference between the reading obtained for a given mass and the zero reading.

5. Form the table:

Mass m (kg)	Elongation δL (m)

6. Taking into account equation 4.1 and the fact that $F = mg$ and $A = \pi r^2$, the following relation between elongation δL and applied mass m can be derived.

$$\delta L = \frac{gL}{\pi r^2 Y} m$$

The slope Z of the dependence of elongationδL vs mass m is

$$Z = \frac{gL}{\pi r^2 Y}$$

Young's modulus Y can thus be calculated as

$$Y = \frac{gL}{\pi r^2 Z}$$

7. Determine the Young's modulus using the above expression. To calculate the slope Z and its uncertainty δZ (for the plot of elongations δL versus mass m) you will use the Method of Least Squares described in Appendix C and a prepared MS Excel workseet on the desktop called "Elasticity.xlsx". Its operation should be self evident. Simply enter the data and the graphs are created and calculations are performed.

8. Determine the absolute uncertainty δY of Young's modulus using the equation

$$\frac{\delta Y}{Y} = \frac{\delta L}{L} + \frac{2\delta r}{r} + \frac{\delta Z}{Z}$$

   Verify if the measured value of the Young's modulus is consistent with the standard value, which will be provided in the lab. That is, verify if $|Y_{standard} - Y| \leq \delta Y$.

4.3.3 Questions

1. Which of the measured quantities contribute the most to the absolute uncertainty δY? How can δY be reduced?

Chapter 5

Standing Waves

Objective

1. study transverse standing waves in a rope and longitudinal standing waves in a spring
2. study sound standing waves in a tube

Apparatus Mechanical wave driver, function generator, ropes, springs, aluminum rod, Vernier computer interface, sound probe, computer, electronic balance, measuring tape and stands

5.1 Introduction

Interference is a fundamental property of waves. It occurs when two or more waves coexist in the same medium and produce a resultant wave. A particularly interesting interference occurs when two identical waves (of the same amplitude and frequency/wavelength) travel in the same medium in opposite directions. The superposition of these two waves under certain condition leads to creation of so-called standing waves. Standing waves can be created only in a medium of finite size, for example, in a rope or spring fixed at both ends or in a solid rod of a finite length. A simple harmonic wave traveling in the positive x-direction can be described by the following wave function:

$$y = A\sin(kx - \omega t)$$

In this wave equation A is amplitude, $k = 2\pi/\lambda$ is wave vector (in rad/m), and $\omega = 2\pi f$ is angular frequency (in rad/s). It is important to distinguish angular frequency $\omega = 2\pi f$ and frequency $f = 1/T$, where T is the period. The wavelength λ, frequency f, and speed v of the wave are inter-related through the equation

$$v = \lambda f$$

The speed of mechanical waves depends on the properties of the medium. For example, the speed of a transverse wave in a rope is given by $v = \sqrt{T/\mu}$, where T is the tension in the rope and $\mu = m/L$ is called linear mass density; m is the mass of the rope and L its length.

The speed of the longitudinal wave moving in the spring is $v = \sqrt{kL/\mu}$, where k is the spring constant per unit length of the spring (in units of N/m), L is the length of the spring, and $\mu = m/L$ is called linear mass density of the spring (m is the mass of the spring and L its length).

When two identical waves travel in opposite directions, they interfere and a new resultant wave is created. According to the superposition principle the net displacement of the resultant wave is equal to the algebraic sum of the displacements due to the individual waves. If the displacements of the two waves are given by equations:

$$y_1 = A\sin(kx - \omega t) \quad \text{and} \quad y_2 = A\sin(kx + \omega t)$$

the resultant wave is

$$y = y_1 + y_2 = A(sin[kx - \omega t) = sin(kx + \omega t)]$$

using the identity

$$\sin a + \sin b = 2\sin\frac{(a+b)}{2}\cos\frac{(a-b)}{2}$$

we obtain

$$y = 2A\sin(kx)\cos(\omega t)$$

The frequency of the resultant wave is the same as the frequency of the individual waves, but the amplitude $2A\sin(kx)$ is different and depends on x.

If the crests or troughs of the two waves traveling in opposite directions occur at the same positions, the waves interfere "constructively" and the resultant wave is bigger than the individual ones. When the crest of the first wave coincides with the trough of the second wave, then we have "destructive" interference between the waves.

The maximum amplitude occurs when $\sin(kx) = 1$, which means that

$$kx = \frac{n\pi}{2}, n = 1, 3, 5, ...$$

substituting

$$k = \frac{2\pi}{\lambda}$$

$$x = \frac{n\lambda}{4}$$

The positions at which the amplitude is a maximum are called **antinodes** and are separated by $\lambda/2$. The minimum amplitude occurs when $\sin(kx) = 0$, which leads to the condition

$$x = \frac{n\lambda}{4}, n = 1, 3, 5, ...\text{(for anti-nodes)}$$

Points corresponding to zero displacement are called **nodes**.

5.1.1 Prelab Exercsie 1

Explain the differences between travelling and standing waves.

5.2 Standing Wave in a Rope and a Spring

5.2.1 Transverse Standing waves in a rope

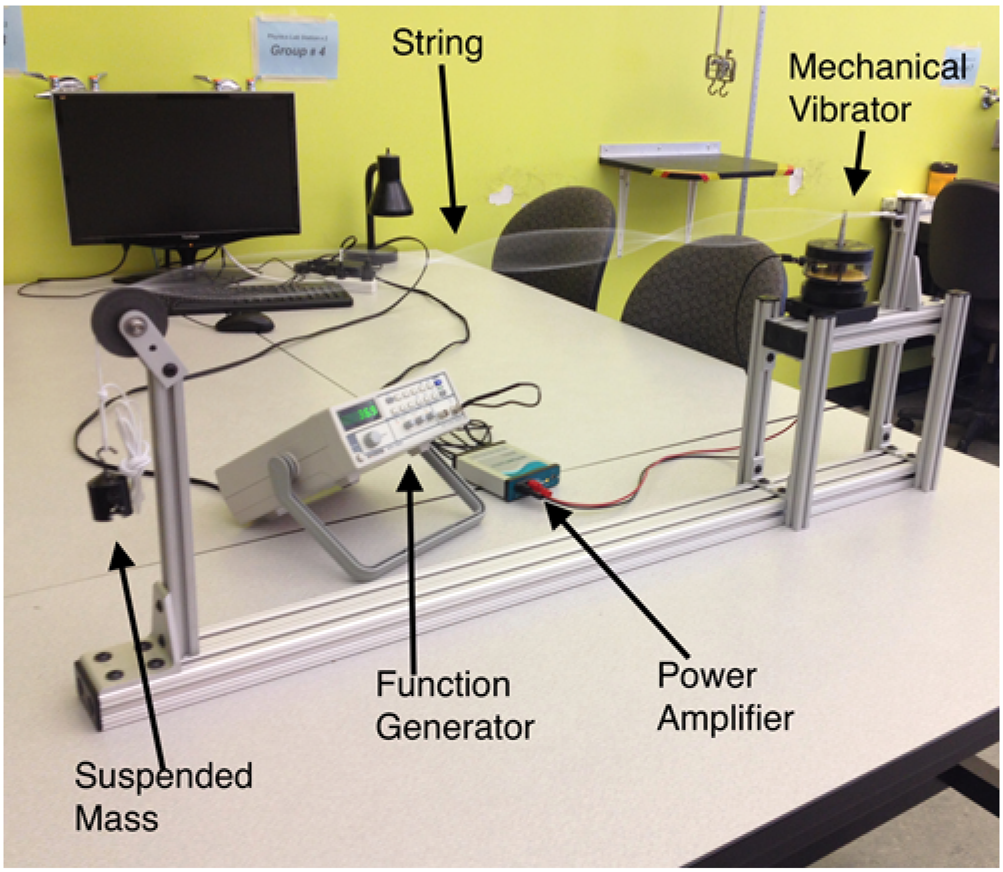


Figure 5.1: The experimental setup to produce standing waves in a rope

The suspended mass creates tension T in the rope, $T = M_{hanging}g$. A mechanical wave driver, whose frequency is controlled by the function generator, creates a harmonic wave at one end. The wave is reflected and inverted at the fixed end and returns to the source where it is reflected again. If the mechanical wave driver sends out a new crest just as the reflected crest reaches it, the new and reflected waves will reinforce each other. The two identical waves traveling in opposite directions interfere and for certain discrete frequencies form a standing wave.

Create at least four standing waves and measure their wavelength (remember that the distance between two adjacent nodes is $\lambda/2$).The frequency of the standing wave is displayed on the function generator.

Use the formula $v = \lambda f$ to determine speed v. Record the results in the table below.

	Wavelength λ (m)	Frequency f (Hz)	Speed $v = \lambda f$ (m/s)
standing wave 1			
standing wave 2			
standing wave 3			

Determine the mean speed of the wave and the deviation from the mean.

Calculate the speed of the wave using the formula $v = \sqrt{T/\mu}$, where tension $T = M_{hanging}g$ and linear mass density $\mu = m_{string}/L$ .The mass of the string can be measured using an electronic balance and length L using a ruler or a measuring tape.

Is the measured speed equal within an experimental uncertainty to the calculated value?

5.2.2 Questions

1. Imagine that you create a second harmonic in a short and long rope (both of the same mass density and with the same mass suspended at the end). Which of the three quantities: λ, f and v, would be larger, smaller or the same for both rope?
2. How should the amplitude of the standing wave depend on the frequency of the wave? Does it match with you observations? Can you think of some reasons why or why not?

5.3 Longitudinal Standing Waves in a Spring

Rearrange the standing wave apparatus as shown: Create at least four standing waves in a spring. Measure their wavelengths and record frequencies for which they occur. Use the formula $v = \lambda f$ to determine speed v. Record the results in the table below.

	Wavelength λ (m)	Frequency f (Hz)	Speed $v = \lambda f$ (m/s)
standing wave 1			
standing wave 2			

Determine the mean value of speed and the deviation from the mean. Calculate the speed of the wave using the formula

$$v = \sqrt{\frac{kL}{\mu}}$$

Where L is the length of the spring, k is the spring constant, and $\mu = m_{spring}/L$ is the linear mass density of the spring (m_{spring} is the mass of the spring and L its length). Measure m_{spring} using an electronic balance and L using a ruler or a measuring tape while it is installed in the apparatus. The spring constant can be determined in the same way

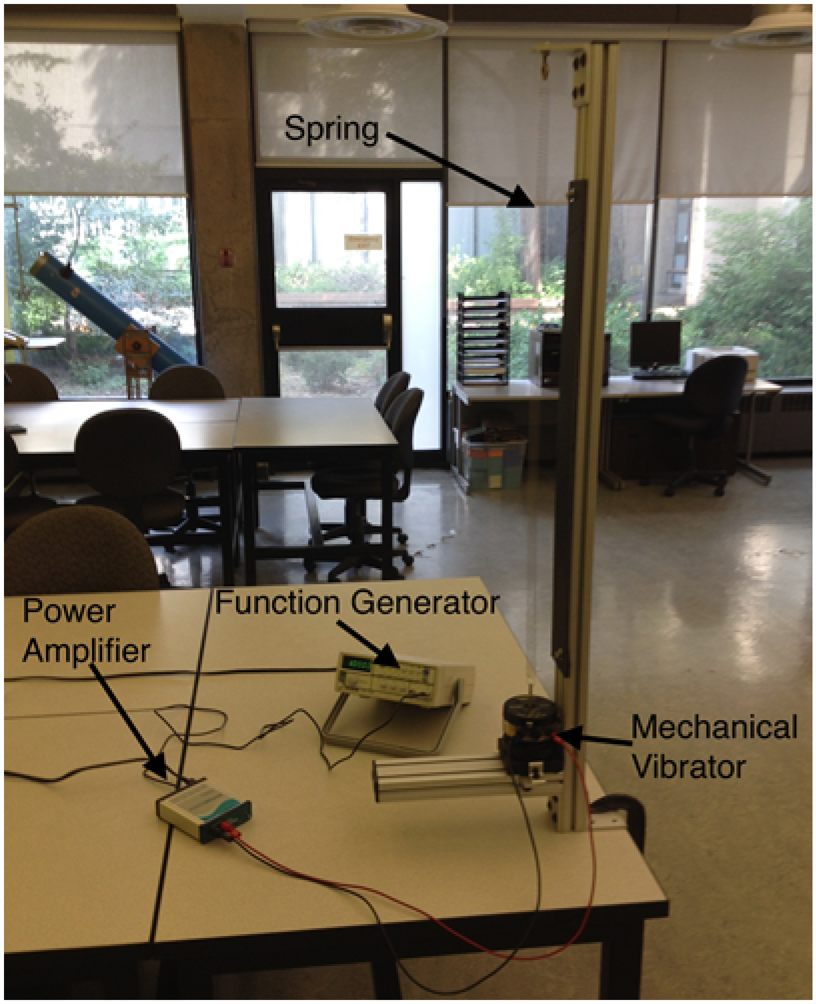


Figure 5.2: Apparatus for studying longitudinal waves in a spring

as in lab Oscillations, that is, by suspending mass $M_{hanging}$ at the end of the spring and measuring spring's elongation d so that $k = M_{hanging}g/d$.

Compare this theoretical calculation of v with your experimentally determined values.

5.3.1 Questions

Imagine that you create a second harmonic in two springs of the same length, and of the same linear mass density, but one soft and the other stiff. Which of the three quantities: λ, f and v, would be larger, smaller or the same for both springs?

5.4 Sound standing waves in the air tube

Sound waves are longitudinal waves. When a sound wave travels through the air, the air particles vibrate along the direction of motion of the wave. This results in series of high- and low-density regions of the air. The simplest kind of sound wave is called a "simple harmonic wave", which corresponds to a sinusoidal variation in the density, as shown in the figure below. Consider a simple harmonic wave generated by a loudspeaker, travelling

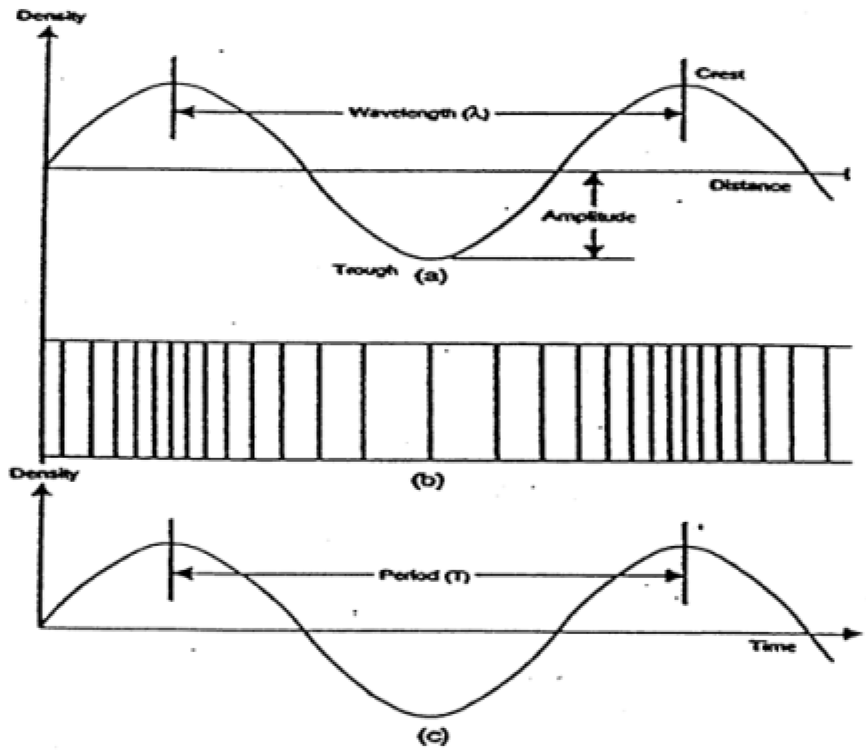


Figure 5.3: Illustration of transverse wave. Density is represented by variation of the shading lines (b). The higher density corresponds to more closely spaced lines. Note that density changes with distance (a) and time at a fixed position (c).

in a glass tube closed at one end. The wave will be reflected and inverted at the closed end of the tube. The reflected wave returns to the source and there it is reflected again. If the speaker is sending out a new crest just as the reflected crest reaches it, the new and reflected waves will reinforce each other and a standing wave will be produced.

The closed end of the tube is always a node, as the air particles at the closed end do not have the freedom to vibrate along the axis of the tube (sound is a longitudinal wave!). The open end of the tube is an antinode, as particles have complete freedom of motion. The wave which is created in the tube is called a **standing wave** and can occur only if the length of the tube is equal to an odd number of the quarter-wavelengths:

$$L = \frac{n\lambda}{4} \quad n = 1, 3, 5, ... \tag{5.1}$$

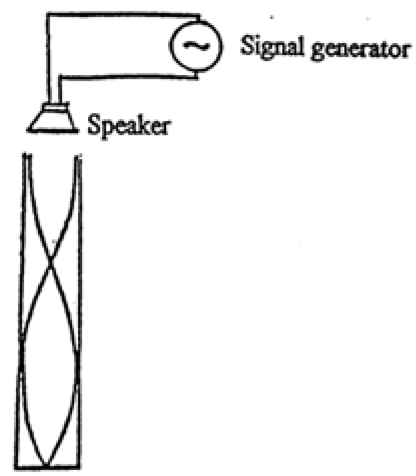


Figure 5.4: Acoustic standing wave in a tube

The wavelength λ, frequency f and the speed of sound v are related via $v = \lambda f$. Note that the n in this equation is not the number of nodes. You will experimentally determine λ and since you will know f can then find the speed of sound v. Note how standing waves are located.

5.4.1 Prelab Exercises 2

The speed of sound in air at your location is $343m/s$. Compute the minimum length of tube, open at one end, you will need to set up the first standing wave for sound with a frequency of $1200Hz$.

5.4.2 Experimental

The experimental set up is shown below.

The length of the air column in the glass tube may be varied by changing the position of the piston. A simple harmonic wave is produced by the loudspeaker attached at the open end of the glass tube. The loudspeaker is driven by the signal generator. A small microphone attached at the top of the tube is connected to an oscilloscope. The signal from the microphone is amplified and displayed on the screen of the oscilloscope. Resonance occurs when the maximum amplitude signal is seen on the oscilloscope.

5.4.3 Measurements and Calculations

1. Power on the oscilloscope, function generator, and microphone.

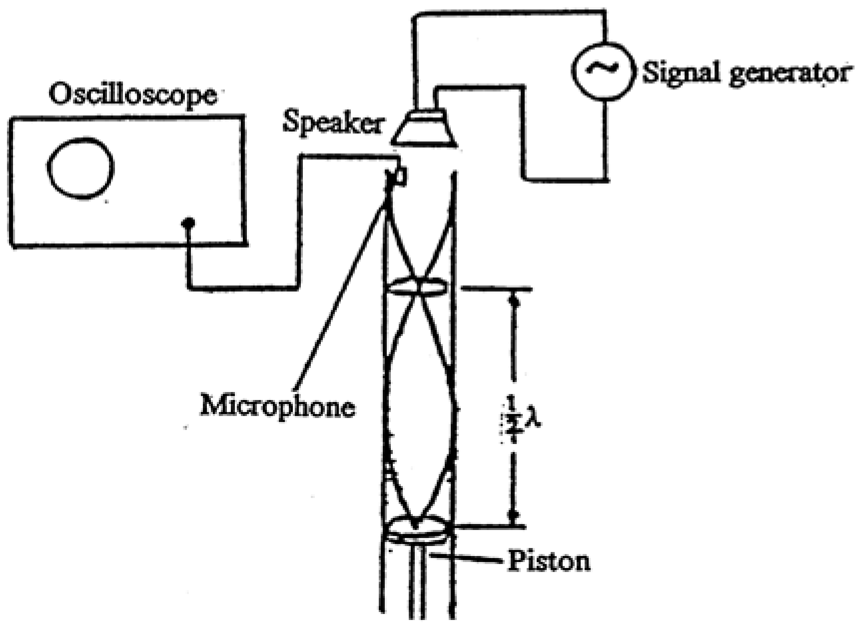


Figure 5.5: Experimental setup for studying acoustic standing waves.

2. Adjust the frequency of the signal generator to about 700 Hz, and the amplitude to low level sound (**barely audible**).
3. Move the piston in the tube starting at the end closest to the speaker until you hear a substantial increase in the sound intensity (maximum amplitude on the screen of the oscilloscope). That corresponds to the first resonance (formation of a standing wave). Find other resonances in the tube.
4. You will record data in a table. A fillable copy of this table is included in Appendix **??** for your convenience.
5. Draw wave envelopes, as in Fig. 5.5, for the first three standing waves obtained for frequency 700 Hz.
6. Adjust the frequency of the signal generator now to 1400 Hz and record all the positions of the piston for which a high intensity sound is detected at the open end of the tube. Enter your positions in the table above and repeat the same calculations as before.
7. Calculate the average value of the velocity v and the average deviation σ (see Appendix A - Statistical Treatment of Random Errors). In your calculations use all the velocities you obtained for both frequencies since the speed of sound does not depend on the frequency.
8. Compute the percentage difference between the speed of sound you determined experimentally and the standard value for the speed of sound, 343 m/s (at room temperature and zero altitude).

Frequency 0 (Hz)	Resonance lengths (m)	Difference in resonance lengths = $\frac{\lambda}{2}$ (m)	Wave Length λ (m)	Velocity v = $\lambda f \left[\frac{m}{s}\right]$
	L_1 =	L_2 - L_1 =		
700	L_2 =	L_3 - L_2 =		
	L_3 =	L_4 - L_3 =		
	. . .	. . .		

5.4.4 Questions

1. Explain why speed of sound does not depend on the frequency of sound wave?
2. Explain how the speed of sound can be found keeping the piston at the same position and changing the frequency of the sound wave with the signal generator?

Table 5.1: Transverse standing waves in a String- Data and Speed Calculation

	Wavelegnth λ (m)	Frequency f (Hz)	Speed $v = \lambda f$ (m/s)
standing wave 1			
standing wave 2			
standing wave 3			
standing wave 4			

Calculations:

Mean Speed $\overline{v} =$

Average Deviation: $\delta v = \sigma = \sum_i |v_i - \overline{v}|/N =$

Table 5.2: Longitudinal Standing waves in a Spring- Data and Speed Calculation

	Wavelength λ (m)	Frequency f (Hz)	Speed $v = \lambda f$ (m/s)
standing wave 1			
standing wave 2			
standing wave 3			
standing wave 4			

Calculations:

Mean Speed $\overline{v} =$

Average Deviation: $\delta v = \sigma = \sum_i |v_i - \overline{v}|/N =$

Table 5.3: Acoustic standing waves in a hollow cylinder- data and calculations

Frequency	Resonance Lengths (m)	Difference in Resonance Lengths = $\lambda/2$ (m)	Wavelength λ (m)	Velocity $v = \lambda f$ (m/s)
700 Hz	$L_1 =$	$L_2 - L_1 =$		
	$L_2 =$	$L_3 - L_2 =$		
	$L_3 =$	$L_4 - L_3 =$		
	$L_4 =$	$L_5 - L_4 =$		
	$L_5 =$			
1400 Hz				

Calculations:

Mean Speed $\overline{v} =$

Average Deviation: $\delta v = \sigma = \sum_i |v_i - \overline{v}|/N =$

Chapter 6

Thermal Physics: Thermal Expansion & the Heat Engine

Most materials expand when heated. A typical metal expands only about few percent of the original length when its temperature changes even hundreds of degrees. This small change, however, has to be taken into account when designing instruments or structures exposed to large temperature changes.

Heat engines were at the heart of the Industrial Revolution. The theoretical foundation of the operation of the heat engine was laid down by Nick Carnot in 1824. A heat engine can be defined as a cyclic device that converts thermal energy into work. Various thermodynamic processes are involved in each cycle. Volume, pressure and temperature of the gas change in these processes.

Objective

1. To study thermal expansion of different metals.
2. To study the thermodynamic processes involved in the operation of a heat engine

Apparatus Thermal expansion apparatus, digital multi-meter, thermometer, steam generator, cylinder/piston system, air chamber, hot plate, beakers, weights

6.1 Thermal Expansion

6.1.1 Introduction

We will limit our consideration of the thermal expansion to long tubes made of metals. For such tubes the change in length (ΔL) depends on the change in temperature (ΔT) it experiences; that is, ΔL is proportional to ΔT. For the same change of temperature, a long tube will change its length more than a short one. Thus, we expect that ΔL is proportional to L, where L is the original length of the tube. The two proportionalities can be made into an equation by introducing a constant of proportionality, in this case α:

$$\Delta L = \alpha L \Delta T$$

Where, α is called the temperature coefficient of linear expansion and ΔT can be in either degrees Celsius or kelvins, since the change in temperature is numerically the same for both.

From a microscopic point of view, thermal expansion is explained in the following way. Heating the material causes its atoms to vibrate with greater amplitudes. As thermal energy is added to the material, the equilibrium positions of the rapidly oscillating atoms separate and the solid gradually expands. The weaker the inter-atomic cohesive forces, the greater are the displacements from their equilibrium separations.

Nearly all materials have positive values of the temperature coefficient of linear expansion α. A few important materials are listed in the table below.

Material	Coefficient $\alpha(K^{-1})$
Aluminum	25.0×10^{-6}
Brass	18.9×10^{-6}
Copper	16.6×10^{-6}
Lead	29.0×10^{-6}
Steel (structural)	12.0×10^{-6}
Concrete	12.0×10^{-6}
Water	69.0×10^{-6}
Glass (pyrex)	3.2×10^{-6}

Note that the values in the table above are for *linear expansion*. Generally, the *volume* will expand at a rate of 3 times the linear rate.

Lead, with relatively weak inter-atomic bonds, is soft (has low Young's Modulus), melts at a relatively low temperature (327 °C) and has a high temperature coefficient of linear expansion.

Some ceramics have values of temperature coefficient of linear expansion α near zero or even negative.

6.1.2 Prelab exercise 1

1. The roadway of the Golden Gate Bridge is 1320 m long and it is supported by a steel structure. If the temperature varies from -30 °C to 39 °C, how much does the length change? The result indicates the size of the thermal expansion joints (shown below) that must be built into the structure. Without the joints the surfaces would buckle on very hot days or crack due to contraction on very cold days.

Figure 6.1: Thermal expansion joints in a roadway.

2. Give at least two additional examples where the thermal expansion must be taken into account when designing structures or devices.

6.1.3 Experimental

The thermal expansion apparatus is shown below. It consists of a base with built-in dial gauge and a thermistor, which is a small piece of semiconductor material. The resistance of the thermistor varies strongly with temperature. It can be measured with an ohmmeter and converted to a temperature using the conversion table affixed to the base of the apparatus. Hot steam can be fed directly through the tube to increase the tube's temperature.

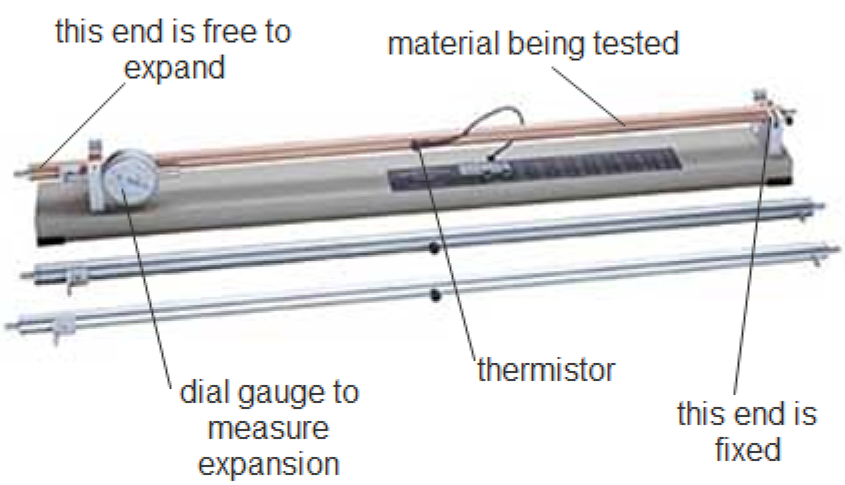


Figure 6.2: Thermal expansion apparatus.

6.1.4 Measurements and Calculations

1. Measure the length of the tube L from the steel pin to the inner edge of the angle bracket as shown in Fig. 6.2.
2. Mount the tube in the base. The steel pin on the tube fits into the slot on the slotted mounting block and the bracket on the tube presses against the spring arm of the dial gauge. Drive the thumbscrew against the pin until the tube can no longer be moved.
3. Attach the themistor lug to the threaded hole in the middle of the tube (the themistor is embedded in the thermistor lug). Align the lug with the tube to maximize the contact between the lug and the tube, as shown below. Place the foam insulator over the thermistor lug.
4. Plug the leads of the digital ohmmeter into the banana connectors in the center of the base. Measure the resistance of the thermistor at room temperature R_{RT} and convert it to temperature T_{RT}.
5. Use flexible tubing to attach the steam generator to the end of the tube. Raise the end of the base at which the steam enters the tube. This will allow any water that condenses in the tube to drain out. Place a container under the other end of the tube to catch the draining water.
6. Turn on the steam generator. When the thermistor resistance stabilizes, record the resistance R_{hot} and convert it to temperature T_{hot}. Measure the expansion of the tube ΔL (each increment on the dial gauge is equivalent to 0.01 mm).
7. Using the equation $\Delta L = \alpha L \Delta T$, calculate α, its uncertainty and the percentage difference.

6.1.5 Questions

1. Discuss possible sources of uncertainty in the experiment. How might you improve accuracy of the experiment?
2. Small gaps are left between railroad tracks sections of 10 meter length. Explain why. Estimate a reasonable size of the gap?

6.2 The Heat Engine

6.2.1 Introduction

The operation of the heat engine involves thermodynamic processes in which volume of gas V, pressure p and temperature T can change. For an ideal gas (the air can be approximately treated as one), the three thermodynamic parameters: V, p and T, are inter-related: if one of the parameters change, then at least one of the remaining two must change as well.

In 1662 Robert Boyle, discovered that keeping the temperature constant, the volume of a gas varies inversely with pressure: $pV = b$, where b is a constant.

In 1787 Jacques Charles discovered that for constant pressure, all gases increase volume when the temperature increases: $V = cT$, where c is a constant.

In 1802 Joseph Gay-Lussac observed that when the volume is kept constant, the absolute pressure of a given amount of any gas varies with temperature: that is, $p = aT$, where a is a constant.

The three laws described above are specific cases of the Ideal Gas Law.

$$pV = nRT$$

where n is the number of moles and R is the Universal Gas Constant $R = 8.31$ J/mol K. For example, if T is constant, then the product nRT is constant. Thus, pV = constant (Boyle's Law).

A thermodynamic process occurs when a system (air in our experiment) changes from one state (one set of p,V and T) to another state (different set of p, V and T). The system can be changed in a variety of ways. Four basic processes are: isothermal (T constant), isobaric (p constant), isochoric (V constant) and adiabatic (no heat is transferred from or to the system; processes which occur suddenly tend to be adiabatic because heat takes a fair amount of time to flow). The thermodynamic processes involving vapours and gases are particularly important, for example, in the operation of steam engine, automobile engine, the jet propulsion airplane engine.

The work done by a heat engine can be evaluated in a particularly simple way for the situation when the system (cylinder of gas) expands against an external pressure, changing its volume (and doing work). Let's assume that a piston of cross-sectional area A is frictionless and weightless. The downward force F (for example, weight mg of a mass m put on top of the piston) is exactly opposed by an upward force produced by the gas: $F = pA$. If the gas is heated, as shown below, the volume increase but the pressure remains the same (isobaric process).

If the displacement of the piston is ΔL, the work done on the surrounding (the atmosphere) by the expanding gas is

$$W = F\Delta L = pA\Delta L = p\Delta V$$

where $\Delta V = A\Delta L$ is the change in volume. When the gas expands, ΔV is positive and the work W is positive. When the gas contracts, ΔV and W are both negative.

The pressure versus volume (or p-V) diagram for an isobaric expansion is shown below. Please note that this is a very crude approximation. The straight lines going from points I to F, F to V_F etc. are curved!

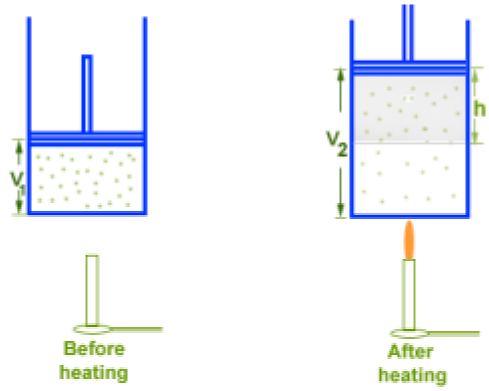


Figure 6.3: An example of an isobaric process.

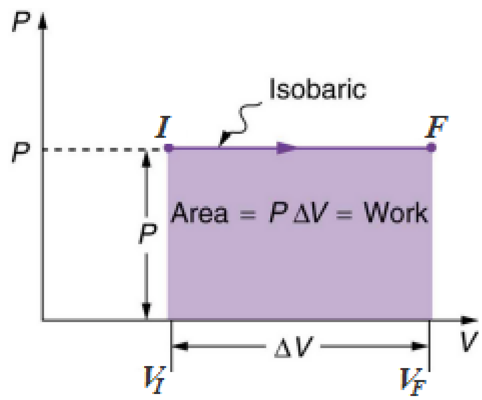


Figure 6.4: Pressure vs Volume plot for an isobaric expansion.

From an initial state "I", the system was transformed at a constant pressure to a different final state "F". Notice that the work done, $p\Delta V$, is the area under the curve. If the pressure varies as the volume changes, the area under the pV curve will still correspond to the work done.

Real engines use so called cyclic thermodynamic processes. After several processes the system returns to the original state and then the process is repeated over and over again. The cyclic process corresponds to a closed figure on the pV diagram. The heat engine transforms the substance (air) through a cycle in which heat is absorbed from a source at high temperature, then work is done by the engine and finally heat is expelled by the engine to a source at lower temperature.

An important characteristic of a heat engine is its efficiency e defined in the following way.

$$e = W/Q$$

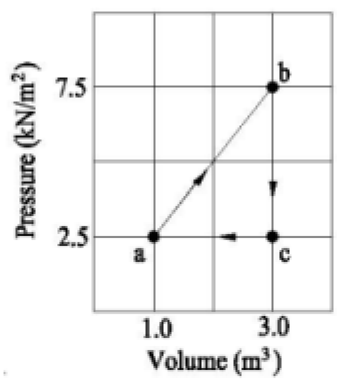
where W is the work done by the engine and Q is the heat delivered to the engine (the ratio of what we get out to what we put in). In practice, heat engines convert only a fraction of the absorbed heat into work (e = 20 - 40%). It can be shown (see the textbook, chapter 21) that the maximum efficiency of a heat engine can be expressed in terms of temperatures.

$$e = \frac{T_H - T_L}{T_H}$$

where T_H is the temperature of the hot reservoir and T_L is the temperature of low temperature reservoir (both in kelvins). This expression is independent of the engine's working fluid. It applies, for example, for an internal combustion auto engine or a steam engine.

6.2.2 Prelab Exercise 2

A sample of an ideal gas is taken through the cyclic process $a - b - c - a$, as shown blow. The temperature of the gas at point a T_a = 298 K.



- How many moles of gas are in the sample?
- What is the temperature of the gas at point b?
- What is the work done by gas during the complete cycle?
- How much heat was added to the gas during the cycle?

6.2.3 Experimental

A cylinder/piston system used in this experiment is shown below. The graphite piston is nearly frictionless and the leakage of air around it is quite negligible. Air from the air chamber (a can) flows through flexible tubing and a valve to the cylinder. The pressure of gas can be changed by placing a mass on the platform on the top of the piston. The temperature of the gas is changed by submerging the air chamber in a beaker containing hot or cold water.

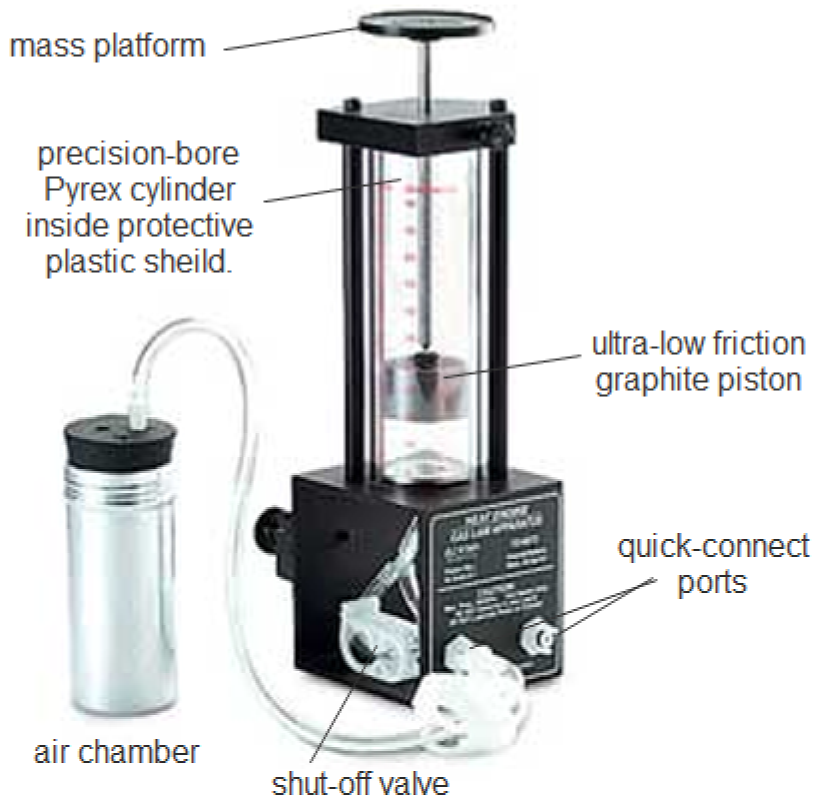


Figure 6.5: Heat Engine Apparatus.

6.2.4 Measurements and Calculations

To verify Charles' Law ($V = cT$) do the following:

1. Connect the air chamber by flexible tubing to one of the valves and close the other shut-off valve. Turn the apparatus on its side, as shown on the next page. In this position, the force acting on the gas is the atmospheric pressure and is equal throughout the range of operation of the piston.

2. Submerge the air chamber in the water contained in the beaker. Heat the water using a hot plate. Record the position of the piston at different temperatures. Plot the dependence of volume V as a function of temperature. The diameter of the piston is $0.0325m$. Does your graph verify Charles' Law? Explain.

The next lab activity is related to operation of a **heat engine**. The closed cycle will be formed by four thermodynamic processes: two isobaric and two isothermic. The area of the closed loop on the pV graphs is equal to work done by the engine.

1. Raise the piston a few centimetres and then connect the flexible tubing to a shut off valve (close the other valve at the same time). The engine cycle is much easier to describe if you begin with the piston resting above the bottom of the cylinder.

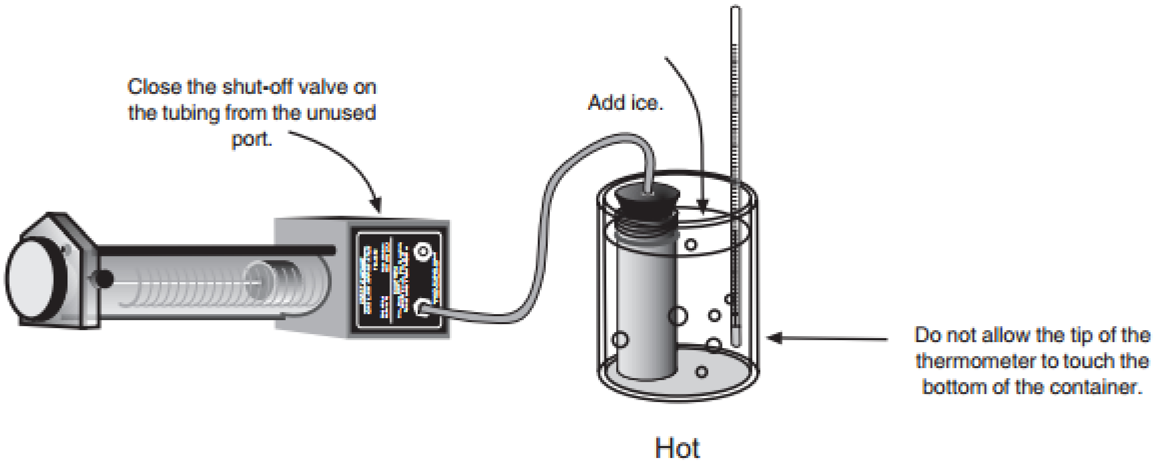


Figure 6.6: Charles' Law set-up.

2. The sequence of operations to follow to complete one engine cycle is illustrated below. These operations, described in words below, *should be completed very quickly to avoid air leakage around the piston.* After each operation, record the position of the piston.

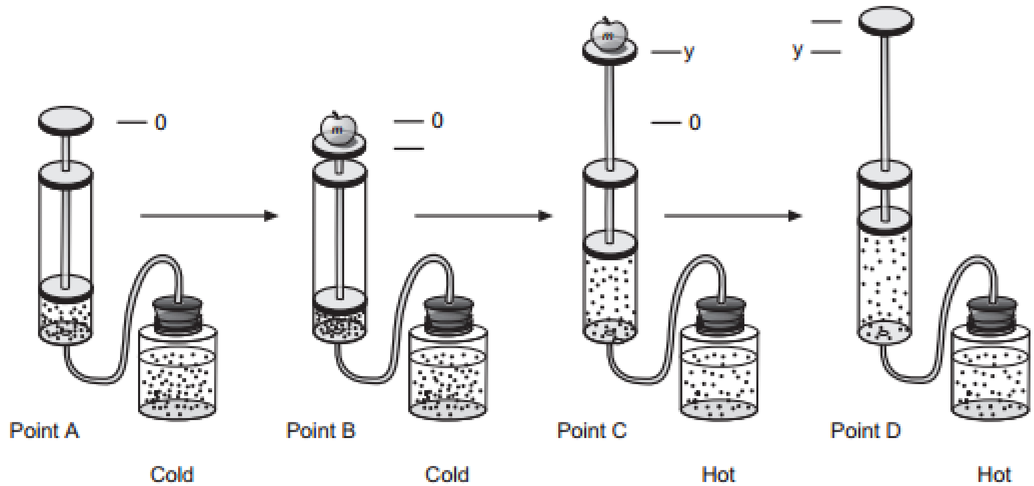


Figure 6.7: A simplified diagram of the mass lifter heat engine at different stages of its cycle.

(a) Put the air chamber into the cold water.

(b) Add mass $M = 200$ g to the platform (on top of the piston).

(c) Place the air chamber in the hot water. The piston should move upward. **This is the engine power stroke**. The engine did work equal to the increase of potential energy $U = (m + M)gh$, where h is the distance moved by the piston.

(d) Remove the mass M from the platform and record the new position of the piston.

The pressure is now smaller and the piston probably moved higher.

Now place the air chamber back in the cold water. The piston should return to the original position (the cycle is completed).

For each of the steps (a –d), calculate volume $V = \pi r^2 h$, where $r = d/2$ (diameter of the piston $d = 0.0325$ m) and h is the position of the piston and pressure $p = (m + M)g/A$, m is the mass of the piston (m= 0.035 kg), M is the additional mass on the platform and $A = \pi r^2$ is the cross-sectional area of the piston.

Your four points on the pV graph should look as shown below.

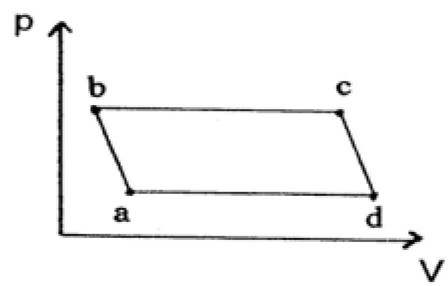


Figure 6.8: Expected pV graph.

Connect the points by straight lines. The area enclosed is then approximately a parallelogram. The thermodynamic process a-b and c-d are isothermal (performed at constant temperature), while processes b-c and d-a are isobaric (at constant pressure due to constant mass M on the platform). By connecting the four points by straight lines we are making an approximation (for small changes of V and p this approximation is quite acceptable).

3. Calculate the thermodynamic work done by the engine. It is equal to the area of the parallelogram. In order to obtain work in joules (J) the units of pressure p should be in Pa (N/m^2) and volume V in m^3. How does this work compare to the increase of the potential energy $U = Mgh$?

4. Determine the efficiency of your heat engine. It depends only on the temperature of the hot and cold water reservoirs.

6.2.5 Questions

1. Several factors contributed to the uncertainty in your heat engine measurements. List these factors and discuss their importance.

Table 6.1: Linear Thermal expansion of a metal tube.

Tube 1 Material:		
Original Length: $L_1 \pm \delta L_1 =$		
Condition	**Thermistor Resistance $\pm$ uncertainty (units)**	**Temperature $\pm$ uncertainty (units)**
Room Temperature	$R_{RT,1} =$	$T_{RT,1} =$
Hot	$R_{H,1} =$	$T_{H,1} =$
–	–	$\Delta T = T_{H,1} - T_{RT,1} =$
Expansion: $\Delta L_1 \pm \delta(\Delta L_1)$		
$\alpha_1 = \frac{\Delta L_1}{L_1(T_{H,1} - T_{RT,1})}$		
Uncertainty: $\frac{\delta \alpha_1}{\alpha_1} = \frac{\delta L_1}{L_1} + \frac{\delta(\Delta L_1)}{\Delta L_1} + \frac{(\delta(\Delta T))}{\Delta T}$		
Percentage Difference of $\alpha_1 =$		

Table 6.2: Linear Thermal expansion of a metal tube.

Tube 2 Material:		
Original Length: $L_2 \pm \delta L_2 =$		
Condition	**Thermistor Resistance ± uncertainty (units)**	**Temperature ± uncertainty (units)**
Room Temperature	$R_{RT,2} =$	$T_{RT,2} =$
Hot	$R_{H,2} =$	$T_{H,2} =$
–	–	$\Delta T = T_{H,2} - T_{RT,2} =$
Expansion: $\Delta L_2 \pm \delta(\Delta L_2)$		
$\alpha_1 = \frac{\Delta L_2}{L_2(T_{H,2} - T_{RT,2})}$		
Uncertainty: $\frac{\delta\alpha_2}{\alpha_2} = \frac{\delta L_2}{L_2} + \frac{\delta(\Delta L_2)}{\Delta L_2} + \frac{(\delta(\Delta T))}{\Delta T}$		
Percentage Difference of $\alpha_2 =$		

Table 6.3: Parameters of a heat engine during a complete cycle. The piston has a mass of 0.035 kg and an area $A_{piston} = \pi r^2$ where the radius r 0.0163 m.

Step	**Condition**	**Temperature of water ± uncertainty**	**Height of piston ± uncertainty**	**Volume** $V_i = A_{piston} \times H_i$	**Pressure** $P = F/A$
Initial	Room temperature	$T_0 =$	$H_0 =$	–	–
(a)	Cold water	$T_1 =$	$H_1 =$	$V_1 =$	$P_1 = \frac{mg}{A_{piston}} =$
(b)	Cold water + 200g	$T_2 =$	$H_2 =$	$V_2 =$	$P_2 = \frac{(m+M)g}{A_{piston}} =$
(c)	Hot water + 200g	$T_3 =$	$H_3 =$	$V_3 =$	$P_3 = \frac{(m+M)g}{A_{piston}} =$
(d)	Hot water remove 200g	$T_4 =$	$H_4 =$	$V_4 =$	$P_4 = \frac{mg}{A_{piston}} =$
(a)	Cold Water	$T_5 =$	$H_5 =$	$V_5 =$	$P_5 = \frac{mg}{A_{piston}} =$

Chapter 7

DC Electricity

In this laboratory session we shall examine the relationships among the motion of electrons (current), the effect which causes them to move (voltage, i.e. potential difference), and the frictional effects which hinder their motion (resistance). Since we cannot use any of our five senses to detect the presence of electrical effects we need one or more detectors to tell us that a current or potential difference exists.

Objectives

1. To learn how to make circuit connections and how to use electrical equipment.
2. To study the principles of operation of a voltage divider, a potentiometer and a bridge.

Apparatus Digital multimeter, batteries, resistors, potentiometers, thermistors, switches, connecting wires.

7.1 Ohm's law. Resistors in series and in parallel

Conservation of charge, mobility of charge, and conservation of energy are the fundamental aspects of this section.

7.1.1 Introduction

An electric current is the flow of electric charges. Batteries are a source of current which flows always in the same direction. This kind of current is called a direct current (abbreviated DC). There are other sources of current which continually reverse the direction of current. Those currents are called alternating currents (abbreviated AC). In this lab you will deal only with DC currents. By convention, the current direction is from positive (higher potential) through the circuit to negative (lower potential) terminal of the battery.

Circuit elements which offer resistance to current are called resistors. According to Ohm's law, current I which flows through resistance R causes an electric potential difference V = IR across the resistor (decrease in electrical energy).

The instrument used to measure current is called an ammeter. The ammeter should be connected in series with the element through which current is measured (Fig. 7.1). Only then does the charge which flows through the element also flow through the ammeter. The internal resistance of the ammeter is very low, so that only a very tiny voltage drop occurs across the ammeter. A voltmeter is an instrument to measure the voltage across an element of the circuit. The voltmeter must be connected in parallel with the given element. This ensures that the voltage drop across the voltmeter is the same as that across the given circuit element. The internal resistance of the voltmeter is very LARGE so that very LITTLE current passes through it. A voltmeter reading is usually made simply by touching two points with the voltmeter leads. (Fig. 7.1). When using an ammeter or a voltmeter in

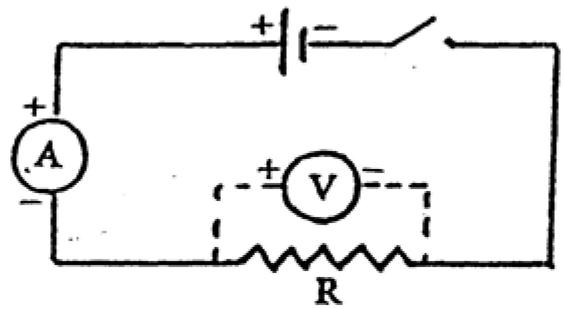


Figure 7.1: Schematic of connecting and ammeter and voltmeter.

DC circuits it is important to pay attention to the polarity of the instruments. The positive (+) side of the voltmeter and the ammeter must be closest to the positive (+) side of the battery.

7.1.2 Experimental

To measure current, voltage and resistance you will use a modern instrument called a digital multimeter. A multimeter is a solid state electronic instrument with a digital display. It can measure current, voltage or resistance. It has a very high internal resistance, $10 M\Omega$ so that practically no current flows through it when connected as a voltmeter. A rotary switch enables one to select the appropriate mode of operation. Fig 7.2 shows the position of the rotary switch and connection of two wires to measure DC voltage, DC current and resistance.

7.1.3 Prelab Exercise

An electrical schematic diagram of DC circuit with two resistors in parallel is shown below. Using the pictorial representation of all the components (shown below) of the schematic

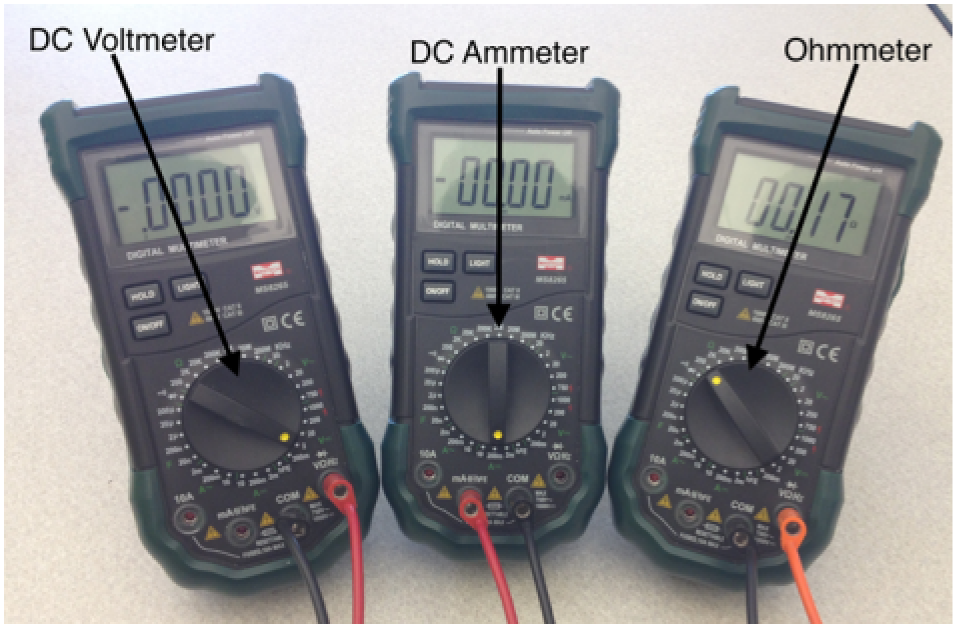


Figure 7.2: Digital multimeter operation.

circuit, connect these components together in pencil (easy to erase) by curved lines which will represent your wires.

Do not use more than 9 wires. The battery, switch and three multimeters cannot have more than one wire attached to each terminal. Two or more wires can be connected to ends of resistors.

Please photocopy or reproduce the diagram and hand it in to your lab instructor.

7.1.4 Measurement and Calculations

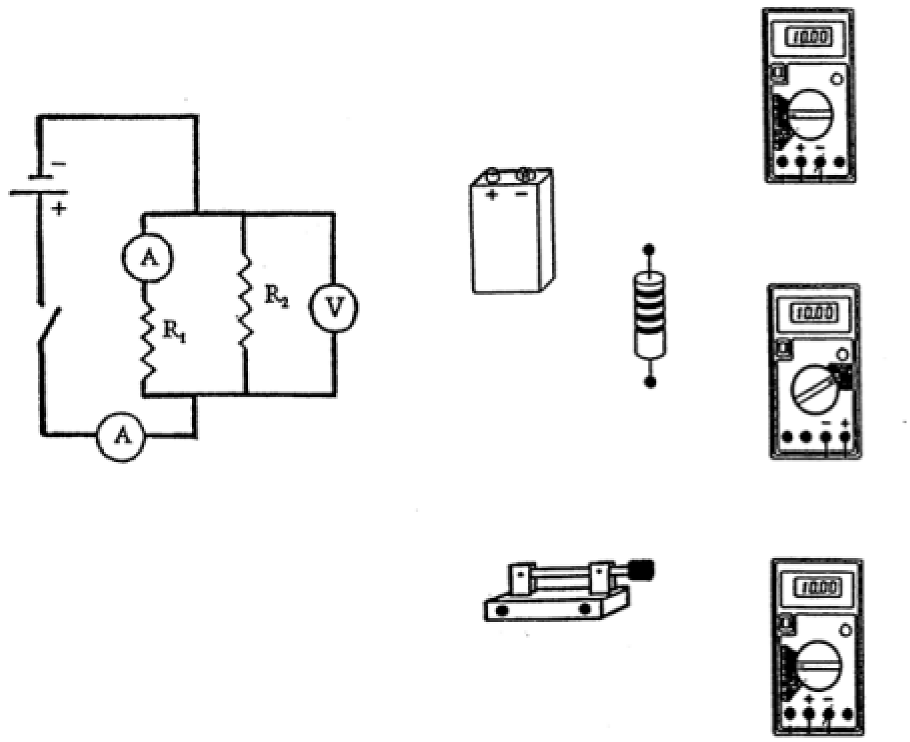
You will be using a solderless breadboard for connecting your circuit. This provides a convenient an clean way of making electrical connections. To understand how the board is wired, refer to the following figures:

Be sure to return the electrical component to the plastic storage box after use. If a component is damaged or missing, please inform the TA so it can be replaced.

1. Connect the circuit: In this circuit two resistors R_1 and R_2 are connected in series.

2. Measure current I at two different point of the circuit as indicated in Fig.7.6. Is the current the same?

3. Measure the voltage V_{ab} across R_1, voltage V_{cd} across R_2, and V_{ad} across both resistors. To make voltage measurements touch the voltmeter leads to appropriate points.



4. Use Ohm's law to calculate

$$R_1 = \frac{V_{ab}}{I}\ ,\ R_2 = \frac{V_{cd}}{I},\ R_s = \frac{V_{ad}}{I}$$

   Verify that $R_s = R_1 + R_2$. Check the colour code chart shown below to determine the resistance R_1 and R_2 (based on conservation of energy).

   To use the colour code chart correctly you have to match the colour of the band with the digit associated with this band. For example, if the first band would be red, the second - green, the third - red and the fourth - silver, then the value of the resistance would be $25 \times 10^2 = 2500\Omega$ and its precision 10%. (i.e. $0.1 \times 2500 = \pm 250\Omega$). Verify that the measured resistances are within the precision limits given by the colour codes.

5. Connect the circuits in which resistors R_1 and R_2 are in parallel (side-by-side). Start with the circuit (a), the proceed to (b) and (c). The best way to connect these circuits is to first connect the loop drawn with a thick line in each case and then attach the second resistor.

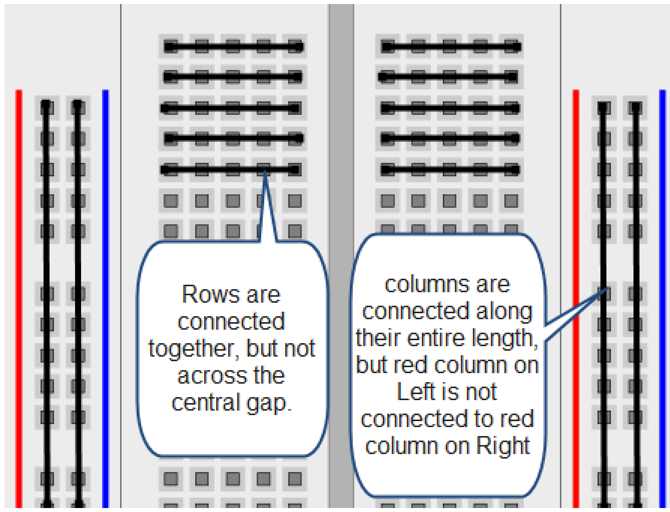


Figure 7.3: Solderless breadboard layout

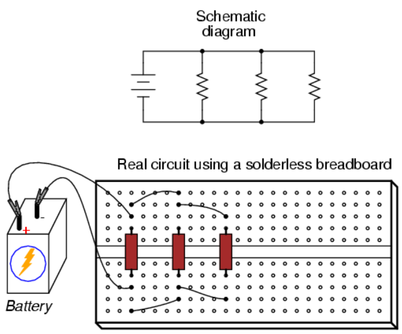


Figure 7.4: Solderless breadboard parallel example

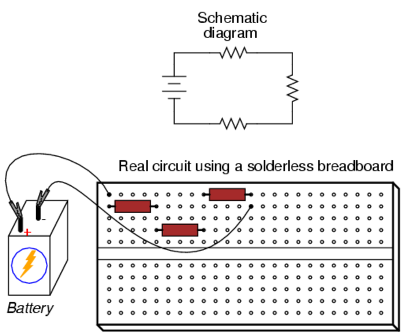


Figure 7.5: Solderless breadboard series example

6. Measure currents I (Fig.7.8a), I_1 (Fig.7.8b), I_2 (Fig. 7.8c). How are the currents related?

7. Measure the voltage V between points A and B i.e. across R_1 and R_2 in parallel. Use any of the circuits in Fig. 7.8. The circuits are equivalent as the ammeter has very

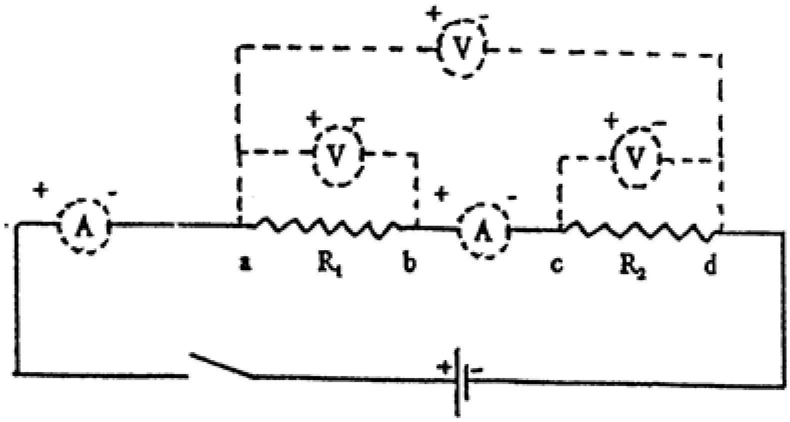


Figure 7.6

BAND 1	BAND 2	BAND 3 MULTIPLIER	BAND 4 TOLERANCE
0 BLACK	0 BLACK	BLACK × 1	NONE + or - 20%
1 BROWN	1 BROWN	BROWN ×10	SILVER + or - 10%
2 RED	2 RED	RED × 100	GOLD + or - 5%
3 ORANGE	3 ORANGE	ORANGE × 1,000	
4 YELLOW	4 YELLOW	YELLOW × 10,000	
5 GREEN	5 GREEN	GREEN × 100,000	
6 BLUE	6 BLUE	BLUE × 1,000,000	
7 VIOLET	7 VIOLET	SILVER × .01	
8 GRAY	8 GRAY	GOLD × .1	
9 WHITE	9 WHITE		

Example: Yellow - Violet - Brown - Gold
4 + 7 x 10 = 470 Ohms 5% Tolerance

Figure 7.7: Resistor colour code chart

low internal resistance and does not change the current.

8. Calculate $R_p = V/I$ the equivalent resistance of R_1 and R_2 in parallel. Use the value of R_p, and values R_1 and R_2 measured in point 4, to verify that

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R_p = \frac{R_1 R_2}{R_1 + R_2}$$

7.1.5 Questions

1. An ideal voltmeter should have an internal resistance approaching infinity, while an ideal ammeter should have an internal resistance equal to zero. Explain why.

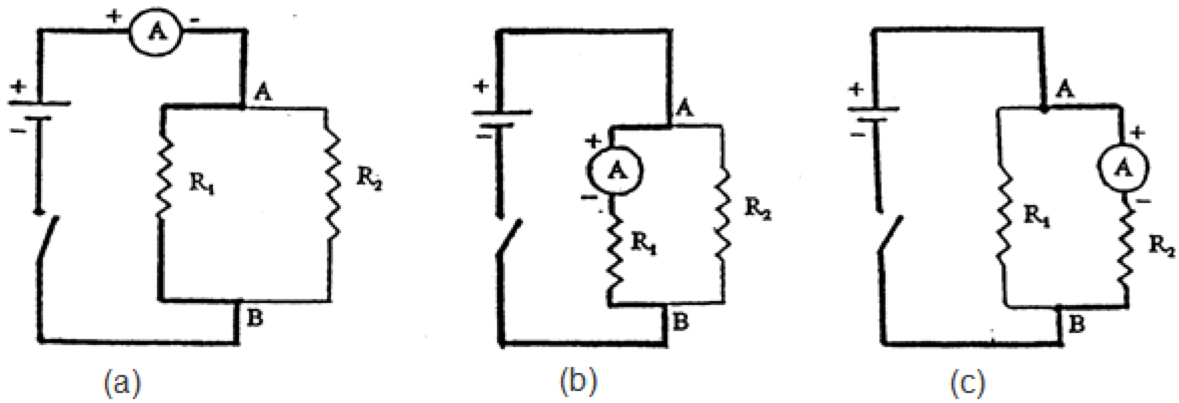


Figure 7.8: Various circuits

2. In the circuit shown in Fig.7.6, you have measured voltages V_{ab} (across R_1) and V_{cd} (across R_2). Is the sum of these two voltages equal to the battery voltage?

3. When the electric current flows through the resistor, its temperature will increase. Does this effect change the resistance of the resistor? On one of the side tables there is a resistor attached to an ohmmeter. Use the hot heating iron provided to verify your answer experimentally.

7.2 Voltage divider and Bridge Circuit

The voltage divider is shown below. It consists of a battery and two resistors, R_1 and R_2. The voltage divider allows one to obtain any fraction of the input voltage V_0 as the output

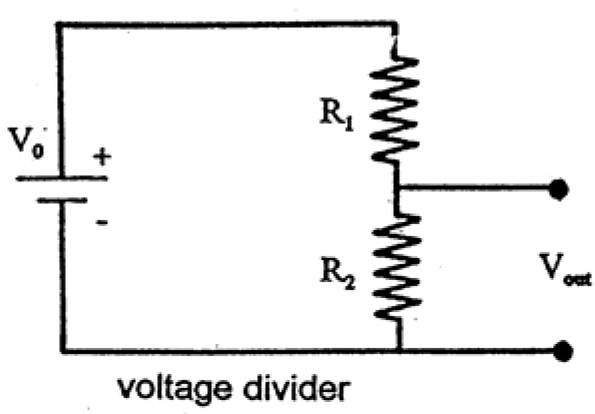


Figure 7.9: Voltage divider circuit.

voltage V_{out} . The output voltage can be calculated as follows.

$$V_{out} = IR_2$$

To find I:

$$V_0 = I(R_1 + R_2)$$

$$I = \frac{V_0}{R_1 + R_2}$$

Thus

$$V_{out} = V_0 \frac{R_2}{R_1 + R_2}$$

Depending on R_1 and R_2 , V_0 can change between 0 and V_0.

$$\text{When } R_1 \to 0. V_{out} \to V_0$$

$$\text{When } R_2 \to 0. V_{out} \to 0$$

A voltage divider in which R_1 and R_2 can be varied continuously is called a potentiometer shown in Fig. 7.10. The sliding contact (point A) can be moved between the two ends of the resistor.

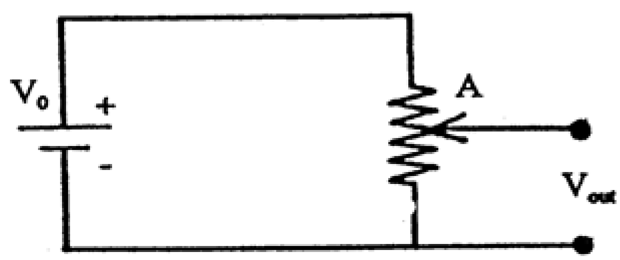


Figure 7.10: Potentiometer

A bridge circuit is a combination of two voltage dividers "bridged" by a voltmeter. The bridge circuit is shown below. A sensitive voltmeter compares the voltage V_1 set by the voltage divider with the voltage V_2 set by the potentiometer. The bridge is said to be 'balanced' when the voltmeter reads zero. In most applications, the bridge is used to detect small changes in resistances R_1 or R_2 . In this experiment, the role of the resistor R_2 is played by a thermistor, whose resistance depends strongly on the temperature. As the temperature of the thermistor change, the bridge becomes unbalanced and the voltage V_1 develops. The voltage V_1 is proportional to the temperature and can serve as a very sensitive thermometer. The electrical signal produced by the bridge with the thermistor can be, for example, digitized and analyzed by the computer and automate many devices. In this experiment you will calibrate the voltage reading as a function of temperature and then use this calibrated bridge-thermistor arrangement to measure the unknown temperature.

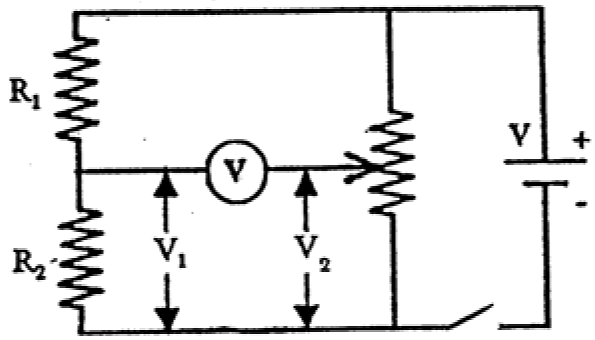


Figure 7.11: Bridge circuit.

7.2.1 Measurement and Calculations

1. Use the potentiometer to build the circuit shown in Fig. 7.12. By changing the position of the sliding contact verify that the voltmeter reading changes between zero and the voltage of the battery.

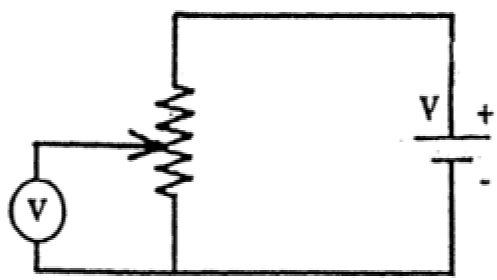
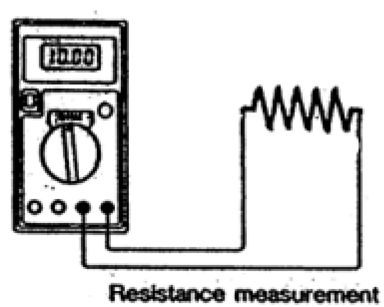


Figure 7.12: Voltage divider circuit.

2. Set up the multimeter as an ohmmeter (see figure below) and measure the resistance of a thermistor at room temperature. Then hold the thermistor tightly in your hand and note any change in its resistance. Is the change measurable?



3. Build the bridge circuit with the thermistor in place of R_2 as shown in Fig. 7.13. Set the voltmeter to its most sensitive range (DCV set to 200mV).

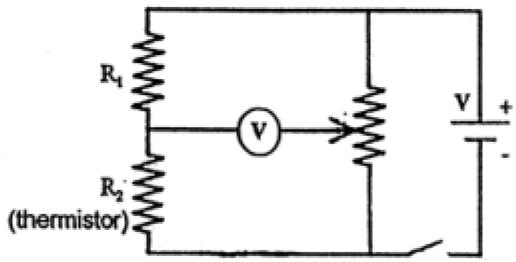


Figure 7.13: Bridge circuit with thermistor.

4. Balance the bridge by changing the position of the sliding contact on the variable resistor (voltmeter reads zero).
5. To calibrate your circuit's output, we need some known temperatures.
   - Room temperature: $22°C$
   - Ice water: $0°C$
   - boiling water: $100°C$ (use digital sensor to get precise value)
   - Mixture of hot and cold water $\sim 50°C$ (use digital sensor to get precise value)
6. Plot the resistance versus temperature for the 4 temperatures. Sketch in a line of best fit.

Chapter 8

Charge/Mass Ratio of Electron

Experiments in the 19th C. caused great changes in the concepts of Physics, changes that we are still exploring today. In today's lab session we shall examine two of those experiments, experiments which led to the concept of the FIELD (instead of action-at-a-distance) and to the ideas of fundamental particles and the "graininess" of matter. These same experiments along with a few others, led to a search (yet to be fully realized) for a mechanism which would unify electricity, light, magnetism and gravity.

Objective

1. To determine the charge/mass ratio for an electron.

Apparatus Complete unit to measure charge/mass ratio, bar magnet, compass needle, air coils, digital multimeter, signal generator, graph paper.

8.1 Charge/Mass ratio for electrons

8.1.1 Introduction

The relationship between magnetism and electricity was discovered in 1819 by the Danish physicist H. Oersted, who found that an electric current in a wire causes a deflection of a nearby compass needle. Later Ampere found that magnetic fields exert a force on a current-carrying wires.

The magnetic field produced by the electric current can interact with the magnetic field of a permanent magnet. It was discovered that the magnetism of a permanent magnet results from tiny currents formed by electrons spinning and rotating around the nucleus of an atom.

The electric current is caused by moving electric charges. Thus we can expect that a magnetic force acts on moving charges. This force is

$$\vec{F} = q\vec{v} \times \vec{B}$$

where q is the electric charge which includes a + or - sign, $\vec{v}$ is the charge velocity and $\vec{B}$ is the magnetic field. The direction of the force $\vec{F}$ is determined by the cross product $\vec{v} \times \vec{B}$ and whether the charge is negative or positive. Fig. 8.1 shows how to apply a right-hand rule to find the direction of for a positively charged particle (turn the vector $\vec{v}$ into $\vec{B}$ using the four fingers of the right hand, with the palm facing vector $\vec{B}$;the thumb shows the direction of $\vec{F}$). For a negatively charged particle the direction is reversed.

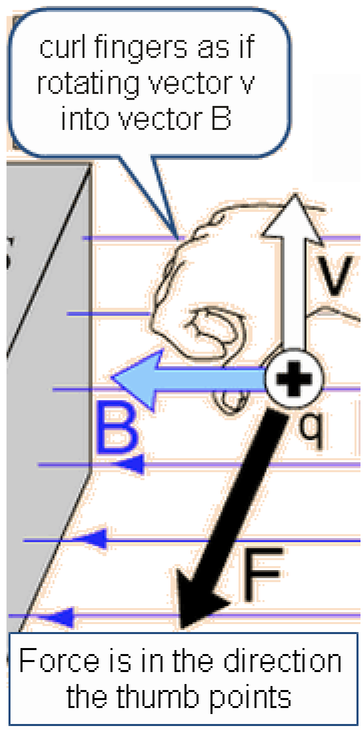


Figure 8.1: Right-hand rule for Faraday's Law

The magnetic force $\vec{F}$ acting on a moving charged particle is always perpendicular to both the velocity $\vec{v}$ and magnetic field $\vec{B}$. Consider the case when an electron of charge e is injected into a magnetic field (Fig. 8.2) whose direction is into the page.

The magnetic field in Fig. 8.2 is confined to the rectangular region (difficult to do in practice) and perpendicular to the page (into the page). When an electron enters the magnetic field the force F of size $= evB$ starts to act on the electron. The force deflects the electrons. As the direction of $\vec{v}$ changes, so does the direction of $\vec{F}$. As a result the electron moves in a

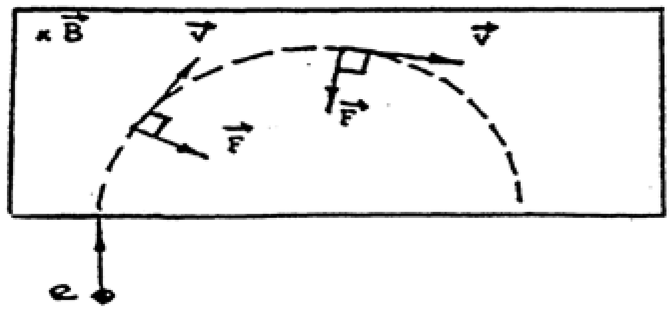


Figure 8.2: Charge moving in a magnetic field

circle perpendicular to the direction of $\vec{B}$. The magnetic force F is a centripetal force, so that

$$evB = \frac{mv^2}{r} \quad \text{where m is the mass of an electron}$$

$$r = \frac{mv}{eB} \tag{8.1}$$

The purpose of this experiment is to determine the value of the ratio e/m. From Eq. 8.1.

$$\frac{e}{m} = \frac{v}{rB} \tag{8.2}$$

8.1.2 Experimental

Electrons are produced by the electron gun in a partially evacuated glass tube. The tube contains helium vapour (at a very low pressure) to make the electron beam visible. After collisions with the electrons, helium atoms get excited and radiate visible light. Electrons are emitted by a heated cathode (thermal emission) and accelerated in an electric field between the cathode and the anode. The increase in the kinetic energy of the electrons $mv^2/2$ comes from the change of potential energy eV of the electrons in the electric field.

$$\frac{1}{2}mv^2 = eV + \frac{1}{2}mv_0^2 \quad (v_0 \approx 0m/s)$$

$$v = (\frac{2eV}{m})^{\frac{1}{2}} \tag{8.3}$$

The magnetic field is produced by a constant current flowing in two coils surrounding the tube. The coils, called Helmholtz coils, produce a relatively uniform magnetic field in the central volume of the tube. The magnitude of the magnetic field depends on the number of turns, the radius of the coils and the magnitude of the current.

For our coils

$$B = 10.27 \times 10^{-4} I \tag{8.4}$$

Figure 8.3: Electron e/m apparatus

1. Electron Tube
2. Helmholtz Coil
3. Sliding Index
4. Scale
5. Angular Index
6. Fixed Index
7. CCW current Indicator
8. CW current Indicator
9. Dark Box
10. Deflection V Control
11. Deflection V polarity switch
12. Accelerating V Control
13. Current Direction Switch
14. Current Control
15. Power Switch

If the current I is in amperes [A] then the magnetic field is in Tesla [T].

Combining equations 8.2, 8.3 and 8.4 we obtain

$$\frac{e}{m} = \frac{V}{52.73 \times 10^{-8} I^2 r^2} \tag{8.5}$$

The apparatus is shown in Fig. 8.3. The e/m tube is mounted on a rotatable socket and is located between two Helmholtz coils. The accelerating voltage is controlled by the second knob from the left. The knob and the switch on the left-hand-side of the control panel play no role in the experiment and should not be used. The current control knob serves to change the current in the Helmholtz coils. The direction of the current can be changed between clockwise to counter-clockwise using a switch. The tube, Helmholtz coils, voltmeter, ammeter and a reversing switch are "integrated" into one compact unit.

The diameter of the electron beam path is measured using a scale and a moveable eyepiece, which is a hollow tube fitted with cross wires. The entire scale is movable up and down and should pass through the center of the circular path of electrons.

8.1.3 Prelab Exercise 1

1. In Fig. 8.2, the electron moves in a circle of radius $r = 0.041cm$. The magnetic field $B = 0.74T$. Determine the speed of the electron. The mass and charge of the electron can be found in the textbook. How many times is the speed of the electron smaller than speed of light $c = 3.0 \times 10^8 m/s$?

8.1.4 Measurement and Calculations

1. Set the apparatus controls as follows: Deflection Voltage to minimum, Deflection Polarity to OFF, Accelerating Voltage to minimum, magnetizing current to minimum, magnetizing current direction to OFF.
2. Turn the power switch to ON, and allow the tube to warm up (verify the heating element attached to the cathode starts glowing).
3. Slowly increase the accelerating voltage to about 200 V. You should now see a straight blue/green beam of electrons.
4. Set the magnetizing current direction to CW and adjust the Current Control of the B-field coils to 1 Ampere. Now, the electron beam is immersed in a magnetic field which is parallel to the axis of the coils.
5. Being very careful not to crack the delicate glass electron tube, bring a bar magnet near the tube and observe its effect on the beam. Using the right-hand rule (Fig. 8.1) determine the sign of charged particles.

6. Turn the switch controlling the current in the Helmholtz coils into the clockwise position. By gripping the black base of the electron tube, rotate the socket of the tube until the path is a closed circle. Observe the effect of changing the magnitude of the current. Change the direction of the current to the counter-clockwise. Record your observations.

7. Using a compass needle, find the direction of the magnetic field inside the Helmholtz coils for the clockwise and counter-clockwise direction of the current. Draw a clear diagram.

8. Set the current in the Helmholtz coils to about $1.0A$. Vary the electron acceleration voltage from 250 to 150 V in steps of $20V$ and record the diameter of the circular beam for at least six different voltages. Record the outer diameter of the beam in each case, since the inner diameter is due to slowed-down electrons.

9. **Switch all control dials to minimum and shut off the main power switch.**

10. Plot the radius squared versus the electron acceleration voltage. Fit a line to determine the electron charge to mass ratio.

8.1.5 Questions

1. Why do electrons spread out (the beam gets broader) as they move along the circular path?

2. Sketch the trajectory of electrons when their velocity is not perpendicular to the magnetic field produced by the Helmholtz coils. Verify your prediction by rotating the e/m tube. Be extremely careful and use safety glasses. Sketch also the trajectories of electrons with decreasing speeds in the same magnetic field.

3. Does the magnetic field due to the Earth affect your result of the e/m ratio? Explain!

4. Compare trajectories of electrons and protons moving with the same speed in the magnetic field produced by the Helmholtz coils?

Chapter 9

Polarization of Light & Lenses

Objective

1. To study polarization of light and optical activity
2. To determine the focal length of various converging and diverging lenses.

Apparatus glass block, glucose solution in clear containers, polarizer, optical bench, lens holders, lenses, screen, light sources.

9.1 Polarization of Light

Light is an electromagnetic wave. A simple electromagnetic wave is shown below at an instant of time. The wave propagates in the z-direction. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ vibrate in mutually perpendicular directions.

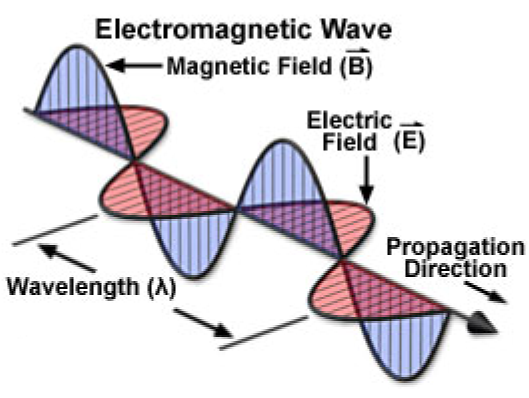


Figure 9.1: Electromagnetic wave propagation.

An electromagnetic wave is said to be polarized if the $\vec{E}$ vector always vibrates in the same direction (y-axis in the figure above). The wave is unpolarized if the $\vec{E}$ vector vibrates in all possible directions perpendicular to the direction of propagation.

A common light source, such as an electric bulb, emits unpolarized light. This is due to the fact that the light source contains millions of atoms and each atom emits light with its own direction of $\vec{E}$ vector.

Light can be polarized by four different methods: by reflection, by selective absorption, by scattering and by double-refraction. In the lab you will polarize light by reflection and analyze it by selective absorption. The fact that light waves can be polarized is evidence of the transverse nature of light waves.

Polarization of light by selective absorption Some materials called polarizers transmit only waves whose $\vec{E}$ vector vibrates in the direction parallel to a certain direction (called the transmission axis) within the polaroid. Waves with E vectors in other directions are absorbed.

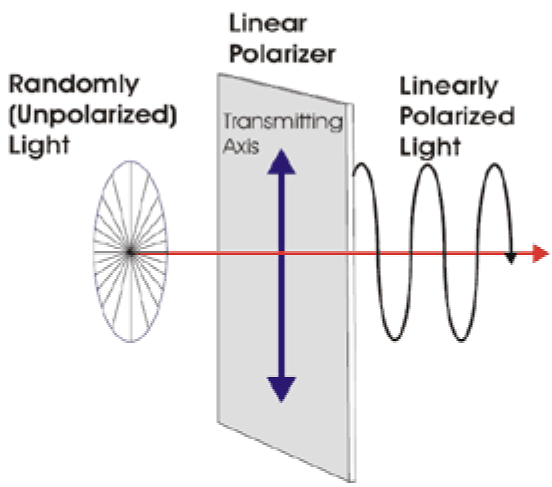


Figure 9.2: Effect of a polaroid on unpolarized light.

Polaroids contain some long-chain molecules aligned in one direction. When light is incident with $\vec{E}$ parallel to the chains, electric currents are set up in the chains, and light is absorbed. If $\vec{E}$ is perpendicular to the chains, the light is transmitted. The direction perpendicular to the chain is thus the transmission axis of the polaroid.

Polarization of light by reflection When unpolarized light is reflected from a dielectric material (for example, glass or plastic), the reflected light is partially polarized. The degree of polarization depends on the angle of incidence and the index of refraction of the dielectric. The figure below shows the case when the reflected light is fully polarized with $\vec{E}$ vibrating in the direction perpendicular to the page (represented by dots). The refracted ray of light is only partially polarized.

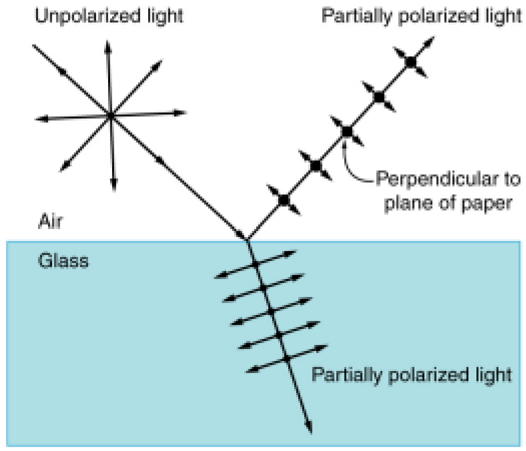


Figure 9.3: Polarization of light by reflection

The reflected light is completely polarized when the following relation is fulfilled:

$$\tan\theta_B = n$$

Where θ_B is the angle of incidence called Brewster's angle and n is the index of refraction of the material that reflects light. This relation is true only if light travels in air ($n_{air} = 1$). Brewster's angle θ_B is also a function of the wavelength because n varies with wavelength. This lab uses monochromatic light to avoid this complication.

When polarized light is transmitted through some substances, the plane of polarization is rotated by an angle α. These substances are said to be optically active. For liquid solutions, the angle α depends on the length L of the sample and the concentration c of the optically active substance in the solution:

$$\alpha = kLc$$

where k is a constant characteristic for a given optically active material. Optical activity occurs due to some asymmetry in the shape of molecules. Many biologically important molecules (some proteins, DNA, sugar) are optically active.

9.1.1 Prelab Exercise 1

Briefly explain how the following affect light.

- glass plate
- filter
- sugar solution
- polarizer.

9.1.2 Experimental

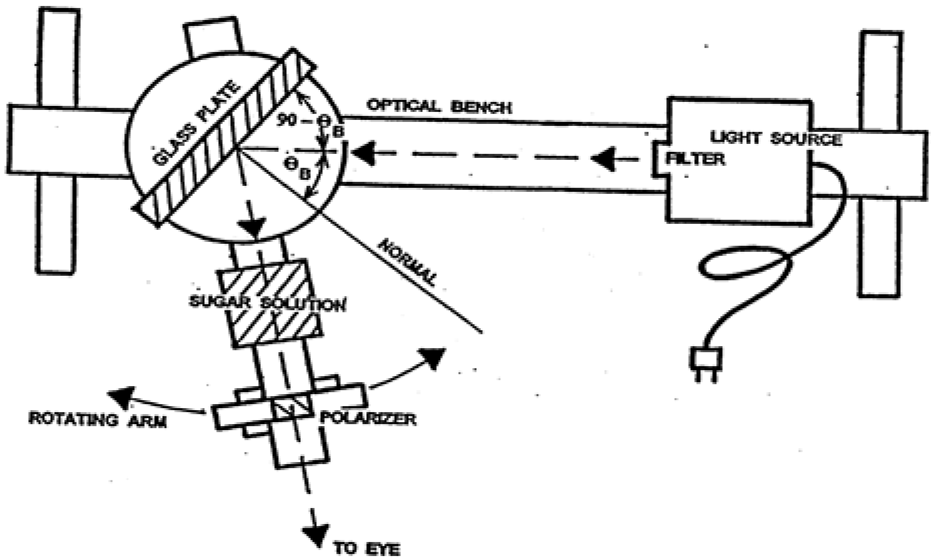


Figure 9.4: Experimental set-up to study the polarization of light

Light is transmitted through a coloured filter to generate monochromatic light. The diaphragm or hole produces a collimated light beam. A glass plate that will reflect the light is permanently mounted on a rotating table which allows it to be set to your calculated Brewster Angle. A polarizer, with its transmission axis indicated by a mark, is mounted in a vertical 360° rotating ring attached to a moveable arm. This arm is positioned along the angle of reflection, an angle equal to the angle of incidence but to the other side of the normal (marked on the rotating table).

9.1.3 Measurements and Calculations

1. Calculate the angle θ_B using the relation $\tan\theta_B = n_{glass}$, where $n_{glass} = 1.52$ is the index of refraction of the glass plate. Position the glass plate at Brewster's angle θ_B as shown in Fig. 9.4.

2. Looking through the polaroid film, rotate the arm an angle θ_B to the other side of the normal until you see the light reflected from the center of the glass plate. Rotate the polaroid film and observe changes of the transmitted light intensity. Find the orientation of the polaroid film for which the intensity of light is equal to zero. The angle corresponding to this orientation of the polaroid film is a ZERO reading for all subsequent measurements.

3. The light reflected from the glass plate is totally polarized if it is reflected at Brewster's angle. Position the glass plate at the angle different from θ_B by about 10° and verify that the light intensity cannot be reduced to zero for any orientation of the polaroid

film.

4. Determine the direction of vibration of the $\vec{E}$ vector of the reflected light taking as a reference the axis of rotation of the glass plate. The transmission axis of the polaroid film is indicated by the white arrow head on the ring that holds the film.

5. Keeping the polaroid film at the zero intensity of the transmitted light, put a container with the glucose solution on the rotating arm between the glass plate and the polaroid film. The glucose solution rotates the plane of polarization by an angle α. Measure the angle α in degrees using the ring's graduated scale. Measure the thickness of the container L. The glass window on each side of the container is 2.00 mm thick. Using the relation $\alpha = kLC$, find the concentration C of glucose in the solution. The constant k for glucose is $k = 0.335\ deg\ m^2 kg^{-1}$. Determine the percent uncertainty, $(\delta C/C)100\%$, of the concentration of glucose in the solution. You will need to choose a reasonable uncertainty for k based on the value given.

9.1.4 Questions

1. If the glass plate is replaced by a polished metal plate, would the reflected light off it be polarized? Verify your answer by looking through a polarizer at the light of a desk lamp reflected off the metal plate provided.

9.2 Lenses

9.2.1 Introduction

A lens is a piece of glass or any other transparent material having one or two spherical surfaces. Fig. 9.5 shows ray diagrams for different lenses - converging and diverging. Light rays parallel to the axis of a converging lens are refracted through the focal point F on the opposite side of the lens.

Rays parallel to the axis of the diverging lens are refracted in such a way that their backward continuations pass through the focal point on the same side of the lens. Rays passing through the centre of the lens are undeviated.

The object distance from the lens p, the image distance i and the focal length f are related (for a simple thin lens these distances are measured from the centre of the lens).

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \tag{9.1}$$

By convention the Focal length f is positive for converging lens and negative for diverging lens. Object distance is positive for a real object, and negative for virtual object. Image distance is positive for a real image and negative for a virtual image.

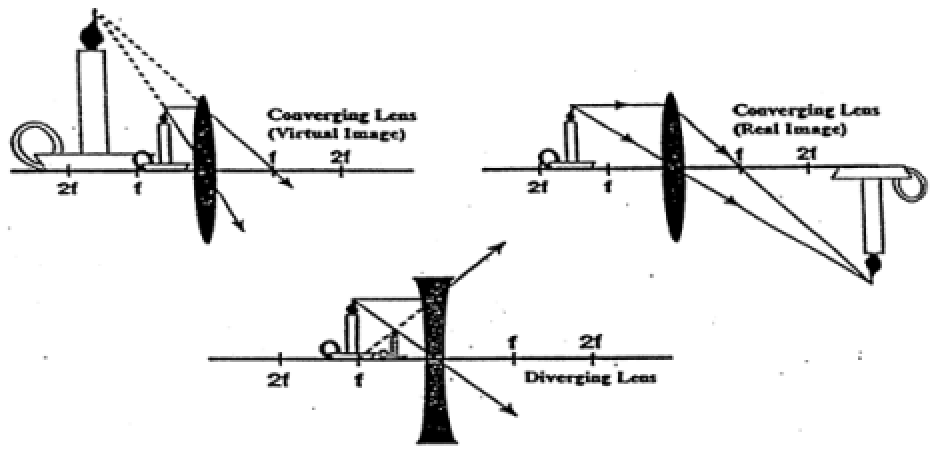


Figure 9.5: Various lens ray diagrams

The image is virtual when it cannot be displayed on a screen. Rays refracted by the lens diverge in that case but their backward continuations intersect giving the location of the image.

The magnification M of a lens is defined as the ratio of the image height h' to the object height h.

$$M = \frac{h'}{h} \tag{9.2}$$

It can be easily shown that $h'/h = -i/p$. Thus magnification can also be calculated as

$$M = -\frac{i}{p}$$

9.2.2 Prelab Exercise 2

1. An object (height h = 16 cm) is located 32.0 cm in front of a converging lens (focal length f = 10 cm). Determine the location and height of the image of the object. Draw an appropriate ray diagram.
2. Give at least five examples of optical instruments which contain lenses.

9.2.3 Experimental

Lenses are inserted in the lens holders, which in turn are mounted on the optical bench (Fig. 9.6). A light box serves as the object and a piece of white cardboard as the screen.

All elements - the lenses, the light box and the screen, can be moved along the optical bench. A scale is attached to the bench to facilitate distance measurements.

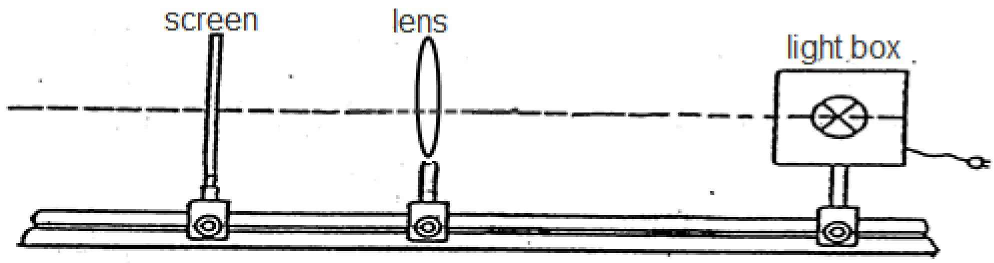


Figure 9.6: Experimental Setup

9.2.4 Measurement and Calculations

1. Use a directly-overhead ceiling light (effectively like an object at infinity) to determine the focal length f of the converging lens.

2. Mount the object box and the converging lens and the screen on the optical bench. The screen and the object box should be on opposite sides of the lens. Put the object box at the distance $p > 2f$. Move the screen along the bench until you obtain a sharp image of the box.

   Measure the distances p and i and then use equation 9.1 to find the focal length f of the lens.

   Measure the sizes of the object h and its image h' and then use equation 9.2 to find the magnification of the lens.

   Is the image erect or inverted? Draw the ray diagram.

3. Repeat the procedure described in the point above for the object box at the distance $f < p < 2f$. Draw the ray diagram.

4. Try to find the image of the object box at the distance $p < f$ on the screen. Is it possible? Where is the image?

9.2.5 Questions

(a) If half of the lens is covered with a sheet of cardboard, what would happen to the image? Explain why.

(b) Under what conditions would the lens form an image that is smaller than the object?

5. Determine the focal length of a diverging lens

   (a) To determine the focal length of a diverging lens we need to form a virtual object for the diverging lens. This can be done by making real image of the object with a converging lens. The figure below shows how the real image (virtual object) is formed. Use a value for $p > 2f$ where f is the focal length of the converging lens.

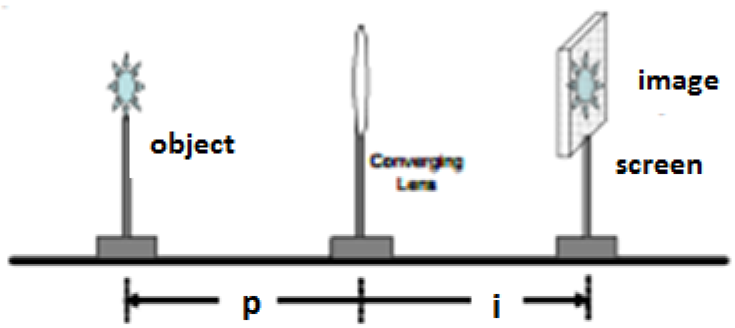


Figure 9.7

The distance of the image i from the converging lens can be found through focusing the image onto a screen. The focal length of the converging lens can be verified using equation 9.1 knowing the object distance p and image distance i.

(b) Add the diverging lens between the converging lens and its image, some distance d from the converging lens - Fig. 9.8. The new real image is formed further from the converging lens. Adjust the position of the screen and diverging lens to form a clear image. The position of the real image formed i' is recorded.

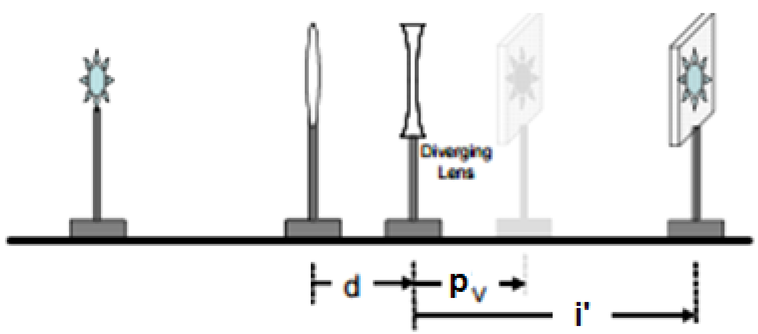


Figure 9.8

(c) Determine the position of the diverging lens' virtual object. This is the distance of virtual object p_v from the diverging lens and is calculated by $p_v = (d–i)$; Note that p_v is negative.

(d) Use equation 9.1, with object distance p_v, and image distance i' to determine the focal length of the diverging lens.

Chapter 10

Diffraction of Light & Spectroscopy

At the beginning of the l9th C. Thomas Young did an experiment that changed things drastically. His double slit experiment could be explained properly if light was a longitudinal wave since only a wave could diffract and have constructive/destructive effects. Moreover, Young showed that colours were related to wavelengths. His experiment and his conclusions created strong controversy. Fifteen years after the "double slit" experiment, Young and Fresnel realized that light had to be a TRANSVERSE WAVE in order to explain polarisation effects. The early part of the 20th century marks another stunning change in our view of the physical world. Quantum ideas and the "discrete" nature of the atomic world have fueled great changes in physics. This lab examines the diffraction of light and the discrete wavelengths produced by various gas discharges.

Objective

1. To measure the wavelength of light emitted by the discharge tube and to identify the gas contained in the tube.

Apparatus Spectrometer, diffraction grating, discharge tubes, power supply.

10.1 Introduction

Many of the properties of light can be explained only if light is treated as a wave. Fig. 10.1 shows laser light incident on a barrier with two slits. A pattern of many bright spots is observed on the screen (not just two spots which one could expect if light travelled along a straight line).

The pattern can be explained by the phenomenon of interference of waves. Recall from the sound experiment that two waves interfere constructively if a crest of one wave occurs at

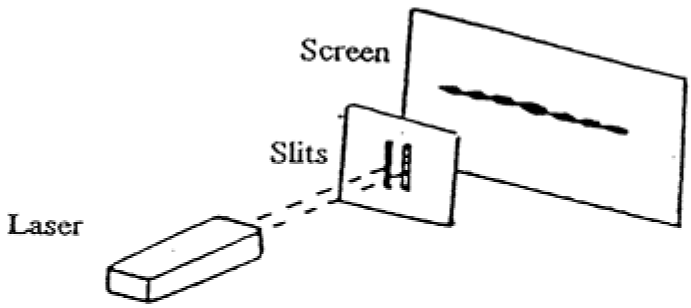


Figure 10.1

the same place as the crest of the second wave. Destructive interference occurs when the crest of one wave coincides with the trough of the other wave. Each slit in Fig. 10.1 can be treated as a source of a wave. The waves from two slits produce a bright spot on the screen when two waves interfere constructively. This happens when the distances travelled by two waves differ by one or two, or three, or $n\lambda$ wavelengths. If the slits are separated by the distance d (Fig. 10.2), then for a difference of one wavelength,

$$\sin\theta_1 = \lambda/d$$

If the distance travelled by the two waves originating from two slits differ by 2λ then a second bright spot occurs on the screen when $\sin\theta_2 = 2\lambda/d$. In general, the n-th bright spot occurs for the angle given by

$$\sin\theta_n = n\frac{\lambda}{d} \tag{10.1}$$

When two slits are replaced by a diffraction grating which contains many slits then the maxima occur at the same positions on the screen as for the two slits, but the peaks are much narrower (Fig. 10.3). Thus the resolution of the pattern of the bright spots is higher when a diffraction grating is used. A diffraction grating is produced by making scratches in a glass plate.

The unscratched parts of the glass are transparent for the light and act as slits. The scratches block the light. A diffraction grating can also be made by taking a photograph of many black lines. The negative of the photograph can serve as a grating.

The laser light contains only one wavelength (monochromatic light). If the laser light in Fig. 10.1 is replaced by polychromatic light (mixture of different wavelengths) then the waves of different λ interfere independently. According to equation 10.1 waves of longer wavelength are deviated more (larger θ). Fig. 10.4 shows the diffraction pattern for the mixture of the violet, blue and red light.

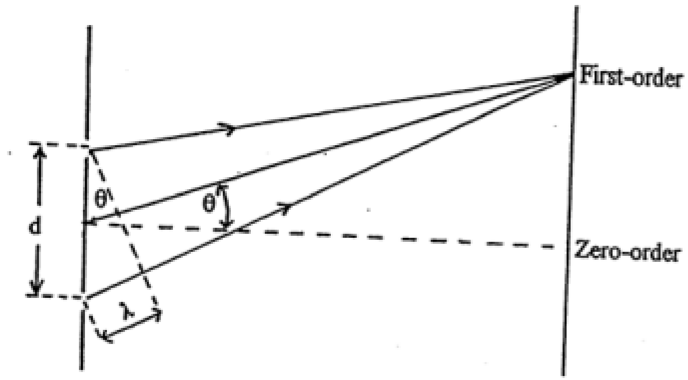


Figure 10.2: Constructive interference from double-slit diffraction.

Figure 10.3: Narrowing of Interference with more slits.

10.2 Prelab Exercises

Read through Experimental section first.

1. Explain the role of the diffraction grating in the spectroscope.
2. Why must the telescope be focused at infinity?
3. The first order maximum for red light ($\lambda = 635.0nm$) is observed at exactly 12 degrees. At what angle would the second order maximum be observed for the same wavelength?
4. Read the angular Vernier scales below. Report the angles in degrees/minutes as well as decimal degrees (for example $45°16'$ and $45.27°$)

10.3 Experimental

To study the diffraction pattern, a spectrometer is used. The spectrometer consists of a collimator, a diffraction grating and a telescope (Fig. 10.5).

The light to be analyzed is sent through a narrow slit of adjustable width. The slit is at the

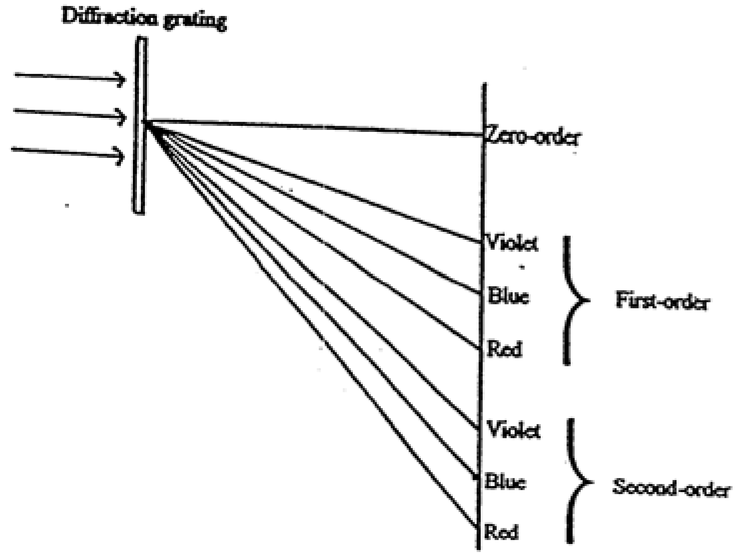
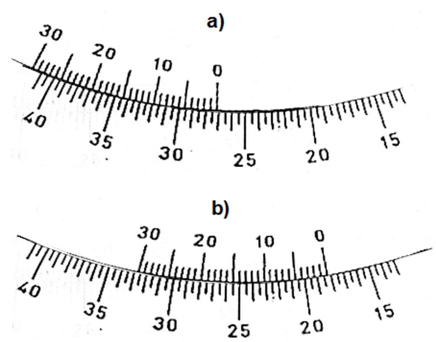


Figure 10.4: Diffraction of multiple wavelengths through a grating.



focal point of the converging lens so that the rays of light emerging from the collimator are parallel. The diffraction pattern is observed through the telescope. The telescope contains two lenses called an objective and an eyepiece. The telescope can be rotated to observe different orders of the bright spots.

An angular Vernier scale, (Fig. 10.6) enables one to read the angle of rotation of the telescope to the nearest minute of arc.

Use the Vernier scale as follows. First, read the angle on the main scale indicated by the zero of the Vernier scale (20.5°) in Fig. 10.6). Then find the division of the Vernier scale which coincides with any division of the main scale (10′ in Fig. 6). Add the reading of the vernier scale to the angle read on the main scale (20°40′). A screw on the bottom of the spectrometer serves to lock the telescope at the given position. A second screw serves to adjust the final position of the telescope. The adjustment screw works only if the lock screw is tightened.

As a source of polychromatic light, a discharge tube is used (Fig. 10.7). The light emitted

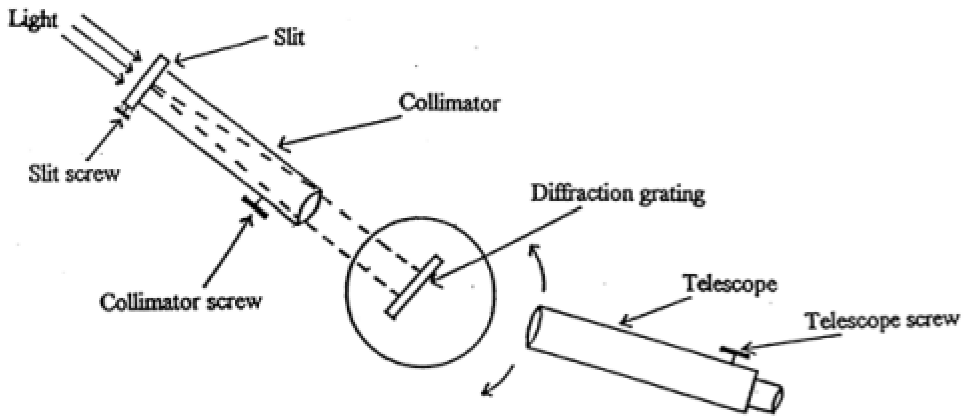


Figure 10.5: Experimental setup.

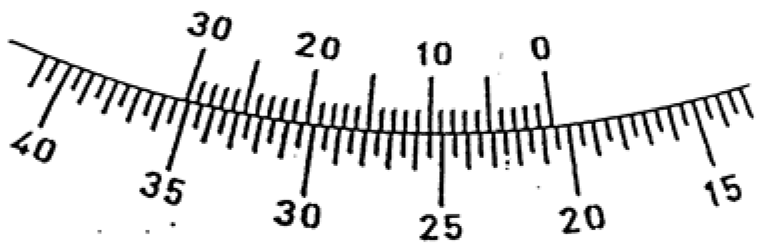


Figure 10.6: Angular Vernier scale.

by the tube depends on the gas in the tube. A very high voltage (5000V) is applied across the tube. **DO NOT TOUCH THE TUBE WHEN IT IS OPERATING**.

10.4 Measurements and Calculations

1. Focus the telescope on a distant object through a window by turning the screw knob mounted on the side of the telescope (Fig. 10.5). The rays of light coming from this distant object are parallel (for our purposes).

2. Mount one of the two gas discharge tubes you will use in the discharge tube power supply and turn it on.

   Position the slit directly on front of the light emitted by the discharge tube and observe it through the telescope.

   Turn the screw knob on the side of the collimator to obtain a sharp image of the slit as seen through the telescope. When the slit is at the focal length of the collimator lens, the light rays that emerge are parallel. Now both the collimator and telescope will be properly focused.

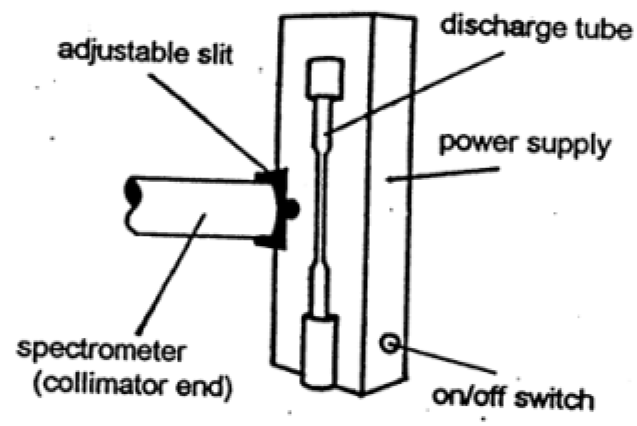


Figure 10.7: Discharge tube apparatus.

The image of the slit should not be too wide since all your spectral lines will also have the same width as the slit. Adjust the slit width by turning the slit screw knob.

3. Place the diffraction grating on the round spectrometer table and make sure it is at right angles to the light rays going from the collimator lens to the telescope lens. Looking through the telescope, position the vertical crosshair so that it goes through the image of the slit, the central maximum. The angle on the vernier scale will be your zero reading. Measure all the angles of the spectral lines observed corresponding to the first, n=1 and second, n=2 order maxima. Use equation 10.1 to calculate the wavelengths for all the different coloured spectral lines.

   The diffraction grating constant, $d = (3.33 \pm 0.2) \times 10^{-6} m$, unless told otherwise.

   Remember that the same sequence of colours or spectral lines will be observed in all orders of maxima, n, although the higher order spectral lines will be fainter and more spread apart. Of course, the wavelength for any given colour will be the same for all n.

4. Match your average wavelength values for each of the spectral linesd with the wavelengths on the emission spectrum wall chart for the different gases. Do not match by colour. Some weak spectral lines you observed may not show up on the chart. Record the label on the gas discharge tube along with the name of the gas you identified.

   Repeat this experiment with the second gas discharge tube provided.

5. The discrete spectra observed for different gases reflect the quantized nature of the electron energy in atoms and molecules. Radiation is emitted when electrons jump to a lower energy state. Bohr showed that the wavelengths of the observed hydrogen emission lines can be fitted to the following formula

$$\frac{1}{\lambda} = R\left(\frac{1}{l^2} - \frac{1}{u^2}\right)$$

where l and u are integers called the quantum numbers of the lower and upper energy states, and R is the Rydberg constant ($R = 1.097 \times 10^7 m^{-1}$).

The red light emitted by the hydrogen atom corresponds to the transition from the $u = 3$ state to the $l = 2$ state. Use the above formula to compute the value of λ for the red light emitted by the hydrogen atom. Evaluate the agreement between your experimental value of λ and the calculated value.

10.5 Questions

1. If monochromatic light enters the collimator's slit, what would you expect the spectrum to look like? Explain.
2. Would a spectrum be formed if the diffraction grating had only 20 lines per centimeter? Explain.
3. How would the spectrum change if the diffraction grating were not at right angles to the incident beam. Rotate the diffraction grating slowly so that it is no longer perpendicular to the incident beam and note any change in angle of the spectral line with respect to the crosshair in the telescope.
4. If the diffraction grating lines are perpendicular to the collimator's slit, would a spectrum be observed? If so, how would it look like (make a sketch)? Verify this by holding the diffraction grating close to your eye and rotating it while looking at the light from your discharge tube.

Table 10.1: Diffraction of Light from Gas Tube - Data and Calculations

Order	Colour	Angle (° *and* ′)	Angle (° *only*)	$\lambda = dsin(\theta_{1,x} - \theta_0)/n$
n=0			$\theta_0 =$	-
n=1			$\theta_{1,a} =$	
			$\theta_{1,b} =$	
			$\theta_{1,c} =$	
			$\theta_{1,d} =$	
n=2			$\theta_{2,a} =$	
			$\theta_{2,b} =$	
			$\theta_{2,c} =$	
			$\theta_{2,d} =$	
average of n=1 and n=2 values		-	-	
		-	-	
		-	-	
		-	-	

Two character code of cell:

Identification of Gas:

Table 10.2: Diffraction of Light from Gas Tube - Data and Calculations

Order	Colour	Angle ($^\circ$ *and* $'$)	Angle ($^\circ$ *only*)	$\lambda = dsin(\theta_{1,x} - \theta_0)/n$
n=0			$\theta_0 =$	-
n=1			$\theta_{1,a} =$	
			$\theta_{1,b} =$	
			$\theta_{1,c} =$	
			$\theta_{1,d} =$	
n=2			$\theta_{2,a} =$	
			$\theta_{2,b} =$	
			$\theta_{2,c} =$	
			$\theta_{2,d} =$	
average of n=1 and n=2 values		-	-	
		-	-	
		-	-	
		-	-	

Two character code of cell:

Identification of Gas:

Appendix A

Statistical Treatment of Random Error

The scatter of measurements can be used to indicate the uncertainty. The table below shows 8 measurements of the width of a piece of wood. One then computes the average value and the difference of each observation from the average to obtain the average deviation.

	Width (cm)		**Deviation from Mean (cm)**
	12.32		0.03
	12.35		0.00
	12.34		0.01
	12.38		0.03
	12.32		0.03
	12.36		0.01
	12.34		0.03
	12.38		0.03
Sum	98.79	Sum	0.17

Mean $\overline{d} = 98.97/8 = 12.35$. Average Deviation $\sigma = 0.17/8 = 0.02$. You would report the measured width as $(\overline{d} \pm \sigma) = (12.35 \pm 0.02)cm$.

In general, the mean of a quantity x measured N times is

$$\overline{x} = \frac{\sum_i x_i}{N}$$

and the average deviation is

$$\sigma = \frac{\sum_i |x_i - \overline{x}|}{N}$$

.

Appendix B

Uncertainty Calculations using Differentials

We would like to estimate the uncertainty in a variable p resulting from the addition/subtraction or multiplication/division of variables x and y. Let the absolute uncertainties be interpreted as differentials dx, dy, etc.

Addition/Subtraction: Taking the differential of $p = x+y$ yields $dp = dx+dy$. Similarly, differentiating $p = x - y$ yields $dp = dx - dy$. The resultant uncertainty is estimated using the maximum (upper) and minimum (lower) bounds. Since the uncertainties are $\pm dx$; $\pm dy$, etc., we choose the sign which giving the greatest uncertainty. i.e. All negative signs are changed to positive signs. **Therefore, for addition and subtraction $dp = dx + dy$ i.e. the final uncertainty is the sum of absolute uncertainties.**

Multiplication/Division Taking the differential of $p = xy$ yields $dp = xdy+ydx$. Dividing by p yields

$$\frac{dp}{p} = \frac{dy}{y} + \frac{dx}{x} \tag{B.1}$$

Similarly, we take the differential of $p = x/y$ and find

$$\frac{dp}{p} = \frac{dx}{x} - \frac{dy}{y}$$

Changing the negative to a positive sign yields

$$\frac{dp}{p} = \frac{dx}{x} + \frac{dy}{y}$$

Therefore, for multiplication and division, the final uncertainty, expressed as a percentage, is the sum of percentage uncertainties of the components.

Variable to a Power Taking the differential of $p = y^a$ where a is a constant yields $dp = ay^{a-1}dy$. This can be rewritten as follows.

$$dp = \frac{ay^{a-1}dy}{y^a} = \frac{ady}{y}$$

Examples The dimensions of a room are:

$$\begin{aligned} \text{length:} &\quad (3.04 \pm .02)m \\ \text{width:} &\quad (3.66 \pm .02)m \\ \text{height:} &\quad (2.74 \pm .02)m \end{aligned}$$

What are the resultant uncertainties in

(a) Floor Perimeter

Perimeter $P = 2l + 2w = 2 \times (3.04 + 3.66) = 13.40m$

Perimeter Uncertainty: $dP = 2dl + 2dw = 2(0.02) + 2(0.02) = 0.08m$

Floor Perimeter: $(13.40 \pm 0.08)m$

(b) Floor Area

Area: $A_{floor} = l \times w = 3.04 \times 3.66 = 11.13m^2$

Taking the differentials gives the following.

$$\frac{dA_{floor}}{A_{floor}} = \frac{dl}{l} + \frac{dw}{w}$$

Area Uncertainty:

$$dA_{floor} = (\frac{0.02}{3.04} + \frac{0.02}{3.66}) \times 11.13 = 0.13m^2$$

Floor Area: $11.13 \pm 0.13m^2$

Appendix C

Method of Least Squares

The method of least squares is used to fit data to a given equation. We consider fitting a straight line as shown in Fig. C.1. Not every data point falls on the line. There is a difference between the fitted y value and the actual y value. The purpose of least squares is to minimize these deviations over all N data points.

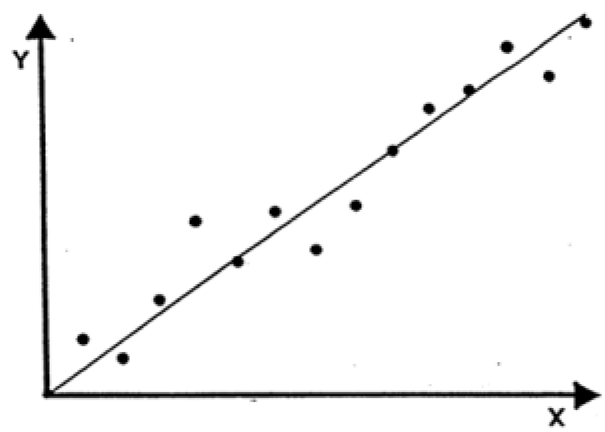


Figure C.1

The equation of a straight line may be given by $y = mx + b$. The difference between the fitted y and the observed y_i is given by $b+mx_i-y_i$. Summing the squares of these differences gives us the total square uncertainty

$$\sigma = \sum(b + mx_i - y_i)^2$$

where $\sum$ means "summing over all data points". It is the sum that we wish to minimize. This is done by setting the parital derivative of σ with respect to b and m equal to zero.

The partial derivative with respect to b is:

$$\frac{\partial \sigma}{\partial b} = 2\sum(b + mx_i - y_i) = 0$$

The partial derivative operator $\partial/\partial b$ means "take the derivative with respect to b only and treat all other numbers as constant".

Expanding the summation and moving the term with y_i to the right of the equals sign gives:

$$\sum b + \sum mx_i = \sum y_i$$

the parameters b and m can be moved from inside the summation, as they do not change during the summations, i.e.:

$$bN + m\sum x_i = \sum y_i$$

The parameter b is multiplied by N, the number of data points, as summing the constant b, N times is bN.

Similarly:

$$\frac{\partial \sigma}{\partial m} = 2\sum(bx_i + mx_i^2 - x_i y_i = 0$$

yields the equation

$$b\sum x_i + m\sum x_i^2 = \sum x_i y_i$$

Solving the two equations gives:

$$\text{Intercept :} \quad b = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{N\sum x_i^2 - (\sum x_i)^2}$$

$$\text{Slope :} \quad m = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{N\sum x_i^2 - (\sum x_i)^2}$$

It can be shown that the slope uncertainty is given by the following expression

$$\text{Uncertainty in Slope :} \quad \delta m = \sqrt{\frac{\sum[(y_i - \overline{y}) - m(x_i - \overline{x})]^2}{(N-2)\sum(x_i - \overline{x})^2}}$$

$$\text{where :} \quad \overline{x} = \frac{\sum x_i}{N} \quad \text{and} \quad \overline{y} = \frac{\sum y_i}{N}$$

Appendix D

Sample Lab Report

Date: Sept. 18, 2015. Name: John Smith Partner's name: Melanie Wong

Lab 1- Uniformly accelerated motion

Objectives : To the determine acceleration and its uncertainty of a glider moving along a frictionless air track and to use it to calculate the gravitational acceleration g.

t_i**(s)**	x_i **(cm)**
0.2	0.5
0.4	1.3
0.6	2.4
0.8	3.8
1.0	5.6
1.2	7.7
1.4	10.2
1.6	13.0
1.8	16.2
2.0	19.6
2.2	23.5
2.4	27.6

$$\delta t = 0.005s \quad , \quad \delta x_i = 0.1cm$$

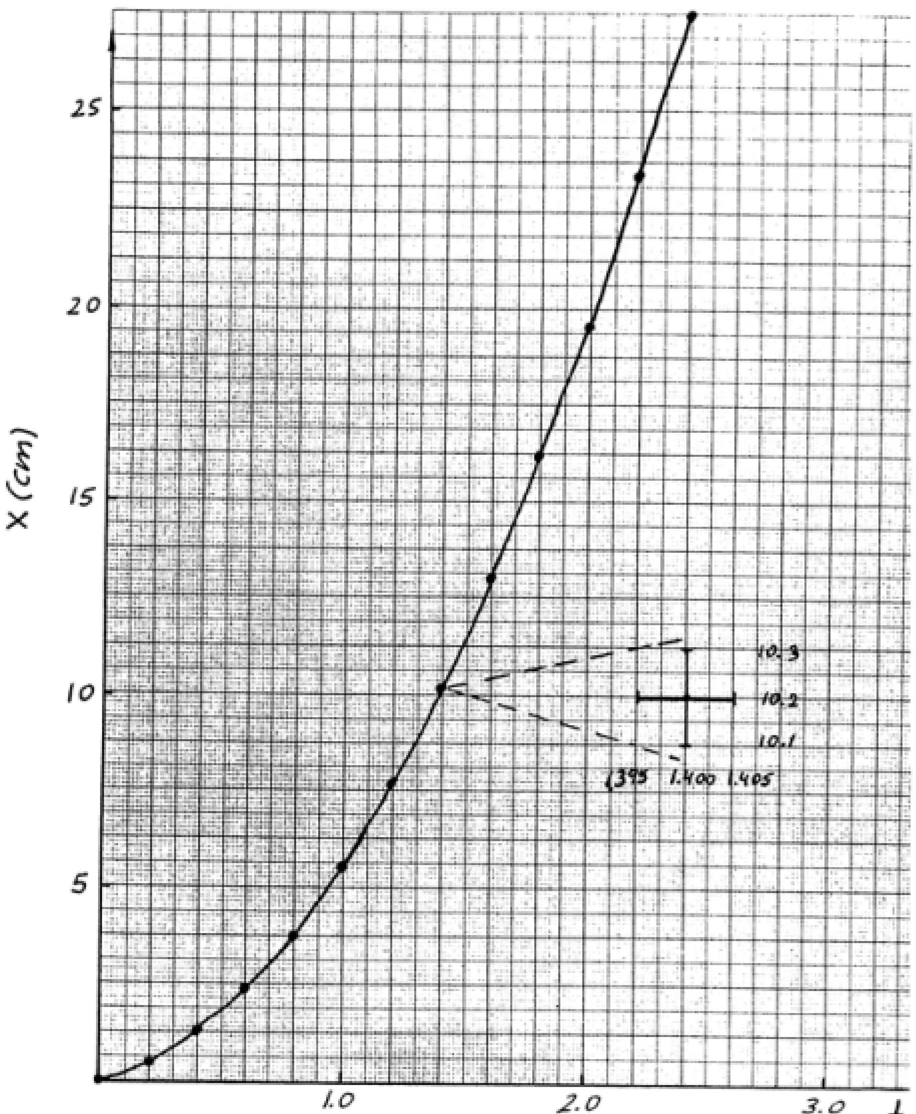


Fig. 1 Position-time graph for a glider moving on a frictionless air track.
Error bars are too small to be shown (smaller than the solid circles).
The equation representing this graph is $x = at^2/2$. Its graph is a parabola.

Figure D.1: Average velocity in consecutive time intervals for a glider moving along an air track. An instantaneous velocity is approximately equal to the average velocity at the median time for each time interval.

t_i (s)	x_i (cm)	$v = \frac{x_{i+1} - x_i}{t_{i+1} - t_i}$ (cm/s)	Median Time t_m(s)
0.2	0.5	$\frac{1.3 - 0.5}{0.4 - 0.2} = 4.0$	$\frac{0.2 + 0.4}{2} = 0.3$
0.4	1.3		
0.6	2.4	$\frac{3.8 - 2.4}{0.8 - 0.6} = 7.0$	$\frac{0.6 + 0.8}{2} = 0.7$
0.8	3.8		
1.0	5.6	$\frac{7.7 - 5.6}{1.2 - 1.0} = 10.5$	$\frac{1.0 + 1.2}{2} = 1.1$
1.2	7.7		
1.4	10.2	14	1.5
1.6	13.0		
1.8	16.2	17	1.9
2.0	19.6		
2.2	23.5	20.5	2.3
2.4	27.6		

Uncertainty Calculations

$$v = \frac{x}{t}\ , \text{ where } x = x_{i+1} - x_i, \quad t = t_{i+1} - t_i$$
$$\delta x = \delta x_{i+1} + \delta x_i = 1mm + 1mm = 2mm = 0.2cm$$
$$\delta t = \delta t_{i+1} + \delta t_i = 0.005s + 0.005s = 0.01s$$

$$\delta v_1 = (\frac{0.2cm}{0.8cm} + \frac{0.01s}{0.2s})4.0cm/s = 1.2cm/s$$
$$\delta v_6 = (\frac{0.2cm}{4.1cm} + \frac{0.01s}{0.2s})20.5cm/s = 2.0cm/s$$

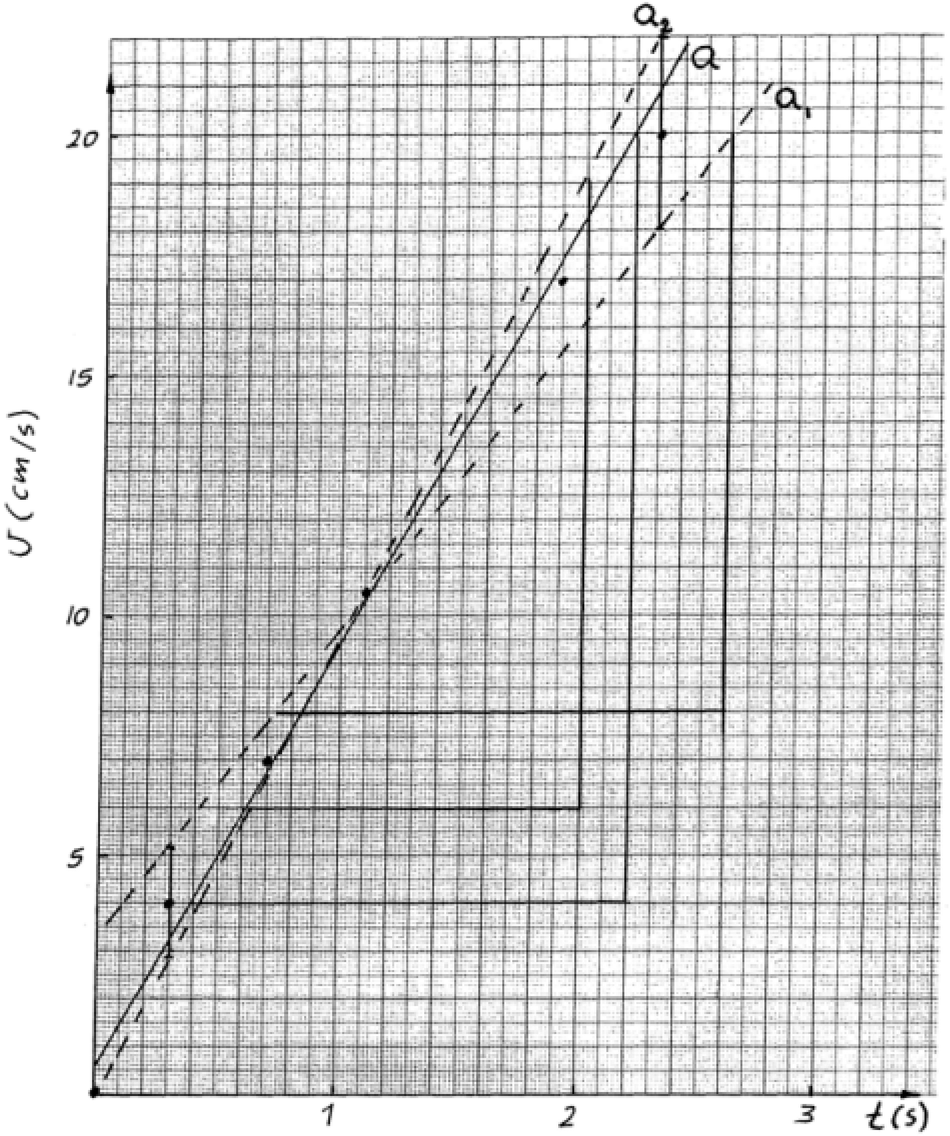


Fig. 2. Velocity versus time graph for a glider moving on the air track. Slopes of the two dotted lines are used to evaluate the error of acceleration.

Acceleration of the Glider

$$a = \frac{\Delta v}{\Delta t} = \frac{20.0cm/s - 4.0cm/s}{2.2s - 0.4s} = 8.9cm/s^2$$

Uncertainty of Acceleration

$$\Delta a = \frac{a_2 - a_1}{2}$$
$$a_1 = \frac{\Delta v}{\Delta t} = \frac{20.0cm/s - 8.0cm/s}{2.6s - 0.72s} = 6.4cm/s^2$$
$$a_2 = \frac{\Delta v}{\Delta t} = \frac{18.0cm/s - 6.0cm/s}{2.0s - 0.62s} = 10.1cm/s^2$$

$$\delta a = \frac{10.1cm/s^2 - 6.4cm/s^2}{2} = 1.8cm/s^2$$

$$\text{Therefore :} \quad a = (8.9 \pm 1.8)cm/s^2$$

Gravitational Acceleration

$$a = g \sin\theta$$
$$\theta = (0.6 \pm 0.1)^{\circ} o$$
$$g = \frac{a}{\sin\theta} = \frac{0.089m/s^2}{\sin 0.6^{\circ}} = 8.5m/s^2$$

Conclusions: The position of the glider versus time graph is parabolic while the velocity versus time graph is a straight line. The magnitude of the gravitational acceleration is slightly smaller than the standard value, which can be accounted for by the friction between the glider and the air track and inaccurate measurement of the angle of inclination of the air track. Within experimental uncertainty, the value of the gravitational acceleration agrees with the standard value of $g = 9.81m/s^2$.