Atomic Physics Assignment 4

5 x 2 = 10 marks

1. Consider two particles having masses m_1 and m_2 at positions $\vec{r}_1$ and $\vec{r}_2$. Assume the particles only interact with a so called central potential that depends on the separation distance $|\vec{r}_1 - \vec{r}_2|$. For simplicity, choose the origin to the center of mass position.

   a) Defining $\vec{r} = \vec{r}_1 - \vec{r}_2$, show $\vec{r}_1 = \frac{m_2}{m_1+m_2}\vec{r}$ $\vec{r}_2 = \frac{-m_1}{m_1+m_2}\vec{r}$

   Note: For $m_1 << m_2$, $\vec{r}_1 = \vec{r}$, and $\vec{r}_2 = 0$

   b) Show Lagrangian $L = \frac{m_1}{2}|\dot{\vec{r}}_1|^2 + \frac{m_2}{2}|\dot{\vec{r}}_2|^2 - U(|\vec{r}_1 - \vec{r}_2|)$

   becomes $L = \frac{\mu}{2}|\dot{\vec{r}}|^2 - U(r)$

   where the reduced mass $\mu \equiv \frac{m_1 m_2}{m_1+m_2}$

2. Evaluate the Bohr radius and Rydberg energy for
   a) Hydrogen
   b) Deuterium
   c) Positronium.

3. Lyman α
   a) What wavelength corresponds to the Lyman α transition?
   b) Does this correspond to a visible, UV or infrared photon?

4. The probability of finding an electron between distance r and r + dr from the nucleus is given by $P(r) = r^2 \int \psi(r,\theta,\phi)\,\psi^*(r,\theta,\phi)\,d\Omega$

   Find the probability and sketch it for the 1s and 2s hydrogen wavefunctions.

5. What are the angular momenta of the following states: G, F, D, P, S?