Quiz 1

Name: _____ Student Number: _____

NO CALCULATORS ALLOWED.

10 X 1 = 10 MARKS

1. sin 195°

$= -\sin 15^\circ$

$= -\sin(45^\circ - 30^\circ)$

$= -[\sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ]$

$= -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}\frac{1}{\sqrt{2}}$

$= \frac{-\sqrt{3}+1}{2\sqrt{2}}$

2. Arcsec 2

$= \arccos\left(\frac{1}{2}\right)$

$= 60^\circ$

3. Find unit vector perpendicular to $\vec{x} = (1, 2, 1)$ and $\vec{y} = (1, 1, 1)$

$$\vec{x} \times \vec{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= (2-1, 0, 1-2)$

$= (1, 0, -1)$

$|\vec{x} \times \vec{y}| = \sqrt{2}$

$\therefore$ unit vector $\perp$ to $\vec{x} \times \vec{y}$ is

$\frac{1}{\sqrt{2}}(1, 0, -1)$

4. Find angle between $\vec{x} = (1, \sqrt{6}, -1)$ and $\vec{y} = (-1, 0, 1)$

$\cos\theta = \frac{\vec{x} \cdot \vec{y}}{|\vec{x}||\vec{y}|}$

$= \frac{-1-1}{\sqrt{8} \cdot \sqrt{2}}$

$= \frac{-2}{4}$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^\circ$

5. What is the period of $y = \cos 3x$?

Period of $\cos x$ is 2π

" " $\cos 3x$ is $\frac{2\pi}{3}$.

6. Derive first four nonzero terms of Taylor's expansion for e^x

$f(x) = e^x \quad f(0) = 1$

$f'(x) = e^x \quad f'(0) = 1$

$f''(x) = e^x \quad f''(0) = 1$

$f'''(x) = e^x \quad f'''(0) = 1$

$$e^x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

7. Evaluate $(26)^{1/3}$ to 1% accuracy.

$$(26)^{1/3} = (27-1)^{1/3}$$

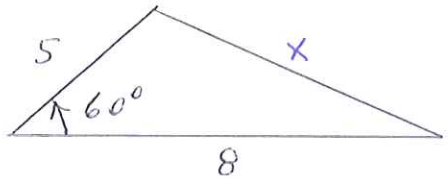
$$= 3\left(1 - \frac{1}{27}\right)^{1/3}$$

$$= 3\left[1 - \frac{1}{3}\cdot\frac{1}{27} + \frac{\frac{1}{3}\left(\frac{-2}{3}\right)}{2!}\left(\frac{-1}{27}\right)^2 + \cdots\right]$$

$$\approx 3 - \frac{1}{27}$$

$$= \frac{80}{27}$$

8. Find length of the triangle side that is not given.



$$x^2 = 5^2 + 8^2 - 2(5)(8)\cos 60^\circ$$

$$= 25 + 64 - 80 \cdot \frac{1}{2}$$

$$= 49$$

$$\therefore x = 7$$

9. Find unit vector pointing from P(2,1) to Q(3,4)

$$\overrightarrow{PQ} = (3,4) - (2,1)$$

$$= (1,3)$$

Unit vector $\parallel \overrightarrow{PQ}$ is $\frac{1}{\sqrt{10}}(1,3)$

10. A light pulse is emitted, reflects off an object and is detected by the receiver located near the laser emitter. Estimate the distance to the object if one round trip takes 5 msec.

$$\text{Distance} = \frac{1}{2} \, 5 \times 10^{-3} \text{ sec} \times 3 \times 10^{8} \text{ m/sec}.$$

$$= 7.5 \times 10^{5} \text{ m}.$$