J. S. WRIGHT

Radiation and Matter
BOOK 1

Waves & Particles

Chapter 1

FACTS WITHOUT THEORY: Transmission of Light

1-1 INTRODUCTION

John Milton, when he wrote of his approaching blindness

"When I consider how my light is spent
Ere half my days, in this dark world and wide"

expressed the tragedy that loss of sight would be for all of us. Sight is the most important way we have of perceiving information about the world around us, and throughout man's history "light" has been considered so important that the word itself has come to echo the deepest emotional and spiritual values of our culture.

We use light in a multitude of ways, yet most of us know surprisingly little about it. The transmission of light, as we shall see in later chapters, has presented physicists with a dilemma which they have only recently begun to solve. In order to appreciate their problem, we must recall a few elementary facts about light.

1-2 RECTILINEAR PROPAGATION OF LIGHT

Transparent media such as glass and air transmit light well; translucent media such as frosted glass and waxed. paper transmit light poorly; and opaque materials such as wood and plaster do not transmit light at all. The transmission of light from the sun to the earth indicates that light does not require a medium for its transmission, that is, it will travel through a vacuum. This fact will prove troublesome when, in Chapters 3 and 4, we attempt to devise two models to explain the transmission of light. In setting up either of these models we take into account a fact indicated by the production of shadows; in any homogeneous medium, (or in a vacuum), where no obstacles are encountered, light travels in straight lines.

The straight lines along which light travels are called rays. In Figure 1.1 the lines AF and AG represent rays coming from a small source A. A bundle of rays, such as that bounded by AF and AG, is called a beam or pencil.

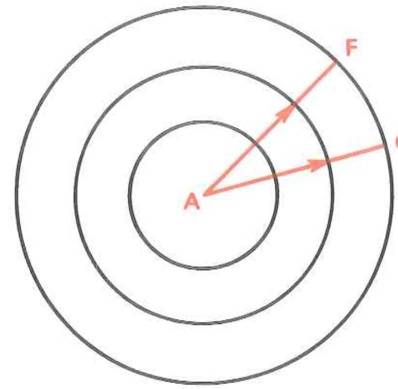


Fig. 1.1. Rays of light from a source *A*.

1–3 HOW WE SEE OBJECTS

In the absence of light we see nothing. If we look directly at a source of light we see the source—the sun, for example—by means of light which travels directly from the source to the eye. Figure 1.2 shows that we see nothing unless the light enters the eye. The light between the source and the cardboard is not visible because it does not reach the eye; some of the light which reaches the cardboard is reflected to the eye and the cardboard is visible as a result. We see all non-luminous objects by means of reflected light.

1–4 REFLECTION OF LIGHT

The line EC in Figure 1.3 represents a ray of light striking a reflecting surface at C. The line CF represents the path of the light after reflection, and CD is perpendicular to the surface at C. EC is called the incident ray, CF is the reflected ray, and CD is the normal (perpendicular) to the surface at C. Angle ECD is called the angle of incidence; angle DCF is called the angle of reflection.

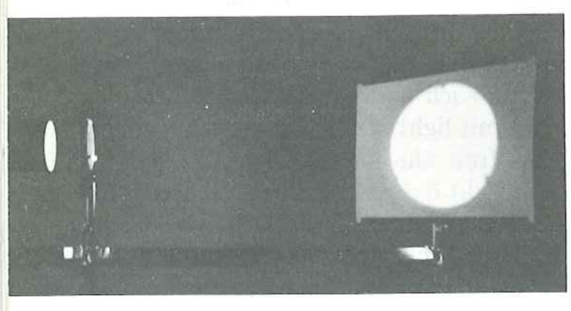


Fig. 1.2. We are aware of light only when it reaches our eyes.

An optical disc (Fig. 1.4) may be used to investigate reflection of light. A ray incident along the disc strikes the mirror at the centre of the disc. The reflected ray is visible on the surface of the disc. Since the normal to the mirror also lies on the surface of the disc, we have illustrated here what is called the first law of reflection of light: the incident ray, the normal, and the reflected ray lie in the same plane. The second law of reflection is also illustrated in Figure 1.4: the angle of reflection is equal to the angle of incidence.

1–5 THE SPEED OF LIGHT IN AIR AND IN SPACE

Everyday experience indicates to us that the time required for light to travel terrestrial distances is very small. Thus, the speed of light, if not infinite, must be very large. In 1676 Olaf Römer made the first measurement of the speed of

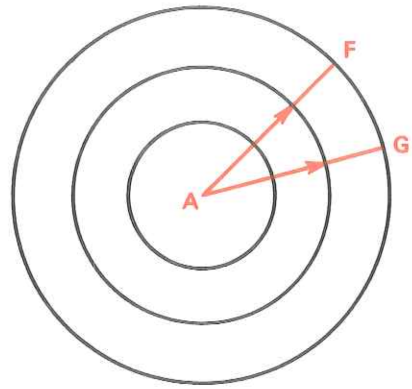


Fig. 1.1. Rays of light from a source *A*.

.e light after reflection, and CD is per-
endicular to the surface at C. EC is called
.e incident ray, CF is the reflected ray,
id CD is the normal (perpendicular) to
.e surface at C. Angle ECD is called the
igle of incidence; angle DCF is called
.e angle of reflection.

An optical disc (Fig. 1.4) may be used
investigate reflection of light. A ray
cident along the disc strikes the mirror
the centre of the disc. The reflected
y is visible on the surface of the disc.
nce the normal to the mirror also lies
1 the surface of the disc, we have illus-
ated here what is called the first law of
flection of light: the incident ray, the
ormal, and the reflected ray lie in the
me plane. The second law of reflection
also illustrated in Figure 1.4: the angle
reflection is equal to the angle of
cidence.

-5 THE SPEED OF LIGHT IN AIR AND IN SPACE

Everyday experience indicates to us
at the time required for light to travel
rrestrial distances is very small. Thus,
e speed of light, if not infinite, must
e very large. In 1676 Olaf Römer made
.e first measurement of the speed of

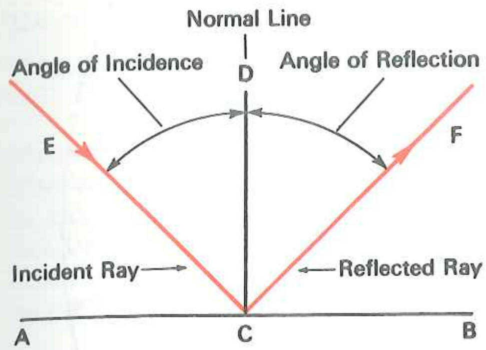


Fig. 1.3. Reflection at a plane surface.

light. His calculations were made as the result of observations of the eclipses of the moons of Jupiter. Römer's information was not accurate enough to permit him to calculate a reliable value for the speed of light. However, he showed that the speed was measurable, and later experimenters, notably Armand Fizeau (1819-1896) and Albert A. Michelson (1852-1931), made measurements of the speed of light over relatively short distances on the surface of the earth. Let us examine Michelson's method briefly.

Michelson and his associates, working in California in the years 1926-1929, measured the time required for light to travel from one mountain to another and back, a total distance of only 44 miles. In order to measure such a short time, they had to develop an ingenious timing device, the essential parts of which are shown in Figure 1.5. A parallel beam of light from a source S fell on an octagonal mirror M, from which it was reflected to a concave mirror C on the other mountain. From the concave mirror the light was reflected to a small plane mirror m, back to the concave mirror, and from there to the octagonal mirror M again. If M remained stationary while the light was travelling from mountain to mountain and back, the returning light was reflected to the telescope T and could be observed through the telescope. The mirror M was then caused to rotate. When the speed of rotation of M was such that section 2 rotated from the position shown in Figure 1.5 to the position initially occupied by section 3 in the time required for the light to make the round trip, light again entered the telescope.

The procedure, then, was to vary the speed of rotation of M until light reappeared in the telescope. The distance travelled by the light (approximately 44 miles) was known; the time required for light to travel that distance was the time required for M to rotate through 45°. This time could be determined quite accurately, as could the distance.

Later Michelson and his co-workers measured the speed of light in a vacuum. They used essentially the same method as that outlined above, in an evacuated underground tunnel a mile long. The light

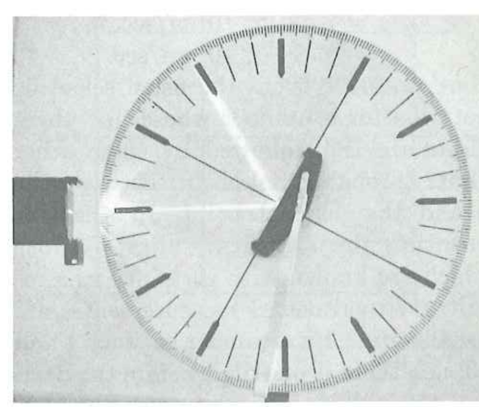


Fig. 1.4. Reflection demonstrated with the optical disc.

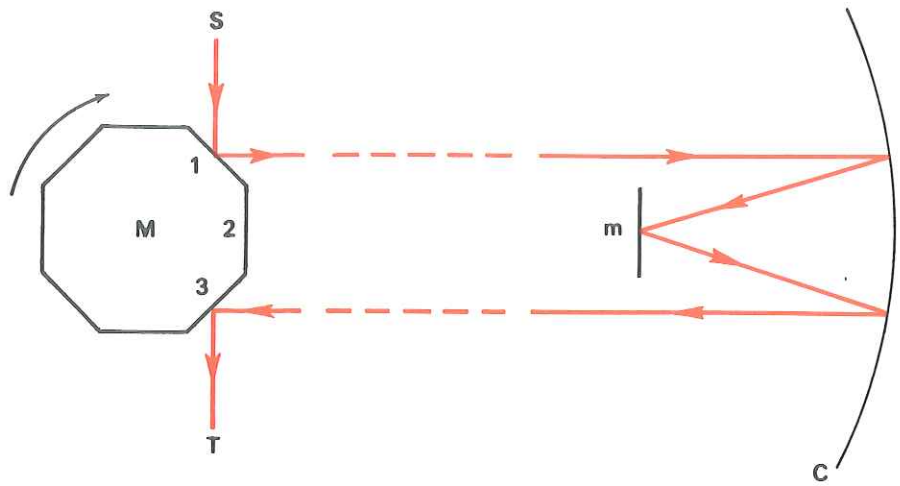


Fig. 1.5. Apparatus used by Michelson in determining the speed of light.

was reflected back and forth in this tunnel, and travelled a total distance of ten miles. They found that the speed in a vacuum was slightly greater than in air. Their accuracy was so great that they were able to state the speed of light in a vacuum to be 299,774 $\pm$ 5 km/sec.

1-6 SCIENTIFIC NOTATION

The speed of light in space is usually given the symbol "c" and has the value

$$c = 3.00 \times 10^8 \text{ m/sec},$$

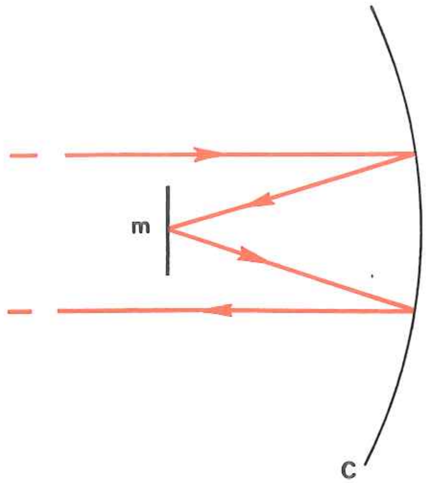
or

$$c = 1.86 \times 10^5 \text{ mi/sec}.$$

Here, 1.86×10^5 is the usual scientific notation for a number whose first three digits are 186, followed by three other digits (probably unknown) between the 6 and the decimal point. In scientific notation the significant digits (those which are known with certainty as a result of experimental measurements) are usually given by a number between 1 and 10, i.e., there is one digit before the decimal point. The exponent of the base 10 is used to indicate the proper position of the decimal point.

Positive exponents of base 10 are used, as above, to simplify the writing of large numbers. Negative exponents of base 10 may be used in a similar manner to simplify the writing of small numbers. Thus 3.24×10^{-4} represents the number 0.000324. The advantage of this notation is obvious if, for example, we wish to calculate the time required for light to travel 6.00 metres at a speed of 3.00×10^8 m/sec. We divide 6.00 by 3.00×10^8 and obtain 2.00×10^{-8}. It is simpler to quote the answer as 2.00×10^{-8} sec than as 0.0000000200 sec.

The order of magnitude of a number is the power of ten closest to that number. Thus the order of magnitude of the number 236 is 10^2, and of the number 0.000961 is 10^{-3}. Frequently an estimate of the order of magnitude of the result of a calculation is the best first step in making that calculation. Such an estimate may be made by performing the necessary arithmetic operations on the orders of magnitude of the numbers involved. Thus



in determining the speed of light.

Positive exponents of base 10 are used, as above, to simplify the writing of large numbers. Negative exponents of base 10 may be used in a similar manner to sim- plify the writing of small numbers. Thus 3.24 $\times$ 10^{-4} represents the number 0.000324. The advantage of this notation s obvious if, for example, we wish to cal- culate the time required for light to travel 6.00 metres at a speed of 3.00 $\times$ 10^8 m/sec. We divide 6.00 by 3.00 $\times$ 10^8 and obtain 2.00 $\times$ 10^{-8}. It is simpler to quote the answer as 2.00 $\times$ 10^{-8} sec than as 0.0000000200 sec.

The order of magnitude of a number s the power of ten closest to that number. Thus the order of magnitude of the num- ber 236 is 10^2, and of the number 0.000961 s 10^{-3}. Frequently an estimate of the order of magnitude of the result of a cal- culation is the best first step in making that calculation. Such an estimate may be made by performing the necessary arithmetic operations on the orders of magnitude of the numbers involved. Thus

the order of magnitude of

$$\frac{12.6 \times 0.082}{91.3} \text{ is } \frac{10^1 \times 10^{-1}}{10^2} \text{ or } 10^{-2}$$

Frequently, too, the number which we wish to describe—the volume of the water in Lake Ontario for example—is so in- definite and variable that to describe it other than by an order of magnitude would not make sense.

1-7 DIRECT PROPORTION

The distance travelled by light (or by a car, or a plane) at constant speed de- pends on the time of travel. Suppose we consider an airplane travelling at 400 mi/hr. The time required for this aircraft to travel 100 mi is 0.25 hr, for 200 mi the time is 0.50 hr, for 600 mi the time is 1.5 hr, and so on. We might tabulate this data as follows:

DISTANCE s (mi)	TIME t (hr)
0	0
100	0.25
200	0.50
300	0.75
400	1.00
600	1.50

We might then plot the information on a graph such as that shown in Figure 1.6; the circled points on the graph give the same information as the table above. Through these points we draw the smoothest curve possible. In this case the "smoothest curve" is a straight line.

The relationship between s and t here is typical of what is called direct pro- portion. The following facts concerning s and t should be noted; similar facts are true for any case where one quantity is directly proportional to another.

(*a*) As t increases, s increases. s is said to be a function of t.

(*b*) If t doubles, s doubles; if t triples, s triples; and so on.

(*c*) The quotient obtained by dividing any value of s by the corresponding value of t is the same as the quotient obtained by dividing any other value of s by its corresponding value of t. This fact may be stated mathematically in two different ways.

(*i*) $\frac{s_1}{t_1} = \frac{s_2}{t_2}$

(*ii*) $\frac{s}{t} = k$, where k is a constant.

i.e., $s = kt$

Here, $k = 400$ mi/hr

and $s = 400t$.

This equation is called the equation of the graph shown in Figure 1.6.

(*d*) The equation above may be arrived at by considering increases in distances

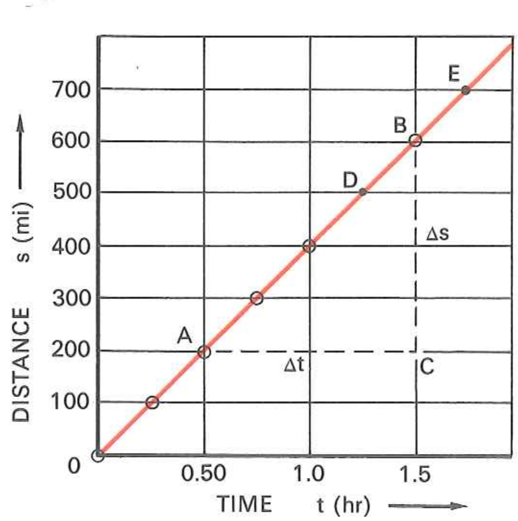


Fig. 1.6. Distance-time graph for an aircraft travelling at 400 mi/hr.

and times, rather than by considering total distances and times. Consider the two points marked A and B on the graph. The increase in distance from A to B, which we shall call Δs (delta s), is BC, and is equal to 400 mi. AC, the corresponding value of Δt, is equal to 1.0 hr. The ratio

$$\frac{\Delta s}{\Delta t} = 400 \text{ mi/hr}$$

or

$$\Delta s = 400\Delta t$$

The ratio $\frac{BC}{AC}$, i.e., $\frac{\Delta s}{\Delta t}$, is called the slope of the straight line graph shown in Figure 1.6. Since the graph is a straight line, the slope is constant.

(*e*) The value of k depends on the units used for s and t. The student should verify that, if s is in mi and t in min, then $k = 6\frac{2}{3}$ mi/min.

(*f*) The mathematical symbol for "is proportional to" is $\propto$. Here, $s \propto t$.

1-8 INTERPOLATION AND EXTRAPOLATION

Consider the point marked D on the graph in Figure 1.6. From the coordinates of this point we conclude that the airplane travelled 500 mi in 1.25 hrs. This will indeed be the case if the speed of the plane remained constant at 400 mi/hr, that is, if we were correct in drawing the graph as a straight line joining the circled points. The process of drawing conclusions from the coordinates of points on a graph intermediate between points whose coordinates represent measured data is called interpolation. Interpolation can sometimes lead us astray. Suppose, for example, that the only information that we had on the plane's flight was that given by the circled points. The plane might very well have changed speed in the interval between $t = 1.00$ hrs and $t = 1.50$ hrs, in which case it might not have covered the 500 mi in 1.25 hrs. The graph for this portion, then, would not have been a straight line.

Consider also the point E on the graph. From the coordinates of E we conclude that the plane would travel 700 mi in 1.75 hrs. The process of drawing conclusions from the coordinates of points on a graph beyond points whose coordinates represent measured data is called extrapolation. Extrapolation involves the same dangers as interpolation, and for the same reasons. However, extrapolation has its uses, for it enables us to make predictions. Sometimes these predictions turn out to be correct; in other cases they may be completely incorrect.

1-9 SPEED OF LIGHT IN MEDIA OTHER THAN AIR

The experimental methods of Michelson, Fizeau, and others can be adapted to the determination of the speed of light in any transparent medium. Foucault, for example, introduced a long tube of water into the path of the light. (See Fig. 1.5, again.) He found that the time of travel of the light was thereby increased, and therefore that the speed of light was less in water than in air. Similar experiments indicate that the speed of light in any transparent material is less than in a vacuum. In air, the speed is only slightly less than in a vacuum. However, the speed in water is $\frac{3}{4}$ of the speed in a vacuum, and the speed in glass is $\frac{2}{3}$ of that in a vacuum.

Because of the fact that the speed of light varies from medium to medium, the time required for light to travel a given distance depends on the medium through which it travels. The time increases as

= 1.50 hrs, in which case it might not ave covered the 500 mi in 1.25 hrs. The raph for this portion, then, would not ave been a straight line.

Consider also the point E on the graph. rom the coordinates of E we conclude hat the plane would travel 700 mi in .75 hrs. The process of drawing conlusions from the coordinates of points on graph beyond points whose coordinates epresent measured data is called extraolation. Extrapolation involves the same angers as interpolation, and for the same easons. However, extrapolation has its ses, for it enables us to make predictions. ometimes these predictions turn out to e correct; in other cases they may be ompletely incorrect.

–9 SPEED OF LIGHT IN MEDIA OTHER THAN AIR

The experimental methods of Michelon, Fizeau, and others can be adapted o the determination of the speed of light n any transparent medium. Foucault, or example, introduced a long tube of ater into the path of the light. (See ig. 1.5, again.) He found that the time f travel of the light was thereby inreased, and therefore that the speed of ght was less in water than in air. Similar xperiments indicate that the speed of ght in any transparent material is less han in a vacuum. In air, the speed is nly slightly less than in a vacuum. However, the speed in water is $\frac{3}{4}$ of the speed n a vacuum, and the speed in glass is $\frac{2}{3}$ f that in a vacuum.

Because of the fact that the speed of ght varies from medium to medium, the ime required for light to travel a given istance depends on the medium through hich it travels. The time increases as

the speed decreases. Let us examine this sort of relationship in greater detail.

1–10 INVERSE PROPORTION

Consider the case of several airplanes, all of which have to make a trip of 1000 miles. The time required for a plane to make the trip depends on that plane's speed. Five possibilities are listed in the table below.

SPEED v (mi/hr)	TIME t (hrs)
500	2.0
400	2.5
250	4.0
200	5.0
125	8.0

The information given in the table is plotted on the graph shown in Figure 1.7. The relationship between v and t here is typical of what is called inverse proportion. The following facts concerning v and t should be noted; similar facts are true for any case in which one quantity is inversely proportional to another.

(a) As v increases, t decreases.

(b) If v is doubled, t is halved; if v is tripled, t is reduced to $\frac{1}{3}$ of its former value.

(c) The product obtained by multiplying any value of v by the corresponding value of t is equal to the product obtained by multiplying any other value of v by its corresponding value of t. This fact may be stated mathematically in two different ways:

(i) $v_1t_1 = v_2t_2$

(ii) $vt = k$, where k is a constant.

Here, $k = 1000$ mi

and $vt = 1000$ or $v = \dfrac{1000}{t}$.

This equation is called the equation of the graph in Figure 1.7.

(d) The graph for inverse proportion is not a straight line, but a portion of an hyperbola. Its slope is not constant, i.e., the value of $\dfrac{\Delta v}{\Delta t}$ depends on what two points on the graph are chosen for the measurements of the changes in speed and time.

(e) The value of the constant k depends on the units used for v and t.

(f) In mathematical notation,

$$t \propto \frac{1}{v}$$

1–11 POWER OF A SOURCE OF LIGHT

The power of a light source is the rate at which that source emits light. A unit commonly used in measuring the power of a source is the candle power, defined originally as the power of a candle manufactured according to definite specifications. The present standard of power is the international standard candle, deter-

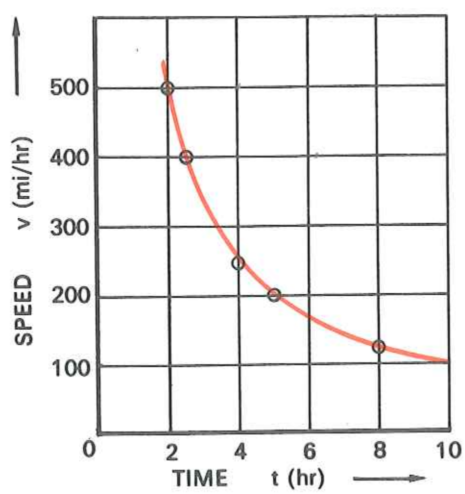
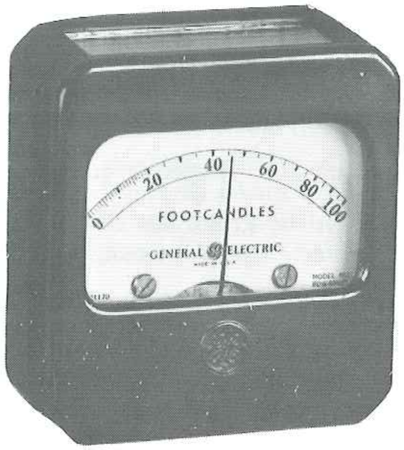


Fig. 1.7. Graph showing the time required for several aircraft to travel 1000 mi at different speeds.



Canadian General Electric Company, Limited

Fig. 1.8. A light meter.

mined in Canada from a set of incandescent lamps maintained by the National Research Council at Ottawa.

The power of a light source, A, may be compared with the power of another source, B, with the aid of a photographic type exposure meter (Fig. 1.8) having a pointer and scale attached. Care should be taken in using the meter to exclude extraneous light and reflected light. If exposure meter readings are taken for each of the two light sources separately, and if the distance from the source to the meter is the same in both cases, the ratio of the meter readings is equal to the ratio of the powers of the two sources. Then, if the candle power of one of the sources is known, the candle power of the other can be calculated.

Suppose that, using this method, we find two light bulbs whose powers are equal. If we now use the two sources simultaneously and close together, and if the distance from the sources to the meter in this case is equal to the distance used for each of the sources separately, the meter reading is double what it was for either of the sources separately. That is, the combined power P is equal to the sum of the individual powers P_1 and P_2.

1-12 INTENSITY OF ILLUMINATION

If we alter the distance between the exposure meter and the light source, the meter reading changes, becoming less as the distance increases. Thus, the meter takes into account both the power of the source and the distance from the source to the meter; it measures intensity of illumination.

The intensity of illumination of a surface at a point, B, is the rate at which light is received by unit area of that surface in the vicinity of B, when the light is incident perpendicular to the surface. If the source, A, is small compared to the distance AB, and if the medium absorbs none of the light, the relationship between the intensity at B and the distance AB can be developed mathematically.

Consider a source, A, radiating light in all directions in three dimensions (Fig. 1.9), at a rate P. At a distance d_1 from A, this energy is distributed over the surface

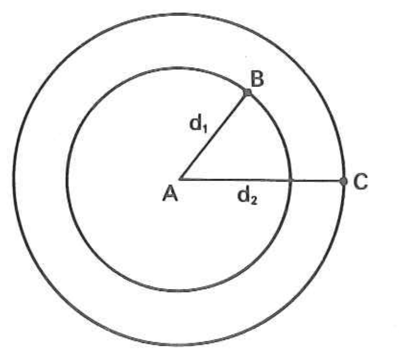


Fig. 1.9. A diagram to assist in deriving the law of inverse squares.

the meter reading is double what it was for either of the sources separately. That is, the combined power P is equal to the sum of the individual powers P_1 and P_2.

1-12 INTENSITY OF ILLUMINATION

If we alter the distance between the exposure meter and the light source, the meter reading changes, becoming less as the distance increases. Thus, the meter takes into account both the power of the source and the distance from the source to the meter; it measures intensity of illumination.

The intensity of illumination of a surface at a point, B, is the rate at which light is received by unit area of that surface in the vicinity of B, when the light is incident perpendicular to the surface. If the source, A, is small compared to the distance AB, and if the medium absorbs none of the light, the relationship between the intensity at B and the distance AB can be developed mathematically.

Consider a source, A, radiating light in all directions in three dimensions (Fig. 1.9), at a rate P. At a distance d_1 from A, this energy is distributed over the surface

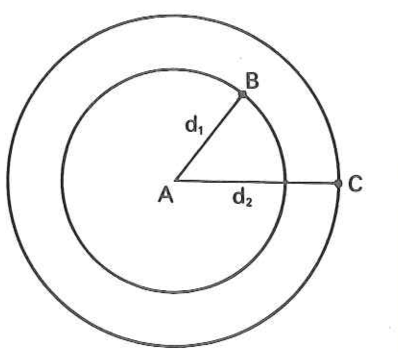


Fig. 1.9. A diagram to assist in deriving the law of inverse squares.

of a sphere of radius d_1, whose surface area is $4\pi d_1^2$. Therefore, the intensity in the vicinity of a point B on this sphere is $I_1 = \frac{P}{4\pi d_1^2}$. Similarly at a point C on the surface of a sphere with centre A and radius d_2, the intensity is $I_2 = \frac{P}{4\pi d_2^2}$.

Hence,

$$\frac{I_1}{I_2} = \frac{\frac{P}{4\pi d_1^2}}{\frac{P}{4\pi d_2^2}}$$

$$\text{or } \frac{I_1}{I_2} = \frac{d_2^2}{d_1^2}.$$

$$\text{or } I \text{ is proportional to } \frac{1}{d^2}.$$

That is, the intensity of the radiation falling on a surface is inversely proportional to the square of the distance from the source to the surface.

1-13 THE INVERSE SQUARE LAW

In the above situation, and in many others encountered in Physics, the value of one variable is inversely proportional to the square of another variable. The equation expressing this relationship is $Id^2 = k$, or $I = \frac{k}{d^2}$. A graph of this relationship is shown in Figure 1.10. For the units used in this graph, what is the value of k, and what is the equation of the graph?

The unit most commonly used in measuring intensity of illumination on a surface is the foot-candle, defined as the intensity of illumination received from one standard candle placed one foot from the surface.

If I represents the intensity of illumination, if P represents the candle power of a source, and if d represents the distance from the source to the surface,

$$I \propto P \text{ when } d \text{ is constant}$$

$$I \propto \frac{1}{d^2} \text{ when } P \text{ is constant.}$$

Hence $I \propto \frac{P}{d^2}$

and $\quad I = \frac{kP}{d^2}$.

From the definition of a foot-candle given above, $k = 1$. Therefore $I = \frac{P}{d^2}$ where P is the candle power of the source, d is the distance in feet from the source to the surface, and I is the intensity of illumination in foot-candles.

The graph for the inverse square law is not a straight line, nor was the graph for inverse proportion (p. 7). Straight line graphs are easy to interpret, and it would be convenient if we could plot the data used in drawing Figure 1.10 so as

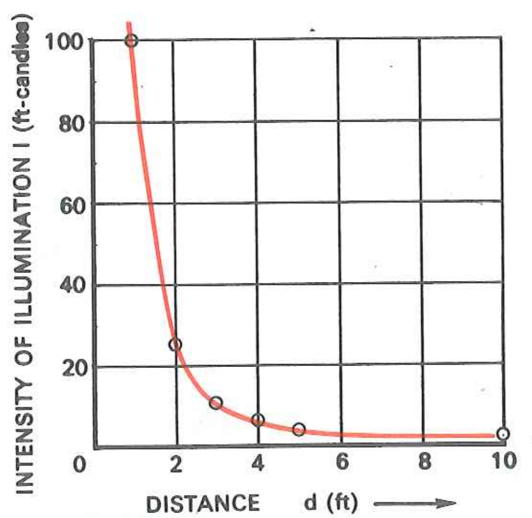


Fig. 1.10 Graph of the relationship $Id^2 = k$.

to produce a straight line graph. If in the relationship $I = \frac{k}{d^2}$ we replace $\frac{1}{d^2}$ by y, we obtain the linear relationship $I = ky$. Then, if we use values of $\frac{1}{d^2}$ as abscissae and values of I as ordinates in drawing the graph, we obtain a straight line (Fig. 1.11). What is the slope of this graph? Compare the slope with the value of k found from Figure 1.10.

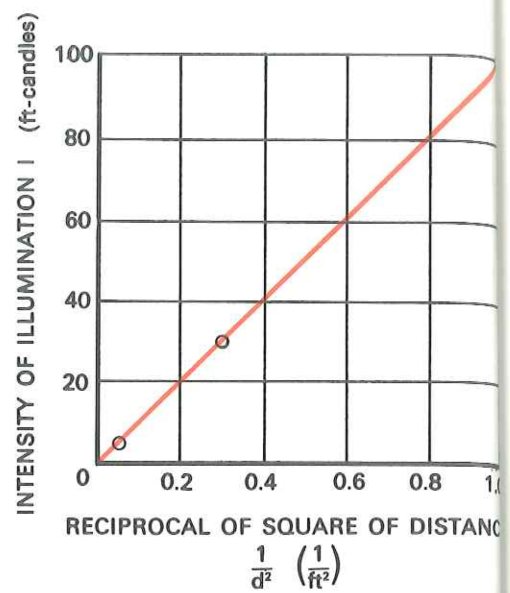


Fig. 1.11. Graph of I plotted against $\frac{1}{d^2}$.

1–14 DIFFRACTION OF LIGHT

Normally, if the source of light is small compared to the distance between the source and the object whose shadow we examine, the edges of the shadow are clearly defined (Fig. 1.12). If the source is larger, the edges of the shadow are less clearly defined (Fig. 1.13) because light reaches parts of the shadow area from some, but not all, of the points on the source. However, even when the source is small, close observation sometimes indicates that the edges of the shadow are fuzzy.

As we look at a long straight source of light through the narrow opening between two fingers held parallel to the source (Fig. 1.14), we observe a pattern similar to that shown in Figure 1.15. The edges of this pattern are in effect the edges of the shadows of the two fingers. These edges are poorly defined, and the whole

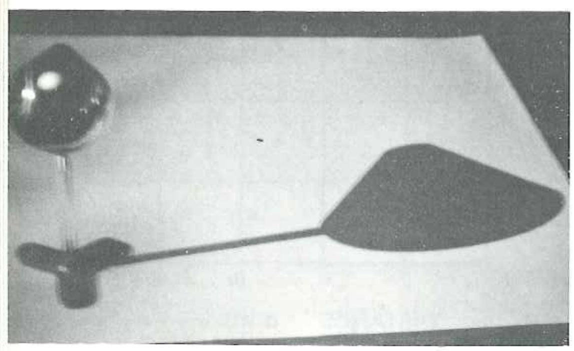


Fig. 1.12. Shadow produced by a small source of light.

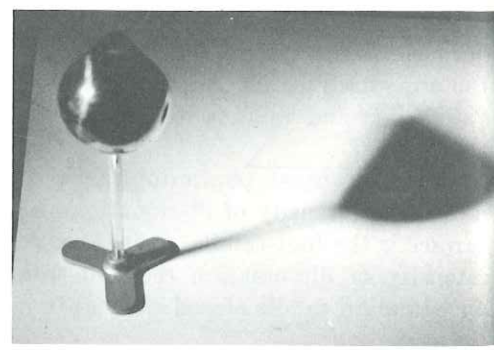


Fig. 1.13. Shadow produced by a large source of light.

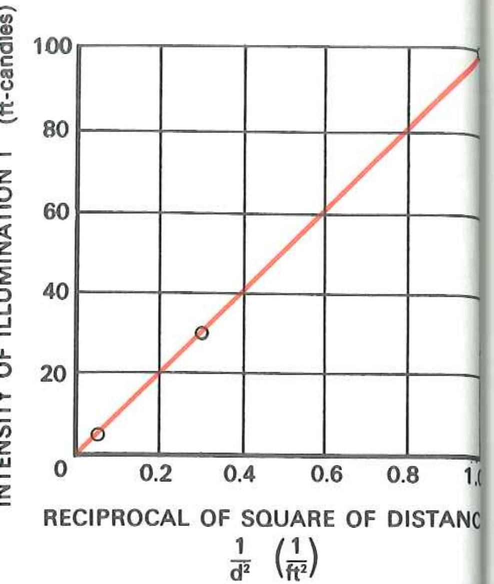
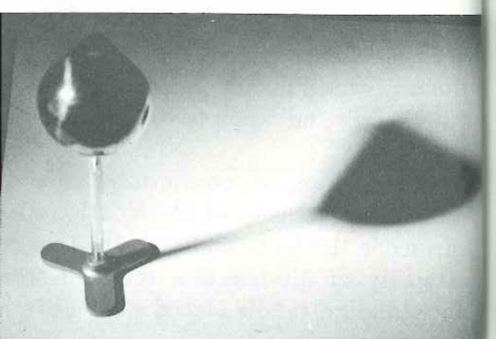


Fig. 1.11. Graph of I plotted against $\frac{1}{d^2}$.

wo fingers held parallel to the source Fig. 1.14), we observe a pattern similar o that shown in Figure 1.15. The edges f this pattern are in effect the edges of he shadows of the two fingers. These dges are poorly defined, and the whole



ig. 1.13. Shadow produced by a large source f light.

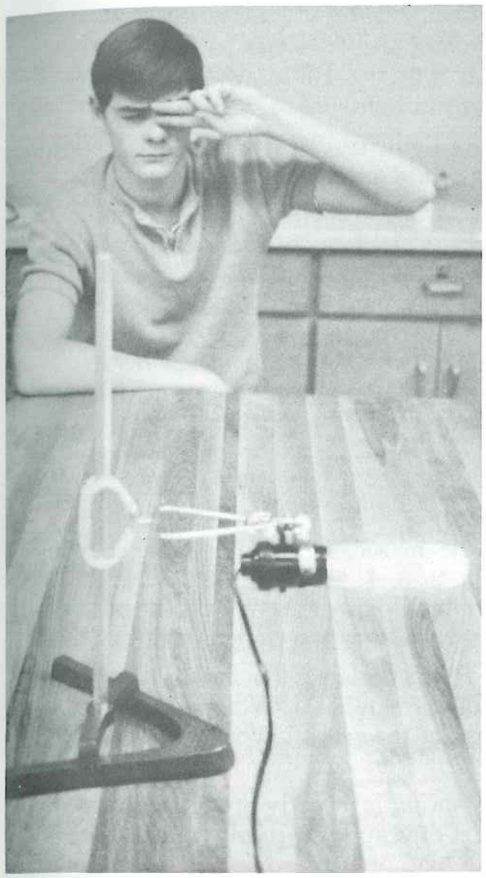
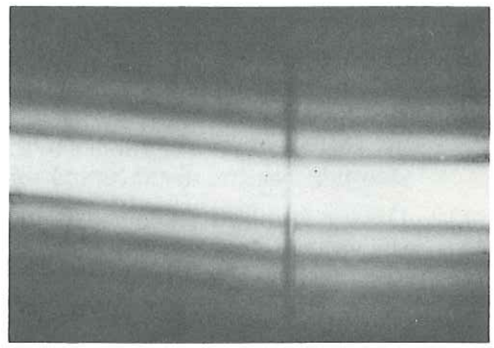


Fig. 1.14. Diffraction of light becomes evident when a narrow source of light is viewed through a narrow opening between two fingers.

pattern is much wider than we have any right to expect. Part of the explanation of this effect is that light bends around corners, that is, it undergoes diffraction. Normally the amount of diffraction is negligible and diffraction may be ignored. However, the existence of appreciable diffraction under certain circumstances is of great importance in determining how light is transmitted.



Physics Department, University of Western Ontario

Fig. 1.15. Diffraction pattern which results when the procedure in Figure 1.14 is followed.

1–15 PROBLEMS

1. An airplane is often visible in the sky up to 30 minutes after sunset. Explain.
2. A horizontal ray of light strikes a vertical plane mirror. The mirror then rotates through 10° about a vertical axis through the point of incidence. Through how large an angle does the reflected ray rotate?
3. In Michelson's experiment to determine the speed of light, the light ray was reflected several times. Michelson's calculations were based on the assumption that the speed of light does not change when the light is reflected. How might he have tested the validity of this assumption?
4. Discuss the action of automobile headlights in illuminating the road in front of the automobile.

5. A light-year is the distance that light travels in a year. State the order of magnitude of a light year (*a*) in miles, (*b*) in kilometres.
6. The distance from the earth to the sun is 9.3×10^7 mi. State the order of magnitude of this distance (*a*) in mi, (*b*) in light-years.
7. A radar signal, travelling at the speed of light, travels from the earth to the moon and back in 2.7 sec. Calculate the distance from the earth to the moon, in metres.
8. Radio waves travel at the speed of light. Calculate the order of magnitude, in sec, of the time required for radio waves to travel across Canada.
9. What is the order of magnitude of the number 4178? (Note that $10^{\frac{1}{2}} = \sqrt{10} = 3.16$.)
10. Simplify each of the following:

(*a*) $3 \times 10^5 \times 10^6$ (*b*) $\dfrac{2.5 \times 10^4}{5 \times 10^{-2}}$ (*c*) $3.9 \times 10^8 \times 5 \times 10^{-3}$

(*d*) $\dfrac{4.2 \times 10^{-6}}{7 \times 10^{-2}}$ (*e*) $\dfrac{6 \times 10^{-2}}{1.5 \times 10^4}$ (*f*) $\dfrac{4 \times 10^{-2} \times 3.2 \times 10^6}{8 \times 10^4}$

11. The speed of light in space is 3.0×10^8 m/sec. Draw up a table showing the distances travelled by light in 0, 1, 2, 3, 4, and 5 sec. Plot a graph of distance against time, similar to the graph in Figure 1.6. What is the relationship between the distance and the time? What is the slope of the graph?
12. Draw a graph in which the perimeter P of a square, in inches, is plotted on the vertical axis, against the length L of one side, in ft. What is the relationship between P and L? What is the slope of the graph? What is the equation of the graph?
13. For moderate loads, the extension of a spring is directly proportional to the load hanging from the spring. What sort of graph will result when extension is plotted against load? Suppose that a load of 2 kg. causes an extension of 5 cm. (*a*) What extension will be produced by each of the following loads: (*i*) 1 kg, (*ii*) 0.4 kg, (*iii*) 1.2 kg? (*b*) What load will be required to produce each of the following extensions: (*i*) 2 cm, (*ii*) 4 cm, (*iii*) 0.5 cm?
14. Two quantities, F and a, are related according to the set of values shown below.

F	0	2	3	5	8
a	0	0.8	1.2	2.0	3.2

What is the relationship between F and a? Check by plotting F against a. By interpolation or extrapolation, determine (*i*) the values of a when F is 1, 7, and 10; (*ii*) the values of F when a is 0.6, 1.8, 5.4.

15. Draw a graph in which the area A of a square, in ft^2, is plotted on the vertical axis, against the length L of one side, in yd. If the graph is not a straight line, replot the information so as to obtain a straight line graph. What is the slope of this second graph? What is its equation?

travels in a year. State the order of (b) in kilometres.

ı is 9.3×10^{7} mi. State the order of b) in light-years.

of light, travels from the earth to the the distance from the earth to the

nt. Calculate the order of magnitude, vaves to travel across Canada.

number 4178?

10^4 / $^{-2}$ (c) $3.9 \times 10^8 \times 5 \times 10^{-3}$

$^{-2}$ / 0^4 (f) $\dfrac{4 \times 10^{-2} \times 3.2 \times 10^6}{8 \times 10^4}$

10^8 m/sec. Draw up a table showing , 2, 3, 4, and 5 sec. Plot a graph of ph in Figure 1.6. What is the relation- e? What is the slope of the graph?

P of a square, in inches, is plotted on f one side, in ft. What is the relation- pe of the graph? What is the equation

spring is directly proportional to the rt of graph will result when extension a load of 2 kg. causes an extension of uced by each of the following loads: What load will be required to produce cm, (ii) 4 cm, (iii) 0.5 cm?

according to the set of values shown

	5	8
.2	2.0	3.2

d a? Check by plotting F against a. ermine (i) the values of a when F is a is 0.6, 1.8, 5.4.

quare, in ft^2, is plotted on the vertical in yd. If the graph is not a straight ain a straight line graph. What is the s equation?

16. The area of a circle is proportional to the square of its radius, i.e., $A = kr^2$. What is the value of k if (a) r is in cm and A in cm^2, (b) r is in cm and A in mm^2?

17. In similar triangles, corresponding sides are proportional. What does this statement mean?

18. The areas of similar triangles are proportional to the squares on corresponding sides. What does this statement mean?

19. (a) A beam of light passes from point A to point B in air in a certain time. Calculate the change in time if a plate of glass, 10 cm thick, is inserted between A and B, at right angles to the beam, given that the speed of light in air is 3.00×10^{10} cm/sec and in glass is 2.00×10^{10} cm/sec. (b) Repeat the calculation for the following materials inserted in the beam: (i) 10 cm of water in which the speed of light is 2.25×10^{10} cm/sec, (ii) 10 cm of alcohol in which the speed of light is 2.20×10^{10} cm/sec, (iii) 10 cm of carbon disulphide in which the speed of light is 1.88×10^{10} cm/sec, (iv) 10 cm of diamond in which the speed of light is 1.22×10^{10} cm/sec. (c) Plot a graph of the time required for light to travel through 10 cm of the medium against the speed of light in that medium. (See Fig. 1.7.) (d) Replot the information so as to obtain a straight line graph. What is the relationship between the time and the speed?

20. Two quantities, m and a, are related according to the set of values shown below:

m	1	2	4	10	20
a	10	5	2.5	1	0.5

What is the relationship between m and a? Check by plotting one or more graphs. By interpolation or extrapolation, determine (i) the values of a when m is 0.5, 5, and 40; (ii) the values of m when a is 20, 2, and 0.1.

21. Consider the relationship $E = \frac{1}{2}mv^2$. (a) What is the effect on E of (i) doubling m, (ii) doubling v, (iii) tripling m and halving v? (b) By what factor must (i) m, (ii) v, be changed in order for E to double?

22. Consider the relationship $F \propto \dfrac{mM}{r^2}$. (a) What is the effect on F of (i) changing m by a factor of 2, (ii) changing M by a factor of 8, (iii) changing r by a factor of $\frac{1}{4}$, (iv) making all of the changes indicated in (i), (ii) and (iii)? (b) By what factor must (i) m, (ii) M, (iii) r, be changed in order to change F by a factor of 3?

23. Two angles, i and R, are related according to the set of values shown below:

i	20°	30°	40°	50°	60°	70°
R	13°	20°	25°	30°	35°	38°

What is the relationship between i and R?

24. If the intensity of illumination one foot from a light source is 900 ft-candles, what is the intensity of illumination at a distance of 5 ft from the same source?

25. A 100-watt lamp placed 2 feet from a newspaper provides suitable illumination for reading. How far from a 250-watt bulb should the newspaper be placed in order to receive the same illumination?

26. Describe briefly why an eclipse (*a*) of the sun, (*b*) of the moon, occurs.

27. Does the fact that light undergoes diffraction contradict the statement that light travels in straight lines?

1-16 SUMMARY

Light travels in straight lines (rays) through many materials and through space. A small amount of diffraction occurs when obstacles are encountered. Most objects reflect at least some of the light which is incident on them; the objects become visible because of this reflected light. There are two laws of reflection: (1) The incident ray, the normal, and the reflected ray lie in the same plane. (2) The angle of reflection is equal to the angle of incidence.

The speed of light in space is 3.0×10^8 m/sec, and is less in any other medium than in space. Positive exponents of the base 10 may be used, as they are here, to simplify the writing of large numbers; negative exponents may be used for small numbers.

If $x \propto y$, then $x = ky$, and the graph of x plotted against y is a straight line. If $x \propto \frac{1}{y}$, then $xy = k$, the graph of x plotted against y is an hyperbola, and the graph of x plotted against $\frac{1}{y}$ is a straight line.

For a point source of light, the intensity of illumination on a surface is directly proportional to the power of the source and inversely proportional to the square of the distance from the source to the surface.

ot from a light source is 900 ft-candles,
a distance of 5 ft from the same source?

a newspaper provides suitable illumi-
50-watt bulb should the newspaper be
umination?

the sun, (b) of the moon, occurs.

fraction contradict the statement that

to simplify the writing of large numbers;
negative exponents may be used for small
numbers.

If $x \propto y$, then $x = ky$, and the graph of x plotted against y is a straight line.

If $x \propto \frac{1}{y}$, then $xy = k$, the graph of x plotted against y is an hyperbola, and the graph of x plotted against $\frac{1}{y}$ is a straight line.

For a point source of light, the intensity of illumination on a surface is directly proportional to the power of the source and inversely proportional to the square of the distance from the source to the surface.

Chapter 2

FACTS WITHOUT THEORY: Refraction and Dispersion

2–1 INTRODUCTION

We have already noted that light travels in straight lines in any homogeneous medium where no obstacles are encountered. We have noted, too, that if an obstacle is encountered, the direction in which light travels may be altered by diffraction or reflection. There is a third phenomenon as a result of which the direction of a ray may be changed. It occurs when light passes from one transparent medium to another, and is called refraction.

2–2 REFRACTION, AIR TO WATER AND WATER TO AIR

The photograph (Fig. 2.1), and the drawing (Fig. 2.2) show the path of a ray of light as it passes from air to water, then from water to air again. The water is contained in a rectangular plastic vessel, and the ray is incident obliquely from the lower left of Figure 2.1. The path of the ray may be made visible by chalk dust in the air and by a small amount of fluorescein in the water. Some of the incident light is reflected at the surface of

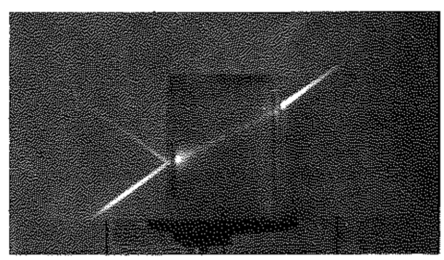


Fig. 2.1. Refraction of light as it passes from air to water, and then from water to air.

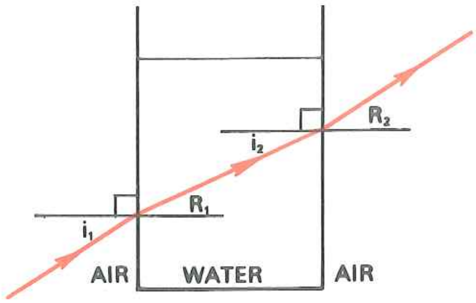


Fig. 2.2. Ray diagram for light passing obliquely from air to water to air.

the water, but some of it enters the water and undergoes a distinct change in direction. This change in direction is called refraction. The path of the light after refraction is called the refracted ray. The angle between the refracted ray and the normal is called the angle of refraction and is given the symbol R.

The refraction at the left surface shows that, when light passes from air to water, it is refracted toward the normal, i.e., the angle of refraction is less than the angle of incidence. When the light passes from water to air (at the right surface), refraction again takes place. In this case the refraction is away from the normal, i.e., the angle of refraction is greater than the angle of incidence.

2–3 REFRACTION, AIR TO GLASS AND GLASS TO AIR

In order to investigate the refraction which takes place from air to glass or glass to air we may mount a rectangular piece of glass on an optical disc. If the angle of incidence is zero, no refraction occurs at either surface. However, if the angle of incidence is not zero, refraction occurs at both surfaces as shown in Figure 2.3. The directions of refraction are the same as for water, but the amounts may differ.

2–4 SNELL'S LAW

So far our discussion of refraction has been qualitative, i.e., it has not involved measurement. We may investigate refraction quantitatively by mounting a thick semi-circular piece of glass on an optical disc as in Figure 2.4. A ray of light incident on the glass at A is refracted, but no refraction occurs at B since the ray is normal to the circumference at this point. When we rotate the disc, the angle of incidence changes and the angle of refraction changes too. A typical set of corresponding values of i and R follows:

i	20°	30°	40°	50°	60°	70°
R	13°	20°	25°	30°	35°	38°

What relationship, if any, exists between i and R? The answer to this question eluded investigators for centuries, and it may well have eluded you, for

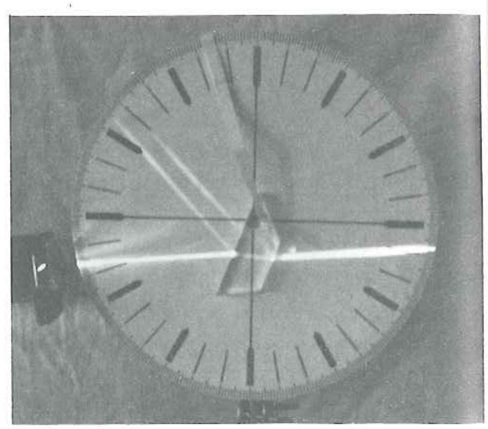


Fig. 2.3. Refraction of light as it passes from air to glass, and then from glass to air. Note the partial reflection at both surfaces.

are the same as for water, but the amounts may differ.

2–4 SNELL'S LAW

So far our discussion of refraction has been qualitative, i.e., it has not involved measurement. We may investigate refraction quantitatively by mounting a thick semi-circular piece of glass on an optical disc as in Figure 2.4. A ray of light incident on the glass at A is refracted, but no refraction occurs at B since the ray is normal to the circumference at this point. When we rotate the disc, the angle of incidence changes and the angle of refraction changes too. A typical set of corresponding values of i and R follows:

i	20°	30°	40°	50°	60°	70°
R	13°	20°	25°	30°	35°	38°

What relationship, if any, exists between i and R? The answer to this question eluded investigators for centuries, and it may well have eluded you, for

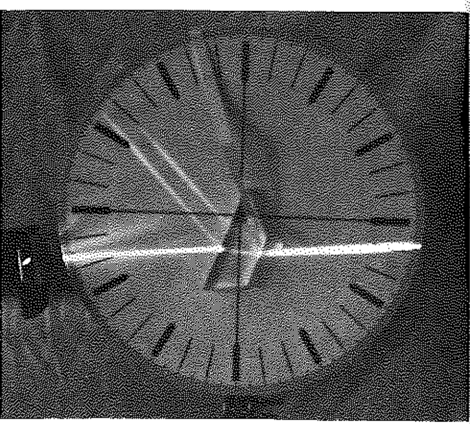


Fig. 2.3. Refraction of light as it passes from air to glass, and then from glass to air. Note the partial reflection at both surfaces.

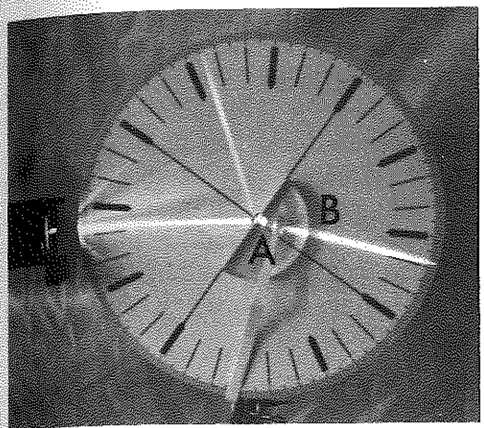


Fig. 2.4. Refraction from air to glass, demonstrated with the optical disc.

these are the values given in question 23 on page 13. The relationship was finally discovered by a German physicist, Willebrord Snell (1591-1676), and it is known as Snell's law. Snell discovered that $\sin i$ is proportional to $\sin R$; i.e., that $\frac{\sin i}{\sin R}$ is a constant.

Let us check this relationship for the set of observed values given above. (If you are not familiar with the trigonometric ratios of angles, see page 83 in the appendix. The appendix also contains trigonometric tables).

i	20°	30°	40°	50°	60°	70°
R	13°	20°	25°	30°	35°	38°
$\sin i$	0.342	0.500	0.643	0.766	0.866	0.940
$\sin R$	0.225	0.342	0.423	0.500	0.574	0.616
$\frac{\sin i}{\sin R}$	1.52	1.46	1.52	1.53	1.51	1.53

Within the limits of experimental error, the value of $\frac{\sin i}{\sin R}$ appears to be constant.

This constant, frequently given the symbol N, is called the relative index of refraction for air and glass. Its value appears to be about 1.5.

2–5 TWO LABORATORY EXERCISES: REFRACTION

Two relatively simple methods for determining indices of refraction are outlined below. You should try both of them.

(*a*) Place a semi-circular plastic dish (Fig. 2.5), half-filled with water, on a sheet of paper, and outline it in pencil. Place a pin B at the centre of the semicircle, and a second pin A on the same side of the diameter as B. Place a third pin C on the other side of the diameter, and apparently in line with A and B as you look through the water. Remove the plastic dish, mark the positions of the pins before removing them, draw the incident and refracted rays and the normal, and measure i and R. Repeat for different values of i by altering the position of

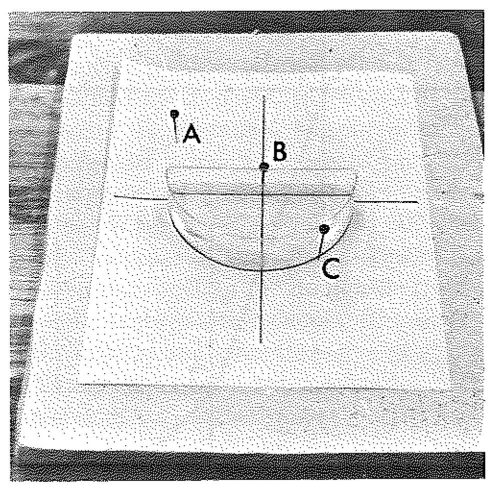


Fig. 2.5. Determination of the index of refraction for light passing from air to water.

pin A. Evaluate *sin* i and *sin* R in each case. Is *sin* i proportional to *sin* R? Check by drawing a graph of *sin* i versus *sin* R. What is the slope of the graph? What is the index of refraction for light passing from air to water? Estimate the number of decimal places to which your result is valid, keeping in mind the sources of error in this exercise, and your precision in measuring i and R. Did you check the case in which $i = 0$? What is the value of zero divided by zero in this case (refer to your graph)?

(b) Lay a rectangular block of plate glass on a sheet of paper and outline it in pencil. Place two pins M and N on one side of the glass (Fig. 2.6). Place two more pins P and Q on the other side of the glass, and apparently in line with M and N as you look through the glass. Remove the block of glass, mark the positions of the pins, and draw in the incident and refracted rays at both surfaces. Draw the normal at each surface, and measure i and R in each case. Compute two indices of refraction: the index for light passing from air to glass, and that for light passing from glass to air. What is true of these two indices? Check your answer by repeating the exercise several times.

2-6 THE LAWS OF REFRACTION

The results of our observations so far may be summarized in two laws of refraction:

(1) The incident ray, the normal, and the refracted ray lie in the same plane. (This law is not necessarily true for some crystalline substances, such as calcite which may have two refracted rays, one of which is not in the same plane as the incident ray and the normal.)

(2) Snell's Law:

$$\frac{\sin i}{\sin R} \text{ is a constant.}$$

This law implies several facts which we have observed, and which are worth summarizing here.

(a) If $i = 0$, then $R = 0$; that is, no refraction occurs when the incident ray is perpendicular to the surface between the two media.

(b) If the speed of light is less in the second medium than in the first, the refraction is toward the normal. R is then less than i, *sin* R is less than *sin* i, and the index of refraction is greater than 1.

(c) If the speed of light is greater in the second medium than in the first, the

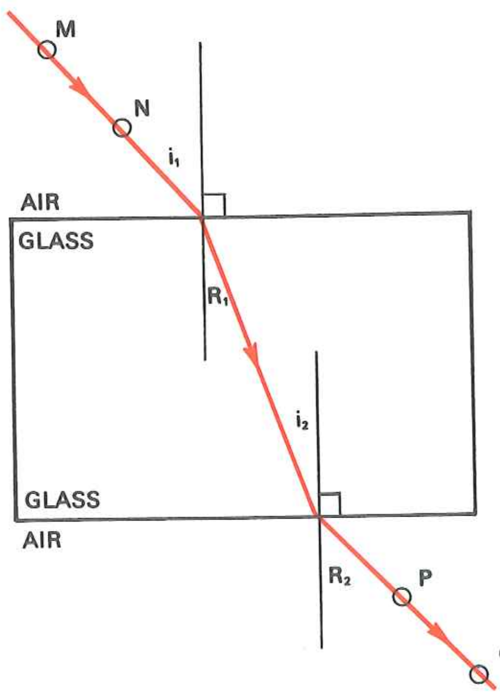


Fig. 2.6. Measurement of indices of refraction, air to glass and glass to air.

Remove the block of glass, mark the positions of the pins, and draw in the incident and refracted rays at both surfaces. Draw the normal at each surface, and measure i and R in each case. Compute two indices of refraction: the index for light passing from air to glass, and that for light passing from glass to air. What is true of these two indices? Check your answer by repeating the exercise several times.

2–6 THE LAWS OF REFRACTION

The results of our observations so far may be summarized in two laws of refraction:

(1) The incident ray, the normal, and the refracted ray lie in the same plane. (This law is not necessarily true for some crystalline substances, such as calcite which may have two refracted rays, one of which is not in the same plane as the incident ray and the normal.)

(2) Snell's Law:

$$\frac{\sin i}{\sin R} \text{ is a constant.}$$

This law implies several facts which we have observed, and which are worth summarizing here.

(*a*) If $i = 0$, then $R = 0$; that is, no refraction occurs when the incident ray is perpendicular to the surface between the two media.

(*b*) If the speed of light is less in the second medium than in the first, the refraction is toward the normal. R is then less than i, $\sin R$ is less than $\sin i$, and the index of refraction is greater than 1.

(*c*) If the speed of light is greater in the second medium than in the first, the

Fig. 2.6. Measurement of indices of refraction, air to glass and glass to air.

refraction is away from the normal. In this case, R is greater than i, $\sin R$ is greater than $\sin i$, and the index of refraction is less than 1.

(*d*) For light passing from one medium to a second medium, the index of refraction is the reciprocal of the index of refraction for light passing from the second medium to the first.

2–7 INDICES OF REFRACTION

The value of an index of refraction depends on the two media involved, and on the order in which these two media occur. The value also depends to a small extent, as we will find later in this chapter, on the colour of the light. Usually the indices of refraction, which are tabulated, are those that apply when light passes from a vacuum to the material listed. They are called absolute indices of refraction. They do not differ significantly from those tabulated below for air, since the speed of light in a vacuum is only slightly greater than the speed of light in air.

INDICES OF REFRACTION OF YELLOW LIGHT FROM AIR TO:	
Water at 20° C	1.33
Water at 90° C	1.32
Alcohol	1.36
Carbon disulphide	1.62
Crown glass	1.52
Flint glass	1.65
Diamond	2.47
Turpentine	1.47

The ratio $\dfrac{\sin i}{\sin R}$ for any two material media is called the relative index of refraction for those two media. Its value is the quotient obtained by dividing the absolute index of refraction of the second medium by that of the first. A partial proof of this fact is given in Question 6 on page 26; you should complete the proof. For example, the relative index of refraction for light passing from water at 20° C to crown glass, using the indices listed in the table above, is calculated as follows:

$$\text{Water } N \text{ glass} = \frac{N \text{ glass}}{N \text{ water}} = \frac{1.52}{1.33} = 1.14$$

2–8 PARTIAL AND TOTAL REFLECTION

When light passes from glass to air, its speed increases and the light is refracted away from the normal. Let us investigate this situation in greater detail with the semi-circular glass block mounted on the optical disc. The arrangement is as shown in Figure 2.7. A ray of light entering the block at A is not refracted, since $i = 0$. However, when the ray reaches B, both reflection and transmission occur. For the reflected portion, the angle of reflection is equal to the angle of incidence. For the transmitted portion, refraction away from the normal takes place. If we rotate the disc so as to increase i, the intensity of the reflected portion increases and the intensity of the refracted portion decreases. When $i = 41°$, $R = 90°$ and cannot increase further. For angles of incidence greater than 41° no light is transmitted; total reflection occurs. Hence, 41° is the critical angle for this type of glass.

The occurrence of total reflection and the size of the critical angle can be predicted. Total reflection can occur only when light is attempting to pass from one medium to a medium in which its speed is greater. In this case R is always greater

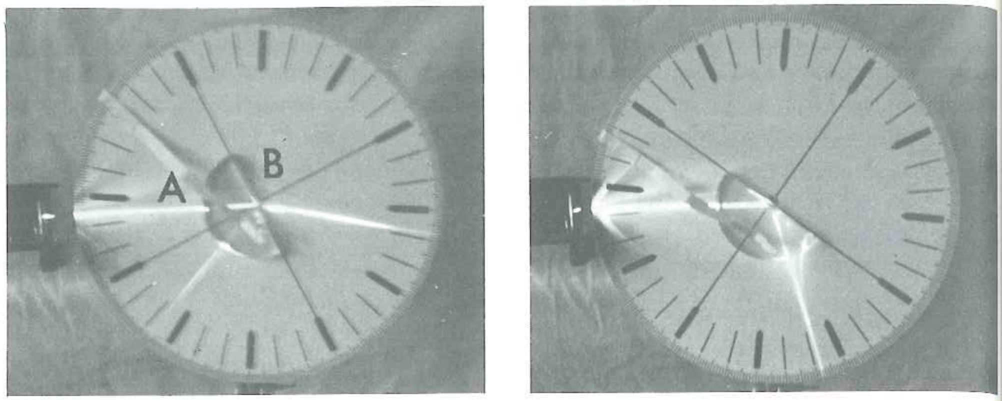


Fig. 2.7. Internal reflection in glass, accompanied by refraction from glass to air *(left)*. Total internal reflection in glass *(right)*.

than i and therefore R reaches its maximum value of 90° when i is still considerably less than 90°.

If the index of refraction from air to glass is 1.52, then the index of refraction from glass to air is $\frac{1}{1.52}$, or 0.66. That is,

$$\frac{\sin i}{\sin R} = 0.66$$

When R attains its maximum value of 90°,

$$\sin R = 1$$

$$\frac{\sin i}{1} = 0.66$$

$$i = 41.3°$$

This is the value of the critical angle for this type of glass.

2–9 REFRACTION BY A PRISM

The triangle ABC in Figure 2.8 represents the cross-section of an equilateral glass prism. The path of a ray of light through such a prism is as shown in the diagram. Refraction toward the normal occurs at the first surface; refraction away from the normal occurs at the second surface. The combined effect of these two refractions is to produce considerable change in the direction of the light. Angle DEF measures this change in direction; it is called the angle of deviation.

Images of objects viewed through a prism are frequently multi-coloured. The great English scientist, mathematician and philosopher, Sir Isaac Newton (1642-1727), made an extensive investigation of the production of these coloured images. Earlier philosophers had attempted to explain the production of the colours by assuming that the colours originated within the prism. Beginning in 1668,

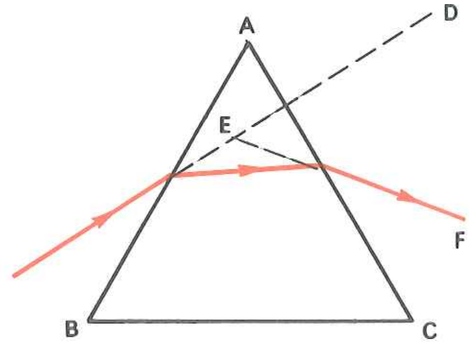
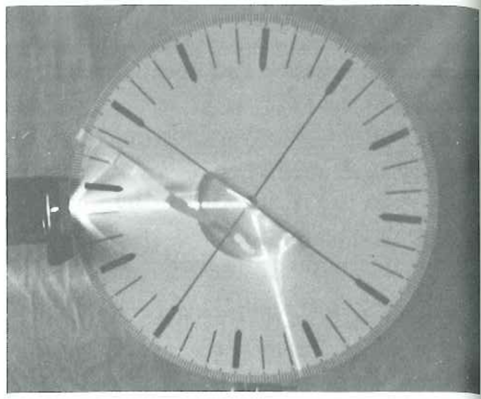


Fig. 2.8. Deviation in a prism.



ed by refraction from glass to air *(left)*. Total

change in the direction of the light. Angle *DEF* measures this change in direction; it is called the angle of deviation.

Images of objects viewed through a prism are frequently multi-coloured. The great English scientist, mathematician and philosopher, Sir Isaac Newton (1642-1727), made an extensive investigation of the production of these coloured images. Earlier philosophers had attempted to explain the production of the colours by assuming that the colours originated within the prism. Beginning in 1668,

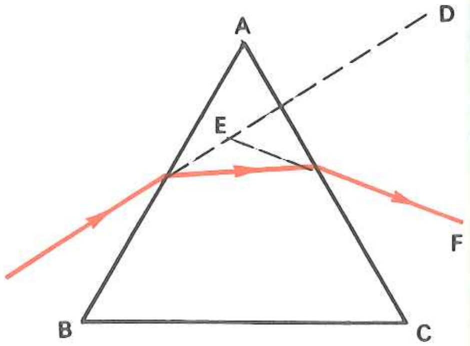


Fig. 2.8. Deviation in a prism.

Newton performed a series of experiments which showed that such was not the case.

2–10 DISPERSION IN A PRISM

Newton permitted a parallel beam of sunlight to fall on a narrow slit, and allowed the narrow beam (ray) emerging from the slit to fall obliquely on the surface *AB* of a glass prism (Fig. 2.9). The light emerging from the prism fell on a screen, forming an image which consisted of brilliant colours ranging from red at one end through orange, yellow, green and blue to violet at the other end. This multi-coloured image is called a spectrum; the spreading of the rays to form a spectrum is called dispersion. In the spectrum produced by a prism when the incident light is "white", the colour change is gradual; there is no sharp dividing line between one colour and the next. Such a spectrum is called a continuous spectrum.

Newton found that, if the incident light was of one colour only, the spectrum contained only that colour. Hence it seemed unlikely that the prism was the source of the colour. Newton suspected that the colours had been present in the incident white light and that the prism had separated them. He confirmed this suspicion by placing a second prism *B* (Fig. 2.10), identical to the first prism *A*, in the position shown. The beam of light emerging from *B* was almost white in colour.

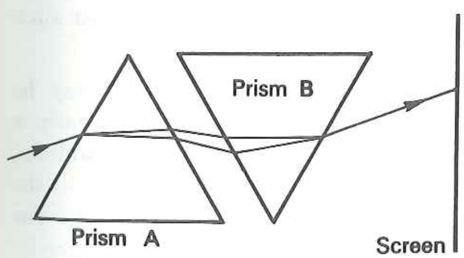


Fig. 2.10. Dispersion and recomposition of white light.

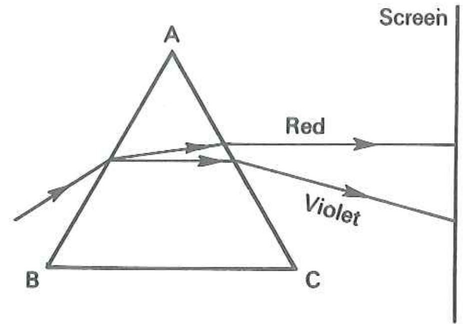


Fig. 2.9. Dispersion of white light by a prism.

Newton concluded that white light is composite in nature, being composed of many colours. When white light is incident on a prism, the components undergo different amounts of deviation, red the least and violet the most. The dispersion or spreading of the colours begins at the first surface, and continues at the second surface. At each surface the red component undergoes a smaller amount of refraction than does the violet component. It follows, then, that the index of refraction for red light is less than that for violet light. For this reason, tables of indices of refraction (see page 19) should specify the colour of the light.

2–11 THE DIFFRACTION GRATING

Instruments designed for the viewing of spectra are called spectroscopes. In many spectroscopes, prisms are used to disperse the light. Equally good instruments can be constructed considerably more cheaply by replacing the prism by a diffraction grating. Good gratings consist of a large number of parallel lines (as many as 20,000 lines per inch) ruled with a diamond point on the surface of glass. Inexpensive gratings are copies or replicas of ruled gratings. Gelatin or some similar material is poured over a ruled grating and allowed to solidify. The gelatin film, when removed, retains an impression of the ruled grating and forms a replica grating good enough for many uses.

The theory of diffraction gratings is complex and need not concern us here. Figure 2.11 is a photograph of a grating spectroscope in use. The end of the spectroscope away from the eye contains a narrow vertical slit. The end of the tube nearest the eye contains the grating; the lines on the grating should also be vertical. If the observer looks through the spectroscope directly at the source of light, he sees an image of the slit, and on either side he sees at least one continuous spectrum, similar to that produced by a prism.

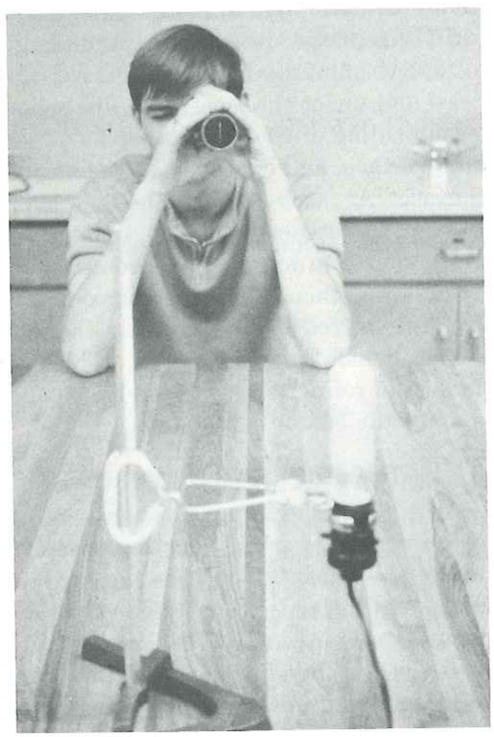


Fig. 2.11. A grating spectroscope in use.

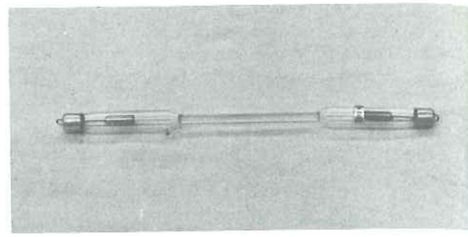


Fig. 2.12. A gas discharge tube.

2–12 LABORATORY EXERCISE: SPECTRA

A diffraction grating spectroscope can be used to examine the spectra of light from various sources. A continuous spectrum is observed if the source is an incandescent solid—for example, the glowing carbon in the flame of a coal oil lamp or the glowing tungsten filament in an incandescent lamp. Incandescent gases and vapours, on the other hand, usually produce an altogether different type of spectrum.

(*a*) The low pressure gas (neon, hydrogen, or argon, for example), in a gas discharge tube (Fig. 2.12) may be made incandescent by connecting the electrodes in the tube to an induction coil. Observe the spectra produced by several such tubes.

(*b*) An incandescent vapour may be produced by heating a salt (usually a chloride) of an element such as sodium, potassium, lithium, or strontium, in the flame of a bunsen burner (Fig. 2.13). The salt may be introduced into the flame by dipping a platinum wire into a concentrated solution of the salt and then holding

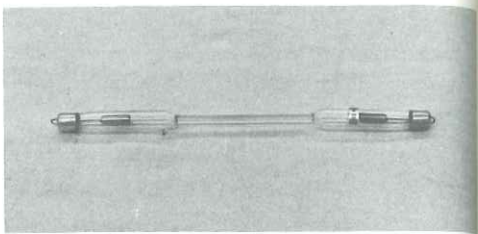


Fig. 2.12. A gas discharge tube.

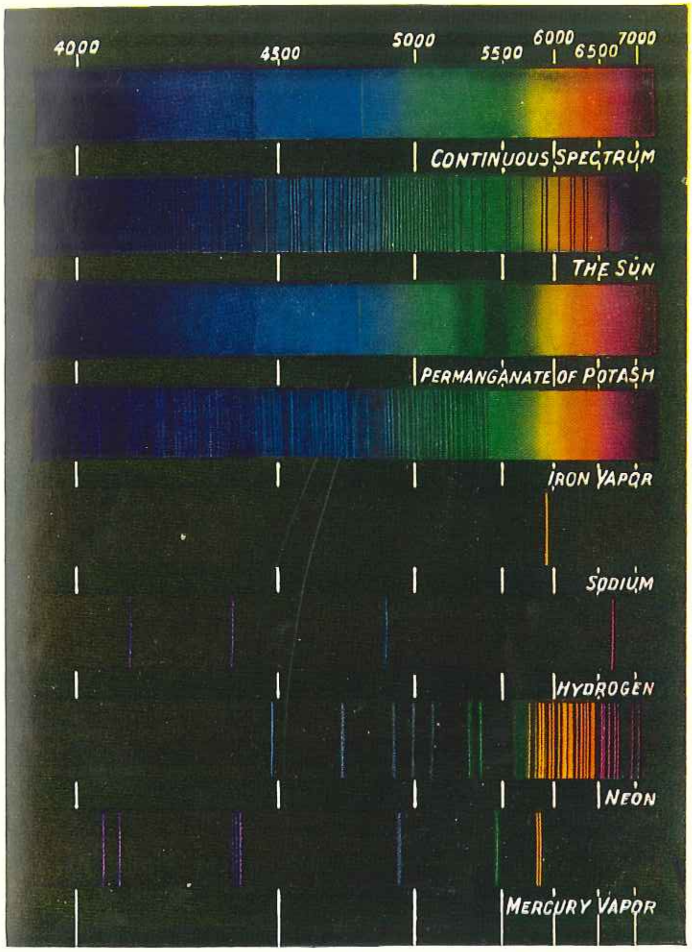
scope directly at the source of light, he sees an image of the slit, and on either side he sees at least one continuous spectrum, similar to that produced by a prism.

2–12 LABORATORY EXERCISE: SPECTRA

A diffraction grating spectroscope can be used to examine the spectra of light from various sources. A continuous spectrum is observed if the source is an incandescent solid—for example, the glowing carbon in the flame of a coal oil lamp or the glowing tungsten filament in an incandescent lamp. Incandescent gases and vapours, on the other hand, usually produce an altogether different type of spectrum.

(*a*) The low pressure gas (neon, hydrogen, or argon, for example), in a gas discharge tube (Fig. 2.12) may be made incandescent by connecting the electrodes in the tube to an induction coil. Observe the spectra produced by several such tubes.

(*b*) An incandescent vapour may be produced by heating a salt (usually a chloride) of an element such as sodium, potassium, lithium, or strontium, in the flame of a bunsen burner (Fig. 2.13). The salt may be introduced into the flame by dipping a platinum wire into a concentrated solution of the salt and then holding



Elizabeth J. Allin

Different types of spectra. From top to bottom: a continuous spectrum; the spectrum of sunlight; the absorption spectrum of a solution of potassium permanganate in water; and discharge tube bright line spectra for iron, sodium, hydrogen, neon, and mercury. The numbers at the top are the wave lengths in Angstrom units.

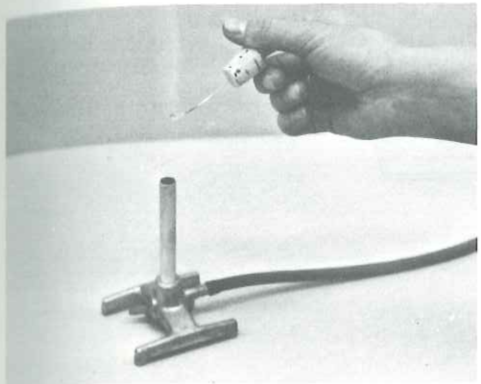


Fig. 2.13. Heating a salt on a platinum wire.

the wire (suitably insulated) in the flame. Special burner attachments, such as that shown in Figure 2.14, may be used. A small piece of asbestos is soaked in a solution of sodium chloride, for example, then placed on the holder which is then rotated into the flame. Observe the flame spectra of several materials. The spectrum which you observe is characteristic of the metal (sodium, potassium etc.) whose salt you used.

2–13 BRIGHT LINE EMISSION SPECTRA

In general, an incandescent gas or vapour has a bright line spectrum. Such a spectrum is not continuous but consists of bright lines of various colours scattered apparently at random through the area where the continuous spectrum would occur if the source were an incandescent solid. The spectrum of sodium vapour (see the colour plate opposite page 22) consists of only two yellow lines very close together—so close in fact that they appear as one in many spectroscopes. A neon spectrum contains many brilliant lines at the red end of the spectrum, and the mercury spectrum shows at least four clearly defined lines. In all cases the spectrum of a gas is unique to that gas; no two gases produce the same sets of lines in the same positions.

2–14 DARK LINE ABSORPTION SPECTRA

A carbon arc, like an incandescent light bulb, produces a continuous spectrum, whereas incandescent sodium vapour produces a bright line spectrum. If the light from a carbon arc is passed through incandescent sodium vapour, a third type of spectrum results. This demonstration is rather difficult to perform in the laboratory, but the following procedure is usually satisfactory.

Heat some powdered sodium nitrite ($NaNO_2$) in a shallow metal dish until the nitrite melts and is vaporizing rapidly. Now drop pieces of blotting paper into the liquid; they will catch fire and burn with a brilliant yellow flame. Shine light from a carbon arc lamp through this flame

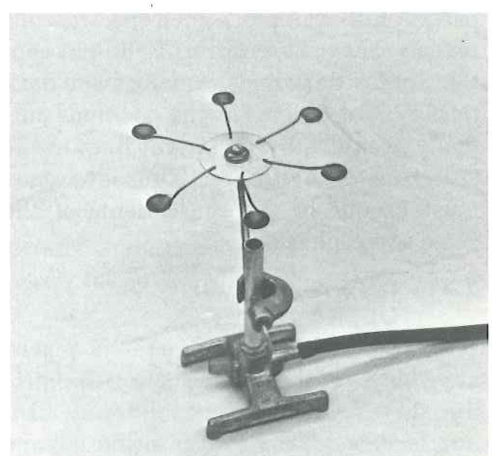


Fig. 2.14. A special burner attachment.

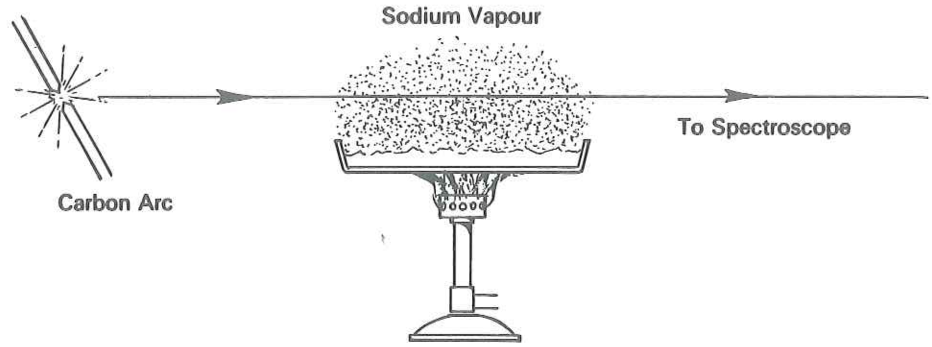


Fig. 2.15. A dark line absorption spectrum is produced when light from a carbon arc passes through incandescent sodium vapour.

to the spectroscope (Fig. 2.15). You should see dark lines in the continuous spectrum in the positions normally occupied by the bright lines in the spectrum of sodium.

In general, when light from a hot source passes through a cooler vapour, the vapour selectively absorbs from the white light those colours which the vapour is capable of emitting. This principle may be used to identify the incandescent gases or vapours through which white light has passed. For example, a good spectroscope reveals that the spectrum of sunlight contains many dark lines. Among these dark lines is a set of lines at the positions normally occupied by the bright lines in the spectrum of hydrogen. Thus hydrogen must be one of the constituents of the sun's atmosphere.

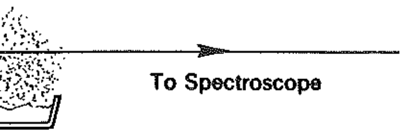
2–15 INFRARED AND ULTRAVIOLET

If a sensitive thermometer is placed anywhere in the visible spectrum produced by an incandescent solid (Fig. 2.16) and is then moved to a position beyond the red end of the spectrum, the temperature, indicated by the thermometer, rises. Evidently radiation is incident on the portion of the screen beyond the red. This radiation is sometimes called thermal radiant energy but more often it is called infrared radiation or simply infrared. Infrared radiation accompanies the light radiated by the sun and is radiated by any object which is warmer than its surroundings.

If zinc sulphide is placed on the screen beyond the violet end of the visible spectrum produced with a quartz prism (Fig. 2.16) it glows with a greenish white visible light. Evidently radiation not visible to the human eye is incident on this portion of the screen. The term ultraviolet light or simply ultraviolet is applied to the region of the spectrum lying just beyond visible violet light.

The sun is the most important source of ultraviolet. A naked carbon arc lamp or a mercury vapour lamp with a quartz window are used as artificial sources of ultraviolet.

Continued exposure to ultraviolet may cause a destruction of the surface cells of the body (sunburn). Intermittent expoır



ıced when light from a carbon arc passes

ṿidently radiation is incident on the rtion of the screen beyond the red. is radiation is sometimes called therıl radiant energy but more often it is lled infrared radiation or simply infrał. Infrared radiation accompanies the ht radiated by the sun and is radiated any object which is warmer than its rroundings.

If zinc sulphide is placed on the screen yond the violet end of the visible specım produced with a quartz prism (Fig. 16) it glows with a greenish white visible ht. Evidently radiation not visible to e human eye is incident on this portion the screen. The term ultraviolet light simply ultraviolet is applied to the gion of the spectrum lying just beyond sible violet light.

The sun is the most important source ultraviolet. A naked carbon arc lamp a mercury vapour lamp with a quartz ndow are used as artificial sources of traviolet.

Continued exposure to ultraviolet may use a destruction of the surface cells of e body (sunburn). Intermittent expo-

sure causes a protective pigment to be produced in the skin, i.e., the skin is tanned. Exposure to ultraviolet radiation results in damage to the structure of bacteria. Many substances, such as zinc sulphide, paraffin oils, calcium tungstate and a solution of quinine, give off visible light when exposed to ultraviolet. They are said to fluoresce, and the emission of such light is called fluorescence.

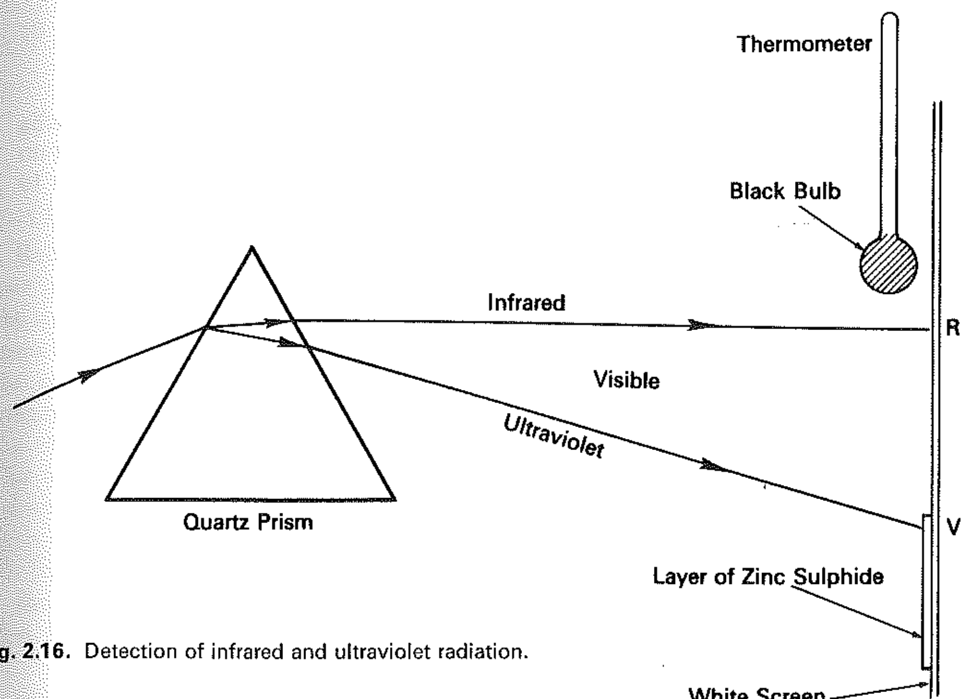


Fig. 2.16. Detection of infrared and ultraviolet radiation.

2–16 PROBLEMS

1. The light from the setting sun follows a curved path as it traverses the earth's atmosphere. Explain.
2. Use the trigonometric tables in the appendix to determine the sine of each of the following angles; (*a*) 30.0°, (*b*) 19.7°, (*c*) 42.6°, (*d*) 79.1°, (*e*) 89.4°, (*f*) 0.2°.
3. What angles have the following sines: (*a*) 0.6347, (*b*) 0.7071, (*c*) 0.0785, (*d*) 0.7570, (*e*) 0.8572, (*f*) 0.9342, (*g*) 0.9629, (*h*) 0.3296?
4. Draw a graph using x as abscissa and $\sin x$ as ordinate, for values of x from 0° to 90°. Take values of x at 5° intervals. Check the accuracy of your graph by interpolating at 37° and 62° and comparing your results with the table of sines in the appendix.

5. Complete the following table.

N	i	R
1.50	30.0°	
1.50		41.0°
1.30	27.0°	
1.30		17.0°
	45.0°	30.0°
	70.0°	87.0°

6. In Section 2-7, we stated that the relative index of refraction for two media is obtained by dividing the absolute index of refraction of the second medium by that of the first. Prove this statement, using Figure 2.17. You have to show that $\dfrac{\sin a}{\sin d} = \dfrac{\sin c}{\sin d} \div \dfrac{\sin b}{\sin a}$.

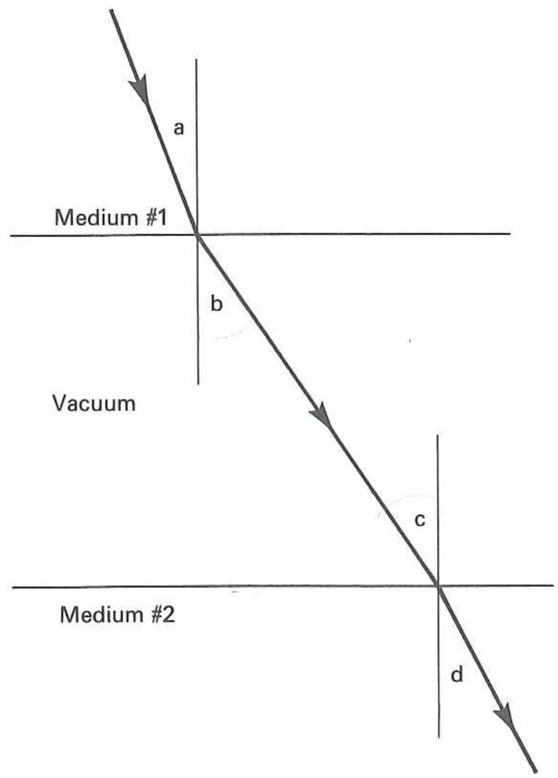


Fig. 2.17. For problem 6.

7. The absolute index of refraction of carbon disulphide is 1.62 and the absolute index of refraction of diamond is 2.47. Calculate the relative index of refraction of carbon disulphide and diamond.

8. Show how two isosceles, right-angled glass prisms may be used in a periscope.

9. Using the table of indices of refraction in Section 2-7, calculate the critical angles for (*a*) alcohol, (*b*) flint glass, and (*c*) diamond.

10. A ray of light is incident on one surface of an equilateral glass prism. If the angle of incidence is 60°, and the index of refraction of the glass is 1.5, calculate the angle of deviation.

11. Describe three types of spectra, indicating the source of each.

2–17 SUMMARY

Refraction of light occurs at the surface between transparent media, because the speed of light in one medium differs from the speed in the other medium. If the speed decreases, the refraction is toward the normal; if the speed increases, the refraction is away from the normal. The incident ray, the normal, and the refracted ray lie in the same plane. The ratio of *sin i* to *sin R* is a constant, *N*, the index of refraction.

Refraction is accompanied by reflection. If the light is attempting to go from a "slow" medium to a "fast" medium, and if the angle of incidence exceeds the critical angle, total reflection occurs.

Both the amount of refraction and the amount of diffraction depend to a small extent on the colour of the light. As a result, dispersion occurs and spectra are produced by prisms and gratings. Continuous spectra are produced by incandescent solids, and line spectra are produced by incandescent gases. Absorption spectra occur when white light from a hot source passes through a cooler incandescent gas.

Chapter 3

NEWTON'S THEORY: Particles of Light

3–1 INTRODUCTION

In Chapters 1 and 2 we have assembled many of the important facts about the behaviour of light. This assembling of facts is a necessary first step in any scientific investigation. We could continue to accumulate facts indefinitely, of course, but sooner or later we should pause to correlate the facts, to fit them into an orderly pattern, and to try to explain them.

How is light transmitted; i.e., how does it travel from a source to a receiver? Obviously we cannot make a direct observation of the mode of transmission. We are forced, therefore, to attempt an explanation by analogy to other visible events or processes. There is always the possibility, of course, that the invisible processes associated with the transmission of light are in no way analogous to anything that has ever been observed directly. Nevertheless, such an analogy or model or theory must be attempted.

Most of the facts of Chapters 1 and 2 were known to Sir Isaac Newton. In 1669 he proposed a theory, the basic assumption of which was that light consisted of a stream of particles—Newton called them "corpuscles"—travelling from the source to the detector.

3–2 TRANSMISSION OF PARTICLES

Newton's corpuscular or particle theory explains very well the fact that light does not require a material medium for its transmission. Particles—bullets, for example—travel better and faster in a vacuum than elsewhere, just as light does. However, a bullet does not travel a

on of light are in no way analogous to
nything that has ever been observed
rectly. Nevertheless, such an analogy
model or theory must be attempted.
Most of the facts of Chapters 1 and 2
ere known to Sir Isaac Newton. In 1669
proposed a theory, the basic assump-
on of which was that light consisted of
stream of particles—Newton called
em "corpuscles"—travelling from the
urce to the detector.

-2 TRANSMISSION OF PARTICLES

Newton's corpuscular or particle theory
plains very well the fact that light does
ot require a material medium for its
ansmission. Particles—bullets, for ex-
mple—travel better and faster in a
cuum than elsewhere, just as light does.
owever, a bullet does not travel a

straight line path as light does, but travels in a curved path under the influence of the force of gravity. The curvature of the path depends on the speed of the bullet: the greater the speed, the less curved the path. We know that the speed of light in a vacuum is 3.0×10^8 m/sec; at this speed we would expect the paths of particles of light to have very little curvature. It is possible, then, that a particle theory can explain rectilinear transmission.

Two light beams will cross without any apparent interaction (Fig. 3.1). In order to explain this fact, we must assume that light particles are so small that the chances of the particles in one beam colliding with those in another beam are negligible. Also, the particles must be small enough not to collide with the molecules of the media through which they pass.

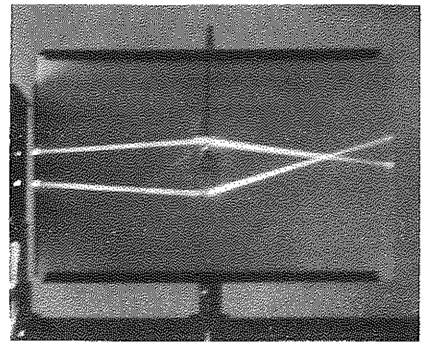
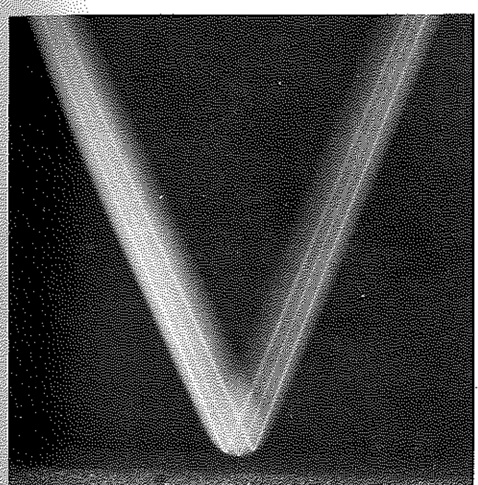


Fig. 3.1. Two light beams cross without interacting with one another.



Physics Department, University of Western Ontario

Fig. 3.2. This time exposure shows the path of a ball before and after being reflected from a smooth surface.

A particle theory, then, accounts for the transmission of light in a vacuum, for the rectilinear propagation of light, and for the fact that light beams can pass through one another. Let us investigate the behaviour of particles further, to see how closely it parallels the behaviour of light.

3-3 REFLECTION OF PARTICLES

Experiments with balls which bounce from smooth surfaces (Fig. 3.2) indicate that the laws of reflection are the same for particles as for light. Indeed, baseball players, tennis players, billiard players and others intuitively base their judgments of the path of the reflected ball on two assumptions: (1) that the angle of incidence equals the angle of reflection, and (2) that the incident path, the reflected path, and the normal lie in the same plane.

3-4 OTHER TESTS OF THE PARTICLE THEORY

Several phenomena associated with light provide further tests for the particle theory.

(*a*) The pressure of light. A stream of particles incident on a surface exerts a pressure on that surface. Our particle model of light would lead us to expect that light incident on a surface would exert pressure—a small pressure to be sure, since we have assumed that the particles are very small. Near the beginning of the twentieth century Hull and Nichols in the United States and Lebedev in Russia were able to detect and measure the pressure of light. They found that, for normal light intensities, this pressure was indeed small, but that it did exist. These findings are obviously in accord with the particle model of light.

(*b*) Absorption of light. When light is incident on the surface of an object, the temperature of that object rises. This fact presents no difficulty to the particle model, for the effect can be considered to be analagous to the heating which occurs, for example, when a piece of lead is hammered with a large particle—a hammer.

We have already noted that a surface may reflect, absorb, or transmit portions of the incident light. Moreover, a dark, dull, opaque surface absorbs most of the incident radiation and reflects very little, whereas the reverse is true for a bright shiny surface. In order to explain this selective absorption, reflection and transmission, we may assume that any surface contains three possible types of regions—absorbing, reflecting, and transmitting—and that the proportion of each type of region varies from one material to another. Do these assumptions seem reasonable?

(*c*) The power of a source of light. The fact that the power of a 100-watt bulb is greater than that of a candle may be explained by assuming that the bulb sends out many more particles of light per second. The particle theory predicts that the power of two identical sources, placed side by side, is double that of either of the sources. Each source sends out x particles per second; the two combined should send out $2x$ particles per second. Measurements with an exposure meter show that source powers actually do add up, as the particle model predicts.

(*d*) The inverse square law. In Chapter 1 we developed the inverse square law for intensity of illumination: the intensity was found to be inversely proportional to the square of the distance from the source. We should note now that the development of this relationship is independent of any assumptions concerning the mode of transmission of light. The particle theory then is consistent with the inverse square law.

Up to this point we have not attempted to explain refraction in terms of particles. Let us make the attempt now.

3–5 LABORATORY EXERCISE: REFRACTION OF PARTICLES

Light undergoes refraction when it passes from one medium to another, i.e., when its speed changes. Moving particles, too, may have their direction of motion changed when their speed changes. To

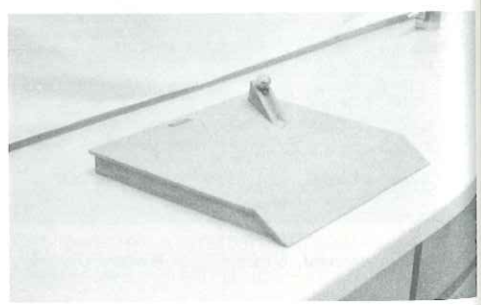


Fig. 3.3. Apparatus for demonstrating refraction of a ball.

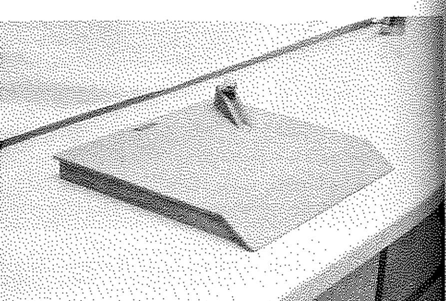
e power of two identical sources, placed e by side, is double that of either of the irces. Each source sends out x particles r second; the two combined should send t $2x$ particles per second. Measurents with an exposure meter show that irce powers actually do add up, as the rticle model predicts.

(d) The inverse square law. In Chapter we developed the inverse square law intensity of illumination: the intensity is found to be inversely proportional the square of the distance from the irce. We should note now that the delopment of this relationship is indendent of any assumptions concerning e mode of transmission of light. The rticle theory then is consistent with e inverse square law.

Up to this point we have not attempted explain refraction in terms of particles. t us make the attempt now.

-5 LABORATORY EXERCISE: REFRACTION OF PARTICLES

Light undergoes refraction when it sses from one medium to another, i.e., ien its speed changes. Moving particles, o, may have their direction of motion anged when their speed changes. To



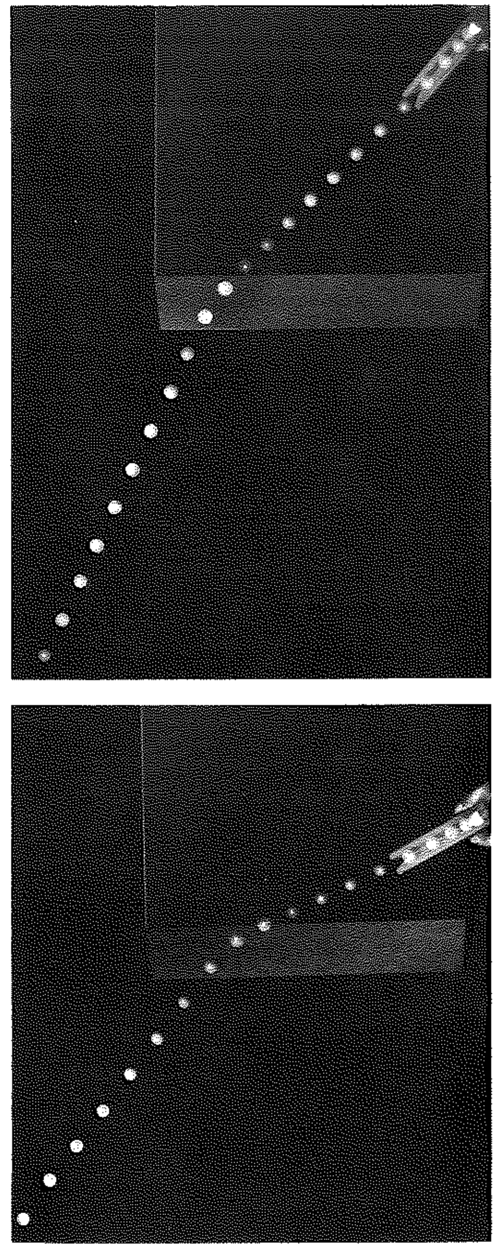
g. 3.3. Apparatus for demonstrating refraction a ball.

investigate this phenomenon, set up the apparatus shown in Figure 3.3. Allow the ball to roll down the small slide, across the upper horizontal surface and down the ramp to the lower horizontal surface. Is the direction of the ball's path on the lower surface different from that on the upper surface? Before you attempt to make any measurements, place a piece of carbon paper over a piece of white paper, on each of the horizontal surfaces. You will then have a permanent tracing of the path of the ball on each of these surfaces. Remove the carbon paper and draw the normals at the points where the ball enters and leaves the sloping ramp. You can then measure the angles of incidence and refraction, and determine the value of $\dfrac{\sin i}{\sin R}$.

Repeat this procedure several times, altering the position of the slide each time in order to change the angle of incidence. However, you must release the ball from the same position on the slide each time. Is the value of $\dfrac{\sin i}{\sin R}$ constant? Should you attribute any variation in the value of $\dfrac{\sin i}{\sin R}$ to experimental error, or is it possible that Snell's law does not apply to particles?

3–6 ANALYSIS OF PARTICLE REFRACTION

Figure 3.4 shows stroboscopic pictures of the ball for two angles of incidence. The ball was released from the same position on the slide in both cases. The distance intervals between successive positions of the ball on the upper level are uniform and equal in both photographs. Therefore the speed v_1 on the



Physics Department, University of Western Ontario

Fig. 3.4. Refraction of a ball for two different angles of incidence.

upper level is the same in both cases. Similarly, the speed v_2 on the lower level is found to be the same in both cases, but v_2 is greater than v_1.

In Figure 3.5, AB represents the ramp, and CD represents the normal to the ramp at O. EO and OG represent the speeds v_1 and v_2, drawn to scale and in the correct directions. EF is perpendicular to AB, and GD is perpendicular to CD. Then $OF = v_1 \sin i$ and $GD = v_2 \sin R$. Measurements on the diagram show that $OF = GD$.

$$\text{Therefore } v_1 \sin i = v_2 \sin R$$

$$\text{and } \frac{\sin i}{\sin R} = \frac{v_2}{v_1}$$

But v_1 and v_2 do not vary from one case to the next.

$$\text{Therefore } \frac{\sin i}{\sin R} \text{ is a constant.}$$

Snell's law, then, seems to apply for particles.

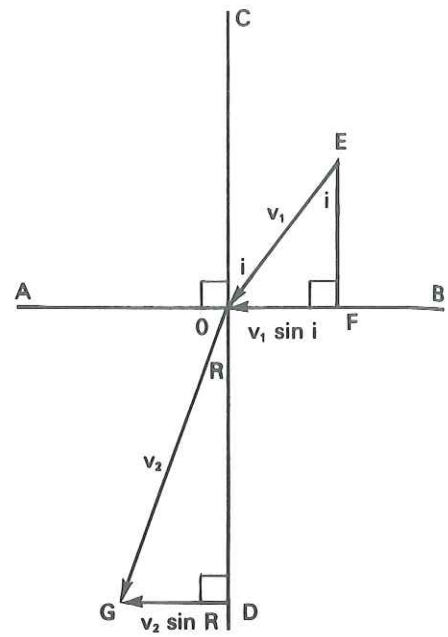


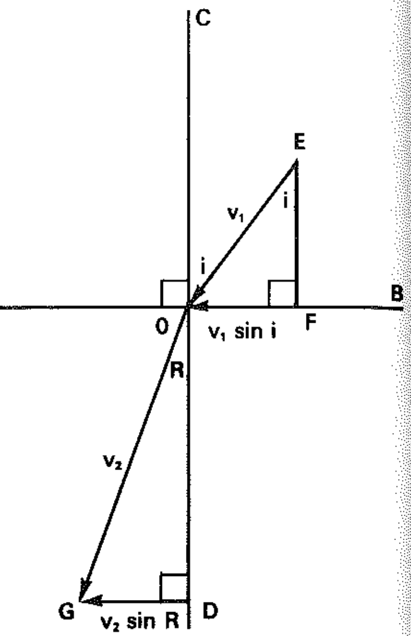
Fig. 3.5. Analysis of particle refraction.

3–7 PARTICLE REFRACTION AND LIGHT

Let us now see whether we can apply this particle model of refraction to light. The path of the ball on the upper level represents the incident ray; the path on the lower level represents the refracted ray; and v_1 and v_2 represent the speeds of light in the two media. The ramp between the two levels represents the boundary between the two media.

As the ball rolls down the ramp it is given a shove by the force of gravity and its speed increases. It is reasonable to assume that particles of light (if they exist) are given a shove as they pass from one medium to another. In either of the media, the particles of light are attracted equally from all sides by the molecules of that medium. But near the boundary, they may very well be attracted to a greater extent by the molecules of the second medium than by the molecules of the first medium. The resulting shove causes them to speed up, and refraction toward the normal results. It is particularly important to notice that our particle model predicts refraction toward the normal when the particle speed increases. This prediction causes difficulty when we compare particle refraction with refraction of light.

Consider the refraction which takes place when light passes from air to glass. The relative index of refraction for air and glass is about 1.5, i.e., $\frac{\sin i}{\sin R} = 1.5$. Then, according to the particle theory, $\frac{v_2}{v_1} = 1.5$. The speed of light in glass,



ig. 3.5. Analysis of particle refraction.

may very well be attracted to a ter extent by the molecules of the nd medium than by the molecules of first medium. The resulting shove es them to speed up, and refraction rd the normal results. It is particu- important to notice that our particle el predicts refraction toward the nor- when the particle speed increases. prediction causes difficulty when we pare particle refraction with refrac- of light.

nsider the refraction which takes e when light passes from air to glass. relative index of refraction for air glass is about 1.5, i.e., $\frac{sin\ i}{sin\ R} = 1.5.$, according to the particle theory, 1.5. The speed of light in glass,

therefore, should be about 1.5 times the speed of light in air. Unfortunately, measurements of the speed of light in air and in glass indicate that the speed of light in air is 1.5 times the speed of light in glass, just the reverse of the prediction of the particle model. The particle model predicted that

$$\frac{sin\ i}{sin\ R} = \frac{v_2}{v_1}$$

In actual fact,

$$\frac{sin\ i}{sin\ R} = \frac{v_1}{v_2}$$

Here our particle model encounters its first real difficulty.

3–8 TO ABANDON OR TO MODIFY?

Newton argued in favour of the particle theory all his life, and many of his contemporaries and successors did too. The failure of this theory to predict the correct direction of refraction in terms of the speeds of light in the two media was not obvious to them, for these speeds were not known at the time.

The particle theory of refraction predicted that the speed of light in water was greater than in air. Later experiments showed that such was not the case. This chain of events occurs frequently in Science. A theory is developed to explain the facts known at the time. This theory is then used to make further predictions. If later experiments disprove these predictions, then the theory must be modified or discarded in favour of a new theory.

We find ourselves in the position of having to modify the particle theory or abandon it. It is conceivable that we could modify the theory by making new and complicated assumptions about particles of light. The questions that we must ask ourselves are: "Would these modifications be worth the effort? Would the particle theory then become unnecessarily complicated?"

Perhaps there is a better—and less complicated—theory.

3–9 PROBLEMS

1. A 60-watt light bulb is weighed, used for two months, and then weighed again. No change in weight is detected. Discuss the implications of this observation with respect to the particle theory.

2. The speed of a ball is usually reduced when the ball is reflected. Does the speed of light change when the light is reflected? Justify your answer.

3. Can you explain the production of (*a*) a continuous spectrum, (*b*) a bright line spectrum, (*c*) a dark line absorption spectrum, in terms of particles? Justify your answer in each case.

4. Can a particle theory explain the existence of infrared and ultraviolet radiation? Justify your answer.

5. How could the apparatus which you used to investigate particle refraction (Fig. 3.3) be used to demonstrate total reflection of particles? Could the same apparatus be used to demonstrate partial reflection and refraction?

6. The relative index of refraction of air and flint glass is 1.65. Calculate the speed of light in flint glass. According to the particle theory, what should be the speed of light in flint glass?
7. The relative index of refraction for air and water is 1.33. Calculate the speed of light in water. If the predictions of the particle theory were correct, what would be the speed of light in water?
8. The relative index of refraction for air and water is 1.33, and the relative index of refraction for air and glass is 1.52. By calculating the speed of light in water and in glass, determine the relative index of refraction for water and glass.
9. Is it possible that sound is transmitted by means of particles? Justify your answer.
10. A block of wood floats on the surface of a swimming pool, 9 ft from the edge of the pool. Discuss three ways in which a man, standing on the edge of the pool, can cause the block to move. Which of these three ways is a possible mode of transmission for light?

3–10 SUMMARY

The analogies used in Science to summarize and explain observed facts are called theories or models. If a particle model is set up to explain the transmission of light, it must be assumed that the particles are very small and travel very fast.

A particle theory accounts quite well for reflection, pressure of light, absorption and heating, and the inverse square law. Particles undergo refraction; as for light, the value of the ratio of *sin i* to *sin R* is constant. However, for particles, $\frac{\sin i}{\sin R} = \frac{v_2}{v_1}$. The particle model also fails to account for partial transmission and reflection of light, and for the differing absorbing and reflecting properties of different surfaces.

It is conceivable that a more sophisticated particle theory might be developed, one which might not have the defects of the simple theory presented here. This will be done if there is no simple alternative.

flint glass is 1.65. Calculate the
e particle theory, what should be

nd water is 1.33. Calculate the
the particle theory were correct,
?

d water is 1.33, and the relative
52. By calculating the speed of
relative index of refraction for

means of particles? Justify your

a swimming pool, 9 ft from the
ich a man, standing on the edge
Which of these three ways is a

nstant. However, for particles,
$= \frac{v_2}{v_1}$. The particle model also fails
count for partial transmission and
tion of light, and for the differing
bing and reflecting properties of dif-
t surfaces.
is conceivable that a more sophisti-
l particle theory might be developed,
which might not have the defects of
simple theory presented here. This
be done if there is no simple
native.

Chapter 4

HUYGEN'S THEORY: Waves of Light

4–1 AN ALTERNATIVE TO NEWTON'S THEORY

Although the failure of a particle theory to predict the correct direction of refraction was not evident, Newton's corpuscular theory was opposed by a few of Newton's contemporaries. One of these was a Dutch physicist, Hans Christiaan Huygens (1629-1695). In 1678 Huygens proposed a wave theory for light, and, though few people agreed with him at the time, support for a wave theory grew slowly but steadily for the next two centuries.

Huygens reasoned that light may be considered as a disturbance travelling outward from its source. He pointed out that a stone dropped into a pool of water creates a disturbance or vibration which travels outward from the point at which the stone enters the water. The transmission of the vibration is called a wave motion. It was natural, then, for Huygens to attempt to explain the behaviour of light in terms of waves. Before we attempt to evaluate Huygens' theory, we must become familiar with waves and with the vibrations which cause and accompany them.

4–2 VIBRATIONS

A vibration is a repeated to-and-fro motion. Consider for example the motion of a weight suspended by means of a short spring (Fig. 4.1) or rubber band. If the weight, originally at rest at A, is pulled down to B and released, it vibrates about its rest position, A, and between the limits, B and C. The repeated motion —from B to C to B—is called one cycle. The frequency, f, of vibration is the number of cycles per second and the period,

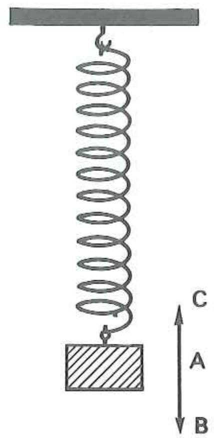


Fig. 4.1. Vibration of a weight attached to a spiral spring.

T, of vibration is the time taken for one cycle. The maximum displacement of the vibrating particle from its rest position, AB or AC in Figure 4.1, is called the amplitude of vibration.

Vibratory motion may also be illustrated by means of a simple pendulum—a weight suspended from a wire or string. Two identical pendulums (Fig. 4.2) may be used to demonstrate what is meant by the word phase in connection with vibrations. If A is pulled aside to Q and B to S, and both are released at the same

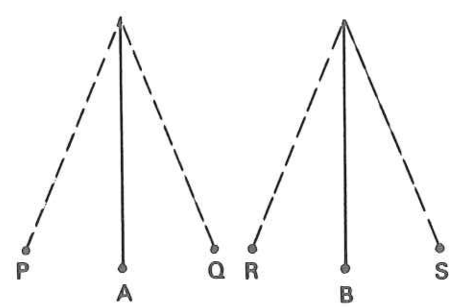


Fig. 4.2. Two identical pendulums illustrate phases of vibration.

instant, A will reach P at the same time that B reaches R. Subsequently, A will reach Q at the same time that B reaches S. A and B are said to be in phase throughout the cycles of movement, since at any instant they are about to move in the same direction from corresponding points in the cycles. If A is drawn aside to P, when B is moved to S, and if both are released simultaneously, then at any instant A and B are in opposite phases of vibration.

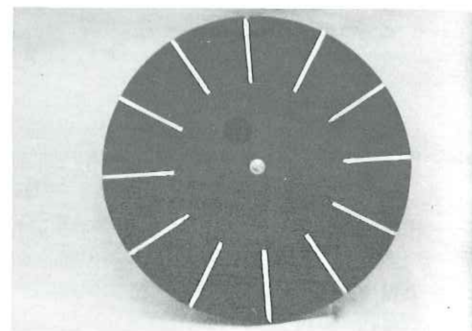


Fig. 4.3. A disc stroboscope.

4–3 LABORATORY EXERCISE: THE STROBOSCOPE

Several methods are available for determining the frequency of a vibration. If the frequency is low, the number of vibrations in a measured time may be counted. Suppose, for example, that a pendulum executes 20 vibrations in 25 sec. Its frequency is obviously 0.8 cycles per second.

If the frequency is higher, counting may be impossible. In this case, a disc stroboscope (Fig. 4.3) may be used. To understand the principle of its action, let us consider the motion of the sweep second hand of a watch. Suppose we look ant, A will reach P at the same time B reaches R. Subsequently, A will h Q at the same time that B reaches A and B are said to be in phase ughout the cycles of movement, since ny instant they are about to move in same direction from corresponding ts in the cycles. If A is drawn aside , when B is moved to S, and if both released simultaneously, then at any ant A and B are in opposite phases ibration.

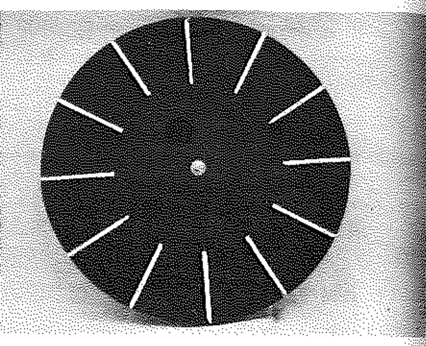


Fig. 4.3. A disc stroboscope.

LABORATORY EXERCISE: THE STROBOSCOPE

veral methods are available for deining the frequency of a vibration. ne frequency is low, the number of ations in a measured time may be ted. Suppose, for example, that a lulum executes 20 vibrations in 25 Its frequency is obviously 0.8 cycles second.

the frequency is higher, counting be impossible. In this case, a disc oscope (Fig. 4.3) may be used. To rstand the principle of its action, s consider the motion of the sweep nd hand of a watch. Suppose we look

at the hand when it is pointing at 12 o'clock, and once a minute thereafter for an hour. Each time we observe the hand it points to 12 o'clock and the watch seems to have stopped. That is, the watch appears stopped if the viewing frequency is 60 per hour. But the watch will appear to be stopped if we view it every second minute, or every third minute, etc.; i.e., if the viewing frequency is 30 per hour, 20 per hour, etc. If we view the second hand more frequently than 60 times per hour, the watch will not appear stopped. We know that the frequency of rotation of the second hand is 60 per hour; therefore we come to the following conclusions:

(*a*) The frequency of the viewed object is equal to the maximum viewing frequency for "stopped" motion.

(*b*) For viewing frequencies other than the maximum, for stopped motion, all we can say is that the frequency of the viewed object is some integral multiple of the viewing frequency.

Now, apply these principles to determine the frequencies of several vibrating objects, starting with one whose frequency is low enough to be determined by counting, e.g., a pendulum or a loaded strip of steel clamped in a vise (Fig. 4.4). Cover all of the slits in the stroboscope except one. Rotate the stroboscope between your eye and the vibrating object, and vary the speed of rotation until the motion appears "stopped." Is this the maximum viewing frequency for "stopped motion"? Did you stop the motion at the limit of the vibration, or at the centre? Where should you stop the motion? Determine the viewing frequency by counting the number of rotations of the disc in a measured time interval. Compare this result

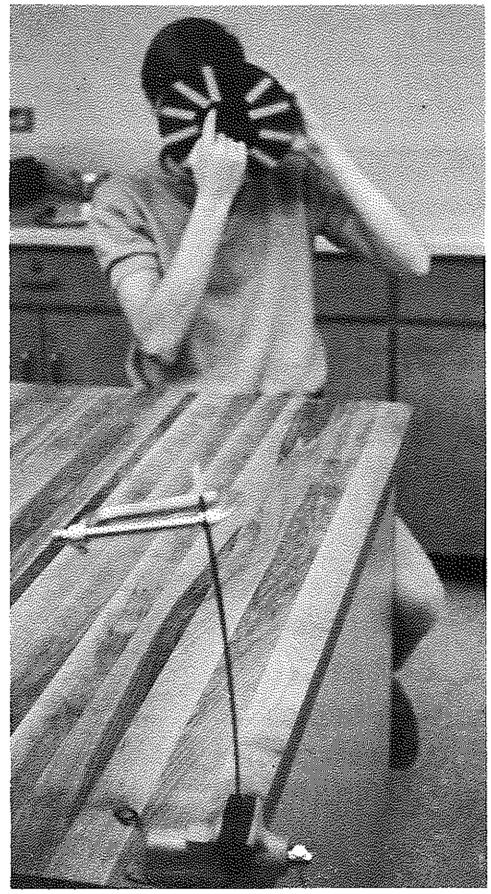


Fig. 4.4. A disc stroboscope in use.

with the frequency of vibration as determined by counting.

Now repeat this procedure for a vibration of higher frequency, the vibration of the clapper of a bell (Fig. 4.5) for example. You will likely have to use more than one of the slits of the stroboscope. Should the slits used be equally spaced? Note that you have to take into account the number of slits used when you calculate the viewing frequency. Since counting is impossible, how can you check your

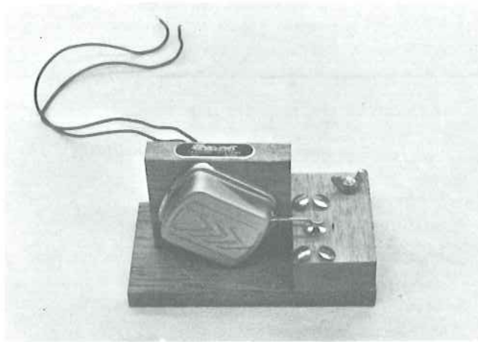


Fig. 4.5. A recording timer.

result? Try the method suggested in Fig. 4.6, i.e., count the number of dots made in 10 sec by the clapper on the strip of paper which you pull underneath the clapper. Does the speed of the paper have to be uniform?

4–4 CHARACTERISTICS OF A WAVE MOTION

Waves, as we normally think of them, require a medium; water waves are impossible without water. Waves are initiated by a disturbance or vibration of some sort; this vibration is transferred to the particles of the medium adjacent to the source, then to the next particles of the medium, etc. Yet close observation of water waves indicates that the medium itself (the water in this case), is not transferred.

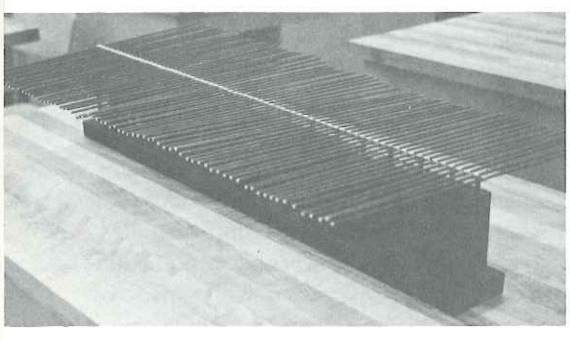


Fig. 4.7. A wave machine.

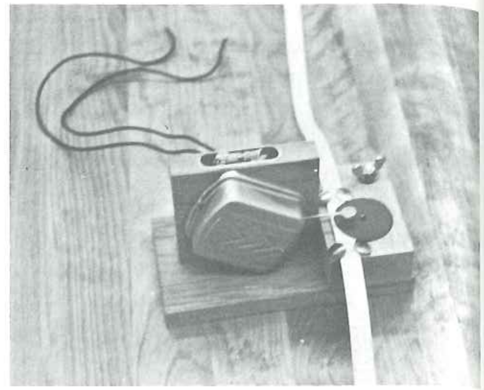
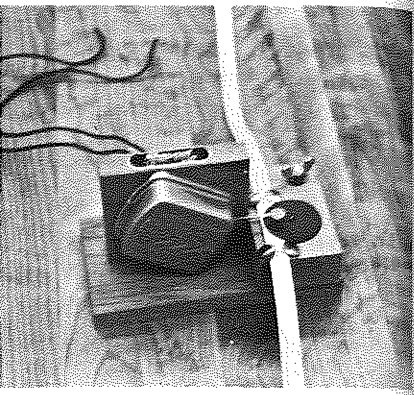


Fig. 4.6. The clapper of the timer strikes the carbon paper on top of the white paper tape. As a result, dots are recorded on the tape as it is pulled through the timer.

The behaviour of the particles of the medium through which the wave travels is difficult to observe with water waves, but may be observed quite readily with the aid of a wave machine (Fig. 4.7) which has been developed by the Bell Telephone Laboratories Inc. It consists basically of a set of transverse rods attached to a central steel "spine". When the rod at one end is caused to vibrate, a wave motion, consisting of a series of crests and troughs (Fig. 4.8), is set up. The effect of such a wave on any one rod is to cause that rod to vibrate up and down, but the rod is not transferred horizontally. In short, the disturbance travels but the medium does not.

If we observe the motion of a single pulse (crest or trough), we note that the shape of the pulse does not change as it



.6. The clapper of the timer strikes the car-
aper on top of the white paper tape. As a
dots are recorded on the tape as it is pulled
h the timer.

ource, then to the next particles of
edium, etc. Yet close observation of
waves indicates that the medium
(the water in this case), is not
ferred.
e behaviour of the particles of the
um through which the wave travels
ficult to observe with water waves,
nay be observed quite readily with
id of a wave machine (Fig. 4.7)
has been developed by the Bell
hone Laboratories Inc. It consists
ally of a set of transverse rods at-
d to a central steel "spine". When
od at one end is caused to vibrate,
ve motion, consisting of a series of
and troughs (Fig. 4.8), is set up.
ffect of such a wave on any one rod
cause that rod to vibrate up and
, but the rod is not transferred
ontally. In short, the disturbance
ls but the medium does not.
we observe the motion of a single
(crest or trough), we note that the
of the pulse does not change as it

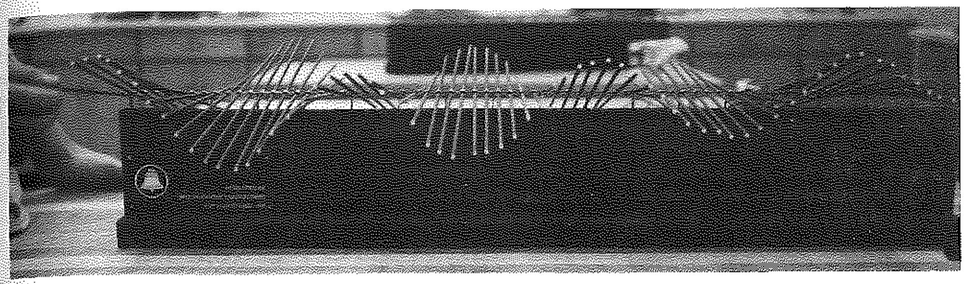


Fig. 4.8. A series of crests and troughs on the wave machine.

travels, and that the pulse travels at constant speed.

4–5 REFLECTION OF PULSES

When a pulse reaches the end of the wave machine, it is reflected back toward the end from which it came. Let us generate a pulse at one end of the machine and observe in detail the reflection which takes place at the other end. Two cases are possible.

(*a*) The rod at the reflecting end may be clamped in position (Fig. 4.9); that end is then said to be fixed. In this case the pulse is reflected without change of shape, but it is reflected upside down (Fig. 4.10), i.e., a crest is reflected as a trough. The fixed end of the machine, of course, remains stationary while the reflection is taking place.

(*b*) If the reflecting end of the machine is free to vibrate, an incident pulse is reflected without a change of shape but this time it is reflected right side up. A crest is reflected as a crest (Fig. 4.11). The free end vibrates while the pulse is being reflected.

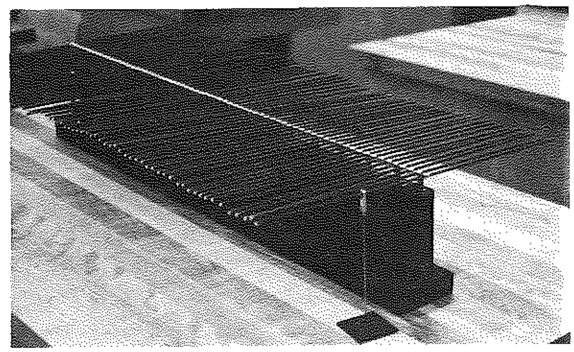


Fig. 4.9. The rod at one end of the wave machine, clamped in place.

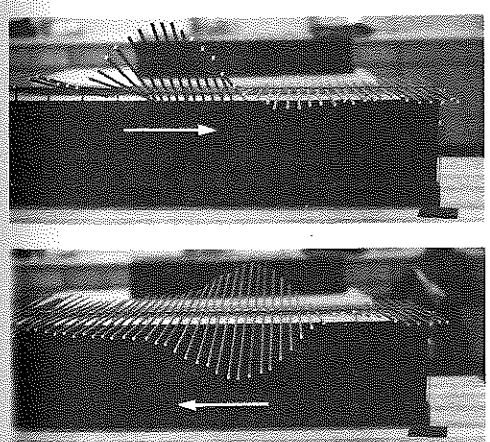


Fig. 4.10. Reflection from a fixed end. A crest travelling from left to right (upper photo) is reflected as a trough (lower photo).

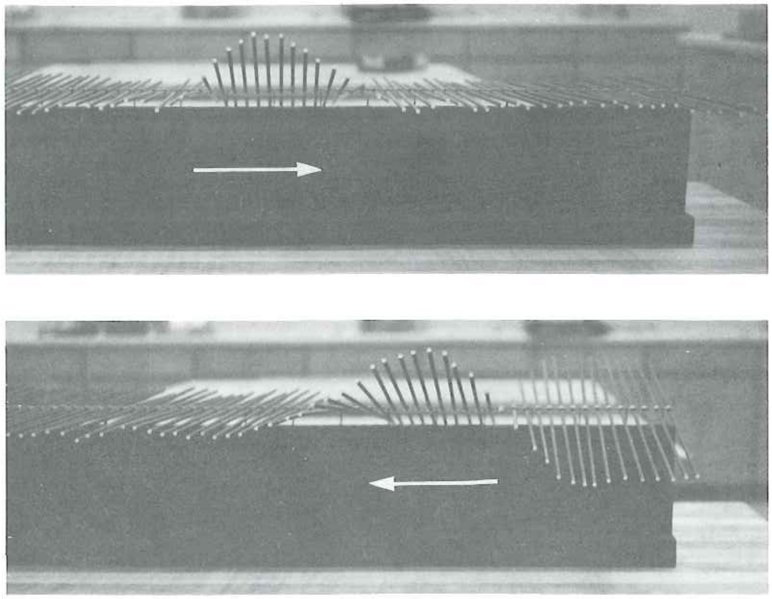


Fig. 4.11. Reflection from a free end. A crest travelling from left to right (upper photo) is reflected as a crest (lower photo).

4–6 LABORATORY EXERCISE: TRANSMISSION AND REFLECTION OF PULSES

You may investigate the production, propagation, transmission and reflection of pulses by using a slinky and a coil spring (Fig. 4.12). Working with your partner, stretch the slinky to a length of about 8 or 10 metres on a smooth floor. Practice generating pulses, crests or troughs, and observe their transmission. What happens to the shape, speed, and amplitude of the pulse as it travels? Does the speed depend on the amplitude? To observe reflection at a fixed end, simply have your partner hold his end of the spring fixed, and generate a pulse at your end. To observe reflection at an end that is essentially free, tie a long thread to the

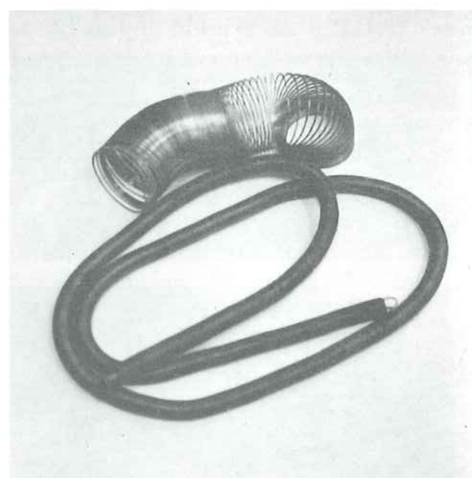
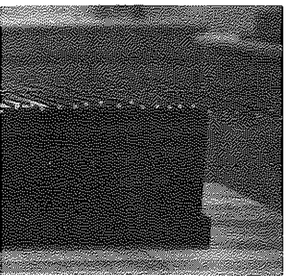
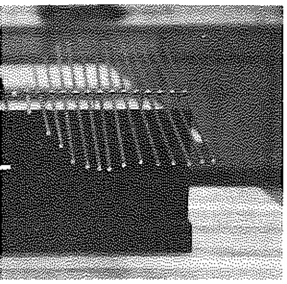


Fig. 4.12. Two springs: a "slinky" (upper) and a coil spring (lower). The coil spring is heavier than the slinky, and of smaller diameter.





left to right (upper photo) is reflected

2. Two springs: a "slinky" (upper) and a
ng (lower). The coil spring is heavier than
y, and of smaller diameter.

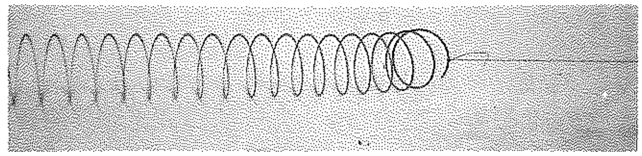


Fig. 4.13. When a long thread is tied to one end of the slinky, that end of the slinky is essentially free.

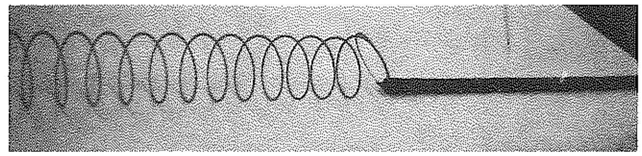


Fig. 4.14. The slinky and the coil spring, hooked together.

end coil of the slinky (Fig. 4.13) and observe pulse reflection at this junction. To observe reflection at a junction that is neither fixed nor completely free, hook the slinky and the coil spring together as shown in Figure 4.14. Observe the effect of the junction on pulses generated (*a*) on the slinky, and (*b*) on the coil spring. Is the partial reflection that takes place similar to that at a fixed end or at a free end? Summarize all of your observations.

4–7 LABORATORY EXERCISE: THE EFFECT OF PULSES ON ONE ANOTHER

Now use the slinky to observe the effect of two pulses which affect the same portion of the spring at the same time. Generate simultaneous pulses at opposite ends of the slinky. Do the pulses pass through one another, or do they meet and bounce back, as two billiard balls do? How do you know? Generate a crest at each end of the spring, and observe the effect at the point where the two crests meet. Repeat for a crest generated at one end and a trough generated at the other end. How does the displacement of the spring at the point where the pulses meet seem to compare with the displacements produced by the individual pulses?

4–8 THE PRINCIPLE OF SUPERPOSITION

In our particle model of light, we had to assume that the particles were so small that there was little likelihood of their colliding. This assumption was necessary because of the fact that light beams cross without evidence of anything resembling a collision. The Laboratory Exercise (Sect. 4.7) provides some qualitative information about what happens when two pulses meet. Figures 4.15 and 4.16 provide greater detail. In the upper photograph in Figure 4.15, the crest on the left is travelling from left to right, and the crest on the right is travelling from right to left. The lower photograph shows the large crest produced when the two crests coincide. The amplitude of this crest, that is, the displacement of the rod from its rest position at this point, seems to be the sum of the amplitudes or displacements of the two separate crests.

The effect of the crossing of a crest and a trough is shown in Figure 4.16. In the upper photograph, the trough on the left (moving from left to right), and the crest on the right (moving from right to left) are just about to meet. The middle photograph indicates that, when the crest and

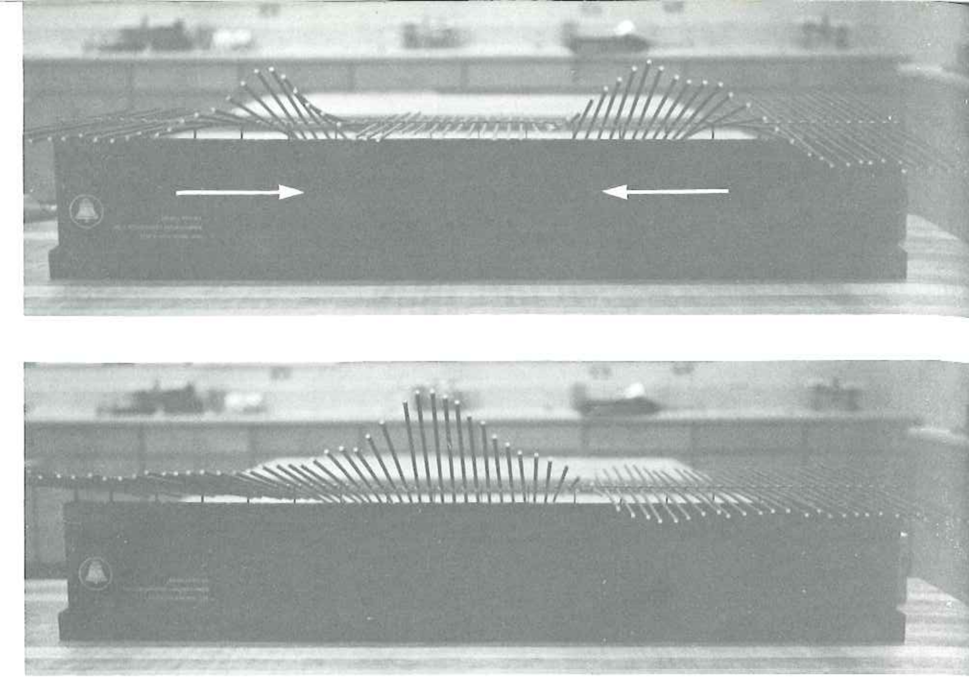


Fig. 4.15. The upper photo shows two crests approaching one another on the wave machine; the lower photo shows the combined effect of the two crests as they cross.

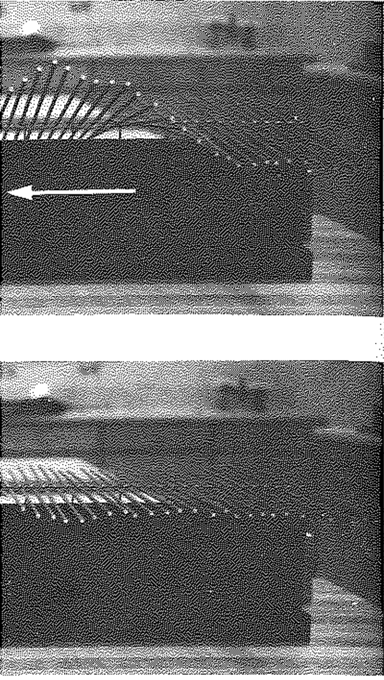
the trough are at the same position on the machine, they cancel one another. The lower photograph shows the positions and shapes of the pulses a moment later. They are travelling in their separate (and original) directions, unchanged in shape. This last photograph makes it obvious that pulses do not collide head on and bounce back as particles do.

We may describe the interaction of two pulses by the following Principle of Superposition: The displacement of the medium at the point where two pulses cross is the algebraic sum of the displacements due to the individual pulses. Thus two crests reinforce one another to form a large crest, two troughs reinforce one another to form a deep trough, and a trough and a crest cancel one another, partially or completely. (Whether the cancellation is partial or complete depends on the relative amplitudes of the crest and the trough.) The interaction of two pulses is called interference. If the two pulses cause displacements in the same direction (as do two crests or two troughs), the reinforcing effect is called constructive interference. On the other hand, if the two pulses cause displacements in opposite directions (as do a crest and a trough), the cancelling effect is called destructive interference.

4–9 TRAVELLING WAVES

A travelling wave is simply a series of pulses—a succession of crests and troughs (Fig. 4.17). Such a wave may be generated on a spring by causing one end of the spring to vibrate. If the frequency of vibration of the end of the spring is constant, the resulting wave is said to be periodic. The wave pattern is regular and symmetrical. The constant length of this recurring pattern—from crest to crest or from trough to trough—is called the wave length. We shall use the Greek letter, λ, as the symbol for the wave length.

The vibration which originates at the end of the spring is transferred from particle to particle along the spring, each particle vibrating with frequency f. As a



ng one another on the wave machine; sts as they cross.

direction (as do two crests or two ns), the reinforcing effect is called uctive interference. On the other if the two pulses cause displace- in opposite directions (as do a crest trough), the cancelling effect is destructive interference.

TRAVELLING WAVES

ravelling wave is simply a series of —a succession of crests and troughs 4.17). Such a wave may be gener- on a spring by causing one end of ring to vibrate. If the frequency of ion of the end of the spring is con- the resulting wave is said to be lic. The wave pattern is regular and etrical. The constant length of this ing pattern—from crest to crest or rough to trough—is called the wave . We shall use the Greek letter, λ, symbol for the wave length.

vibration which originates at the the spring is transferred from par- to particle along the spring, each le vibrating with frequency f. As a

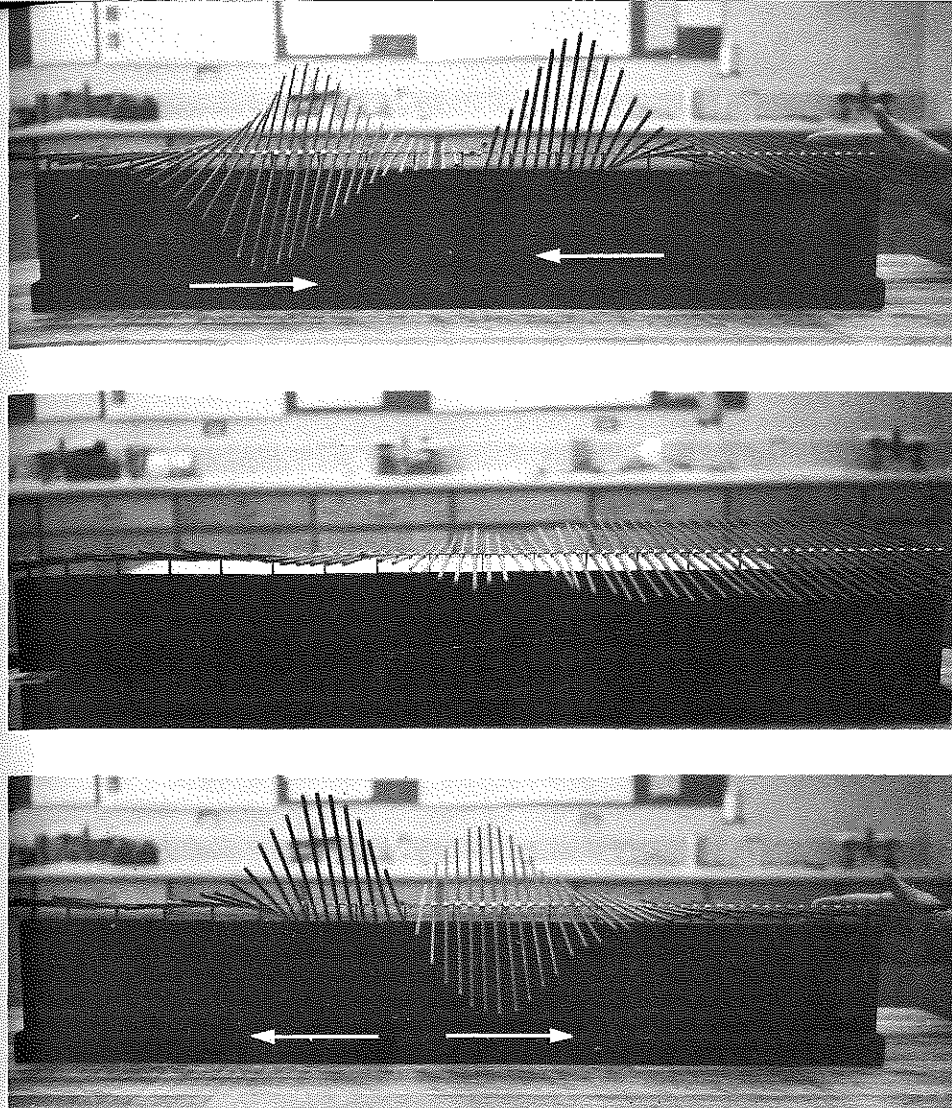


Fig. 4.16. The upper photo shows a crest and a trough about to meet; the middle photo shows their combined effect as they cross; the lower photo shows each continuing after they cross.

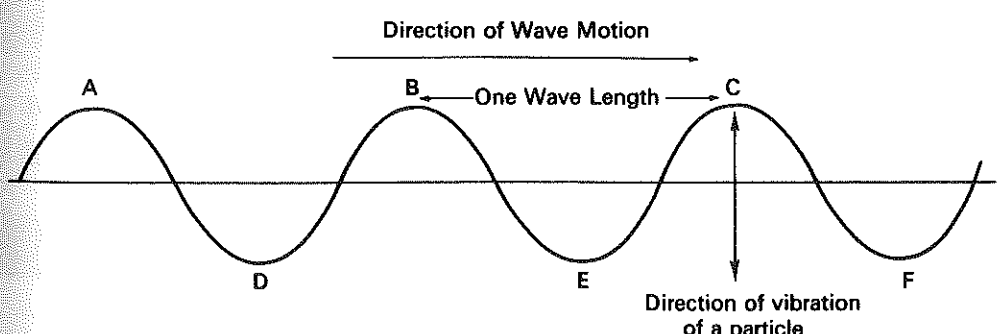


Fig. 4.17. The length of the recurring pattern of a periodic wave is called the wave length.

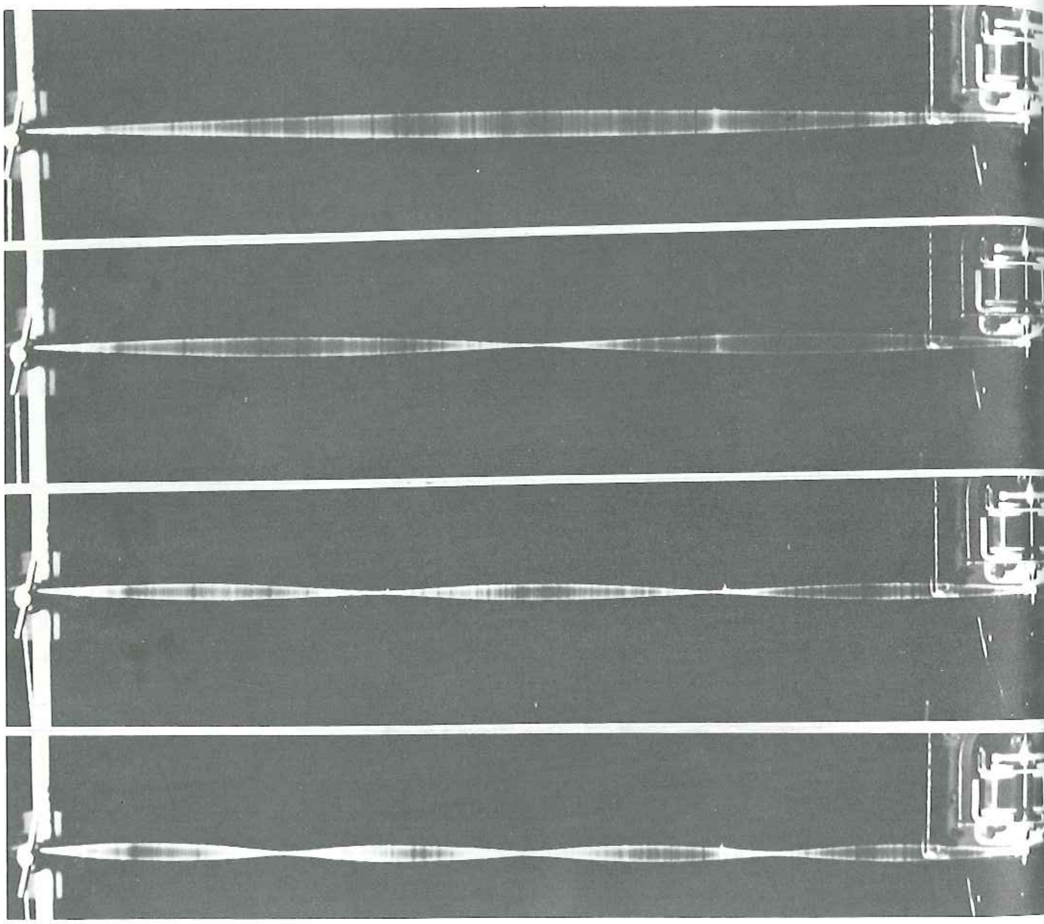
result, the number of crests or troughs or complete wave patterns passing any point on the spring in unit time is also f. Thus the disturbance travels f wave lengths per unit time, that is, the speed, v, of the wave is f wave lengths per unit time. Hence,

$$v = f\lambda \qquad (1)$$

If f is 10 cycles/sec, then the period, T, is $\frac{1}{10}$ sec. In general, $f = \frac{1}{T}$ and equation (1) may be written

$$v = \frac{\lambda}{T} \qquad (2)$$

The relationship expressed in equation (1) is called the universal wave equation. It may be applied to periodic travelling waves of any type.



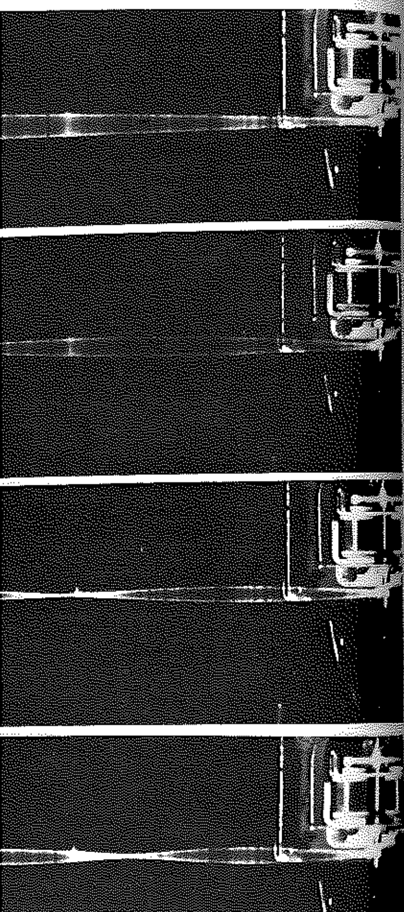
Physics Department, University of Western Ontario

Fig. 4.18. Standing wave patterns produced by the interference of incident and reflected waves on a string.

. In general, $f = \frac{1}{T}$ and equation
be written

$$v = \frac{\lambda}{T} \qquad (2)$$

elationship expressed in equation
lled the universal wave equation,
be applied to periodic travelling
f any type.



Department, University of Western Ontario

rence of incident and reflected waves

4–10 STANDING WAVES

Observation of travelling waves, produced at one end of a spring, is complicated by the fact that the waves are reflected from the other end. The reflected crests and troughs are superimposed upon incident crests and troughs. The incident and reflected waves interfere with one another; under certain circumstances the effect of this interference is to produce patterns called standing waves.

Standing waves differ from travelling waves in that the wave patterns appear to remain stationary. The production of standing waves requires two identical waves moving in opposite directions—a condition which is fulfilled by the arrangement shown in Figure 4.18. For a certain relatively low frequency of vibration of the vibrator, the pattern shown in the upper photograph results; the succeeding 3 patterns result from frequencies which are, respectively, 2, 3, and 4 times as great. For other frequencies, superposition of the incident and reflected waves takes place, but the interference pattern is not a stationary one.

A standing wave pattern is composed of one or more segments; the stationary points at the ends of the segments are called nodes. The distance between nodes in a standing wave pattern is one-half of the wave length of either of the waves which interfere to produce that pattern. At the nodes the sum of the displacements due to the incident and reflected waves is zero at all times. The mid points of the segments are called loops. At the loops the sum of the displacements due to the incident and reflected waves varies from a maximum in one direction to a maximum in the other direction. The particles of the string at the loops vibrate rapidly and vigorously. We must realize that the blurred patterns on the string in Figure 4.18 is due to this rapid vibration. Figure 4.19 shows a three segment pattern and the position of the string at a particular instant.

4–11 WATER WAVES

If light waves exist, they are not likely to be confined to one dimension as are waves on a string, but are likely to bear some resemblance to waves on water.

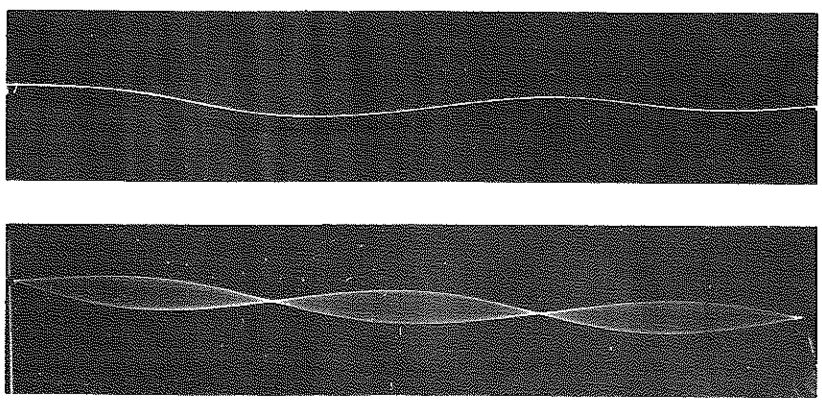


Fig. 4.19. Photographs of standing waves. The exposure time for the upper photo was 0.001 sec; for the lower photo 0.1 sec.

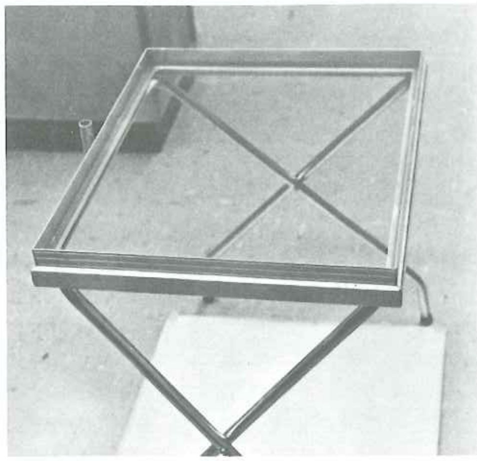


Fig. 4.20. A ripple tank.

Water waves are best studied in the laboratory with the help of a ripple tank (Fig. 4.20). The tank consists essentially of a shallow rectangular frame with a transparent bottom. Waves are produced on a shallow layer of water in the tank. Images of the waves are projected on a screen below the tank by means of a light source placed above the water. The crests of the water waves act as converging lenses to form bright areas on the screen; the troughs act as diverging lenses to form dark areas. For best results, some experimentation is necessary to determine the proper combination of water depth, wave amplitude, screen distance and light intensity.

If a finger or some other object is caused to vibrate up and down in the water at the centre of the ripple tank, a pattern is produced on the screen which at any given moment would resemble the one shown in Figure 4.21. The circular crests and troughs on the water are called wave fronts. A study of these wave fronts and

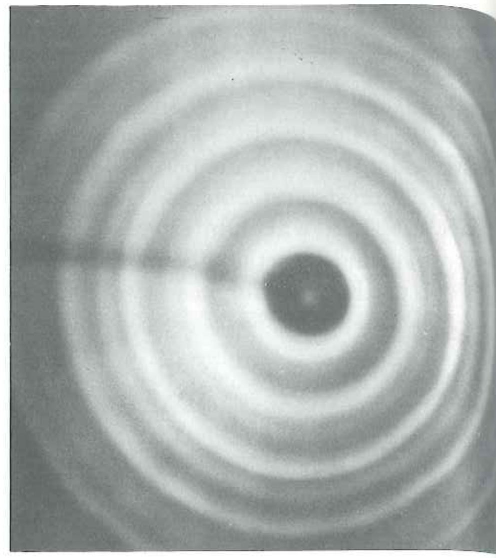


Fig. 4.21. Circular wave fronts.

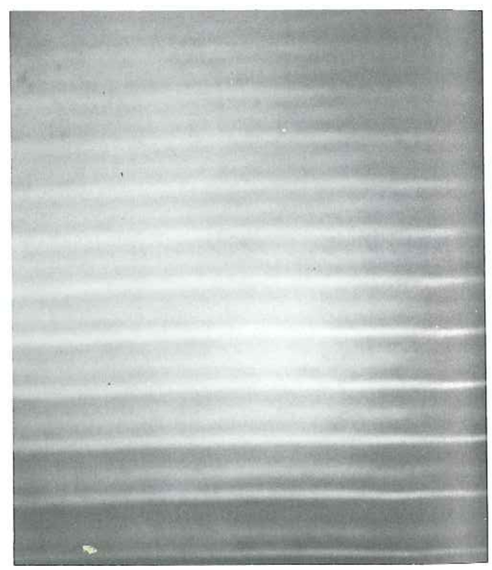
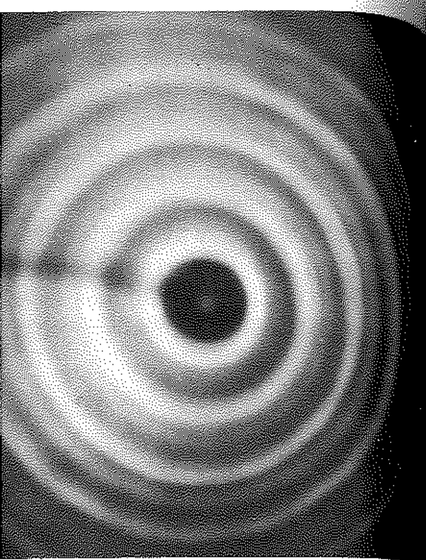
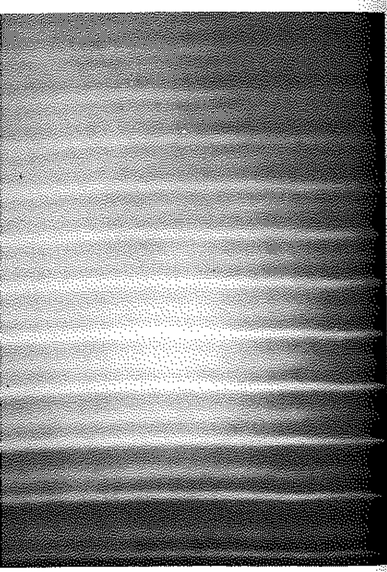


Fig. 4.22. Straight wave fronts.



ig. 4.21. Circular wave fronts.



ig. 4.22. Straight wave fronts.

their motions reveals the following facts:

(*a*) As the wave fronts advance, the vibration associated with them is transferred horizontally in all directions away from the source. However, the water itself is not transferred. A particle of cork floating on the water vibrates mainly vertically; these waves are essentially transverse.

(*b*) All points on a particular wave front are in the same phase of motion.

(*c*) The direction in which the wave travels is always perpendicular to the wave front.

(*d*) A small section of one of these circular wave fronts at a great distance from the source is, for practical purposes, straight.

Straight wave fronts (Fig. 4.22) may be produced more readily by rocking a cylindrical rod back and forth in the water at one end of the tank. Many inexpensive mechanical vibrators are available to produce either straight or circular wave fronts.

4–12 LABORATORY EXERCISE: REFLECTION AND REFRACTION OF WATER WAVES

(*a*) Set up the ripple tank as shown in Figure 4.23. Generate straight or circular wave fronts and observe the patterns they produce on the screen. Vary the frequency of the vibrator, and note the effect on the waves. Could you have predicted this result?

(*b*) Generate straight waves and attempt to calculate their speed from the time required to travel a measured distance on the screen. If this proves impossible, try using a stroboscope to measure their frequency and their wave length on the screen. Then calculate their speed using the relationship $v = f\lambda$. Is this the actual speed of the waves on the water?

(*c*) Allow straight waves to strike a barrier and be reflected from it. Remembering that rays are perpendicular to the wave front, lay rulers on the screen to represent incident and reflected rays. Measure the angles of incidence and reflection. Repeat for various angles of incidence. Do water waves obey the same laws of reflection as light does? (See Fig. 4.24.)

(*d*) To observe refraction in a ripple tank, adjust the water depth to about

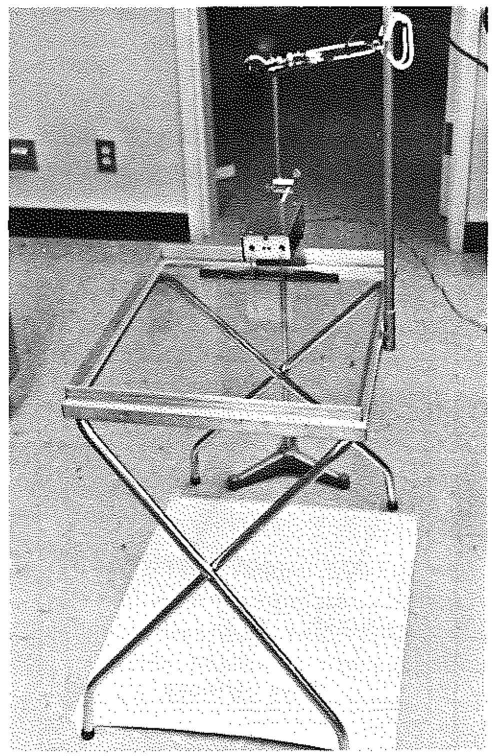


Fig. 4.23. Ripple tank with wave generator in the tank, and light source above the tank.

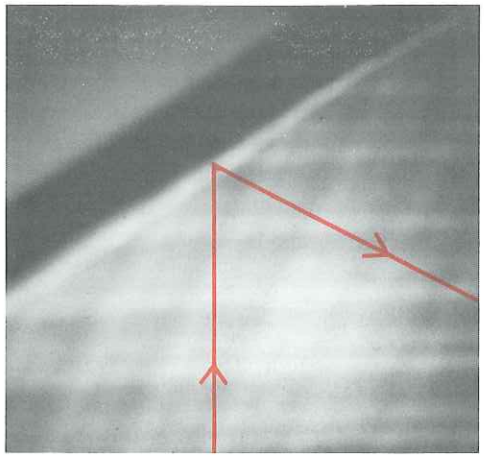


Fig. 4.24. Reflection of straight wave fronts.

1 cm. Place a sheet of transparent material in the tank as shown in Figure 4.25; the upper surface of this material should be no more than 2 mm below the surface of the water. Generate a series of straight wave fronts and allow them to be incident obliquely on the boundary between the deep water and the shallow water. How does the wave length in shallow water compare with that in deep water? What significance has this comparison as far as the speeds in deep and shallow water are concerned? (Since the vibration of a particle in the deep portion adjacent to the boundary causes a nearby particle in the shallow water to vibrate, the frequency of the waves in the shallow water is the same as that in the deep water.) Is the observed direction of refraction of water waves (Fig. 4.26) the same as for light?

Vary the frequency of the waves. Do you find that the amount of refraction depends on frequency; i.e., do your observations agree with those shown in Figure 4.27? (Recall that for light the amount of refraction depends upon the colour of the light, that is, dispersion takes place).

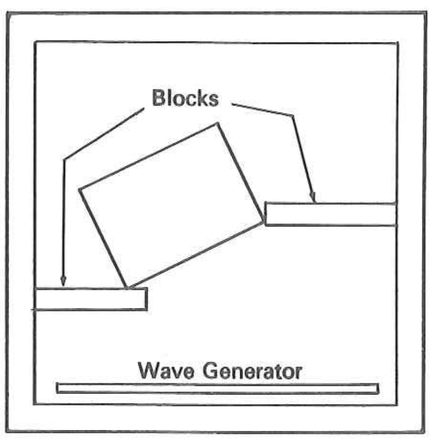


Fig. 4.25. Ripple tank arrangement for illustrating refraction.

Fig. 4.26. Low frequency straight waves are refracted as they pass from deep water (bottom) to shallow water (top). The black marker is parallel to the refracted waves.

ater and the shallow water. How
ne wave length in shallow water
e with that in deep water? What
ance has this comparison as far
speeds in deep and shallow water
cerned? (Since the vibration of a
e in the deep portion adjacent to
ndary causes a nearby particle in
low water to vibrate, the frequency
waves in the shallow water is the
s that in the deep water.) Is the
ed direction of refraction of water
(Fig. 4.26) the same as for light?
 the frequency of the waves. Do
d that the amount of refraction
s on frequency; i.e., do your ob-
ons agree with those shown in
4.27? (Recall that for light the
 of refraction depends upon the
of the light, that is, dispersion
lace).

6. Low frequency straight waves are re-
s they pass from deep water (bottom)
w water (top). The black marker is parallel
fracted waves.

It is unlikely that you will be able to make sufficiently accurate measurements to determine an index of refraction for water waves. We discuss such a relationship in the next section.

4–13 SNELL'S LAW FOR WAVES

Since the frequency of the wave does not change as the wave passes from deep water to shallow water, the relationship $v_1 = f\lambda_1$ applies for the deep water and $v_2 = f\lambda_2$ applies for the shallow water. Thus

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

Now consider Figure 4.28 which shows two successive incident wave fronts and the corresponding refracted wave fronts. (The angles marked i and R are not the angles of incidence and refraction, but are equal to these angles.) From the diagram

$$sin\ i = \frac{\lambda_1}{AB} \text{ and } sin\ R = \frac{\lambda_2}{AB}$$

$$\frac{sin\ i}{sin\ R} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

Since λ_1, λ_2, v_1, and v_2 are independent of the values of i and R,

$$\frac{sin\ i}{sin\ R} = \text{a constant,}$$

and Snell's law applies.

Fig. 4.27. For waves of higher frequency, the amount of refraction is less. The refracted waves are not parallel to the black marker in this case.

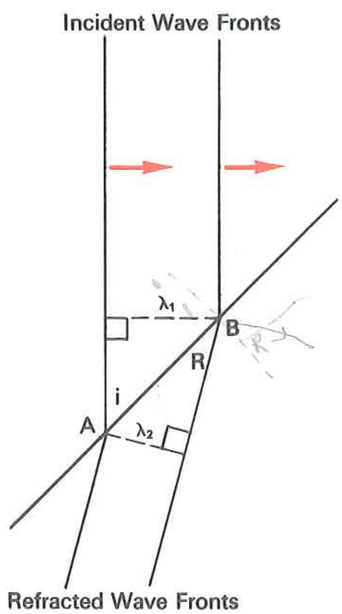


Fig. 4.28. When waves are refracted, the wave length changes. The ratio $\lambda_1 : \lambda_2$ is constant.

4–14 LIGHT: A WAVE MOTION?

We will now summarize the wave phenomena described in this chapter and try to decide whether a wave model is suitable for light.

(a) A wave motion is initiated by a vibration and this vibration is transmitted from particle to particle in the medium. If light is a wave motion, what vibrates in a luminous object to cause the wave? And, since light does not require a medium, if light is transmitted by means of waves, what is it that "waves"?

(*b*) When a wave motion takes place, matter is not transferred. Apparently this is true for light too.

(*c*) Waves may undergo reflection, or partial reflection and transmission, and refraction. Exactly similar phenomena are observed in connection with light.

(*d*) Snell's law applies for waves in the same way as it does for light. Recall that for particles,

$$\frac{\sin i}{\sin R} = \frac{v_2}{v_1}$$

but that for waves and for light,

$$\frac{\sin i}{\sin R} = \frac{v_1}{v_2}$$

(*e*) The wave theory of light can be used quite successfully to explain dispersion. Experiments with water waves in a ripple tank show that the amount of refraction of a water wave depends on the frequency of the wave. This fact, coupled with the fact that different colours undergo different amounts of refraction, suggests that, with each colour of light, there is associated a definite frequency. These frequencies are not measured directly. Instead, wave lengths are calculated from observations in diffraction and interference experiments (Chapter 5), and the formula $v = f\lambda$ is then used to calculate the frequencies. The frequencies thus computed have been found to range from about 3.8×10^{14} cps for red light to about 7.5×10^{14} cps for violet light.

The speed of red light in glass differs from that of violet light. Let

v = speed of light in air
v_1 = speed of red light in glass
v_2 = speed of violet light in glass
N_1 = index of refraction for red light
N_2 = index of refraction for violet light

Then $N_1 = \dfrac{v}{v_1}$ and $N_2 = \dfrac{v}{v_2}$

Hence $\dfrac{N_1}{N_2} = \dfrac{v_2}{v_1}$

Therefore, in glass, the speed of red light is greater than the speed of violet light.

In describing light of a particular colour it is usual to state the wave length rather than the frequency, even though the wave lengths depend on the medium through which the light is travelling. When light changes from one medium to another its speed changes, but its frequency does not change. Therefore, since $v = f\lambda$, the wave length must change. The wave lengths in a vacuum, where the speed is independent of the frequency, range from about 0.000078 cm for red light to 0.00004 cm for the shortest violet. Wave lengths are usually stated in Angstrom units (A), one Angstrom unit being equal to 10^{-8} cm. Thus 0.00004 cm = 4000A.

With the exceptions noted in (*a*) above, a wave model for light seems possible. However, our investigation is not yet complete. Do waves undergo diffraction as light does? And do interference phenomena, such as we have noted briefly for waves, occur for light? In Chapter 5, we will make a detailed study of diffraction and interference of waves and of light.

peed of red light in glass differs
at of violet light. Let

v = speed of light in air
$_1$ = speed of red light in glass
$_2$ = speed of violet light in glass
$_1$ = index of refraction for red light
$_2$ = index of refraction for violet light

$_1 = \frac{v}{v_1}$ and $N_2 = \frac{v}{v_2}$

$\frac{N_1}{N_2} = \frac{v_2}{v_1}$

re, in glass, the speed of red light er than the speed of violet light. scribing light of a particular colour al to state the wave length rather frequency, even though the wave depend on the medium through he light is travelling. When light from one medium to another its anges, but its frequency does not Therefore, since $v = f\lambda$, the wave ust change. The wave lengths in m, where the speed is independent frequency, range from about 8 cm for red light to 0.00004 cm shortest violet. Wave lengths are stated in Angstrom units (A), one m unit being equal to 10^{-8} cm. 00004 cm = 4000A.

the exceptions noted in (*a*) above, model for light seems possible. er, our investigation is not yet e. Do waves undergo diffraction t does? And do interference ena, such as we have noted briefly es, occur for light? In Chapter 5, make a detailed study of diffrac- l interference of waves and of light.

4–15 PROBLEMS

1. Calculate the period of vibration for each of the following frequencies of vibration: (*a*) 200 cycles per second; (*b*) 7 cps; (*c*) 30 c/s; (*d*) 0.1 c/s; (*e*) 500 kilocycles per second; (*f*) 200 megacycles per second.
2. Calculate the frequency of vibration for each of the following periods of vibration: (*a*) $\frac{1}{5}$ sec; (*b*) 0.4 sec; (*c*) 8 sec; (*d*) 4.0×10^{-5} sec; (*e*) 5.0×10^{-8} sec.
3. What is the relationship between the period of a vibration and its frequency? If you were to plot a graph of period against frequency, what sort of graph would you obtain? What function of the period would you have to plot against frequency in order that the graph would be a straight line?
4. A point on a rotating shaft is viewed through an eight-slit stroboscope. The maximum frequency of rotation of the strobe, for which the motion of the spot appears stopped, is 5 revs/sec. Calculate (*a*) the frequency of rotation of the shaft, (*b*) the period of rotation of the shaft.
5. A vibrating metal strip appears motionless at the limit of its vibration when viewed through a 2-slit stroboscope rotating 45 times per minute. If this viewing frequency is the maximum for stopped motion, calculate the frequency of vibration of the strip.
6. The motion of the air valve of a rotating bicycle wheel appears stopped when the wheel is viewed through a 12-slit stroboscope. The stroboscope rotates 10 times in 4 seconds. What are the possible frequencies of rotation of the wheel?
7. One blade of a rotating fan is coloured differently from the other blades. It appears motionless when viewed through a 4-slit stroboscopic disc rotating at 8 revolutions per sec. Calculate the possible frequencies of rotation of the fan.
8. Suppose that the fan in Question 7 had 4 blades, and that the blades were all alike. Calculate the possible frequencies of rotation of the fan in this case.
9. Suppose that, in an experiment with springs of the type shown in Figure 4.12, you hook 2 coil springs to the slinky, one at each end. You then generate a pulse on one of the coil springs. Describe the transmission and reflection which take place when the pulse reaches (*a*) the junction between the first coil spring and the slinky, and (*b*) the junction between the slinky and the second coil spring.
10. Draw the resultant of the black pulse and the coloured pulse in each of the 4 cases shown in Figure 4.29.
11. Draw the resultant of the black pulse and the coloured pulse in each of the 2 cases shown in Figure 4.30.
12. The speed of a wave disturbance is 1120 feet per second and the frequency is 256 cps. Calculate the wave length in inches.

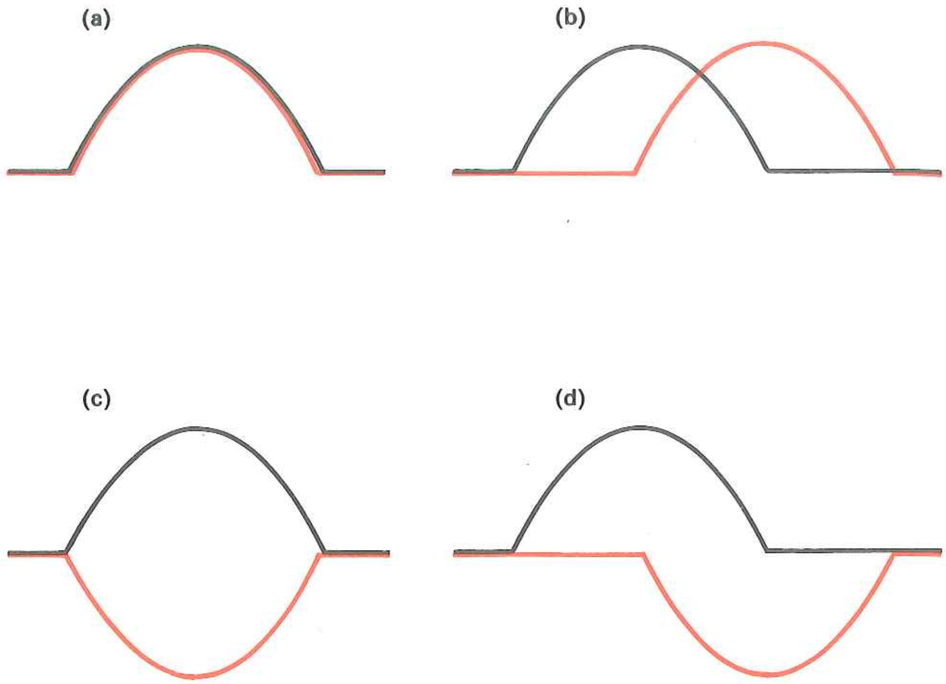


Fig. 4.29. For problem 10.

13. The speed of a wave disturbance is 3×10^{10} cm per sec, and the wave length is 300 metres. Calculate the frequency.

14. One end of a rope is being vibrated transversely. If the frequency is 20 cps and waves 15 cm long are produced, calculate the speed of the disturbance.

15. A source making 400 cps sends out waves of wave length 20 cm. How long will it take the disturbance to travel from the source to an object 160 metres distant?

16. Standing waves are set up in a string by a source making 120 vibrations per second. Seven nodes are counted in a distance of 60 cm, beginning and ending at a node. Calculate the wave length and the speed.

17. Standing waves are set up in a stretched string by means of a tuning fork which makes 128 vibrations per sec. Six nodes are counted in a length of 80.0 cm, one node being at each end of the measured length. Calculate the speed of the wave in the string.

18. The vibrator in a ripple tank produces one crest and one trough every 0.1 sec. The wave length is found to be 2.0 cm. What is the speed of the wave?

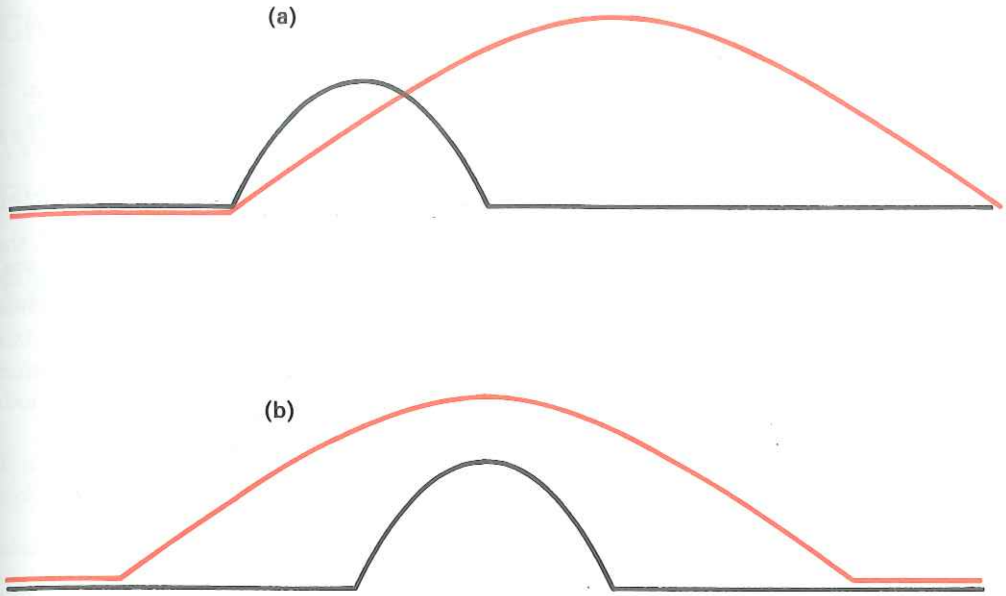


Fig. 4.30. For problem 11.

19. Circular waves having a speed of 30 cm/sec are set up in a ripple tank. Each wave front has a radius 3 cm less than the preceding one. Calculate the frequency of the source.

20. What will be the length of the waves produced by a source of frequency 15 cps if the speed of propagation of the waves is 0.9 ft/sec?

21. Straight waves in the deep portion of the water in a ripple tank have a speed of 24 cm/sec and a frequency of 4.0 c/s. They strike the boundary between deep and shallow water, the angle between a wave front and the boundary being 40°. If the speed in the shallow portion is 15 cm/sec, what angle does a refracted wave front make with the boundary?

22. A wave whose speed in a spring is 4.4 m/sec enters a second spring. The wave length changes from 2.0 m to 3.0 m. Calculate the speed of the wave in the second spring.

23. A certain kind of light has a wave length in air of 0.000055 cm. Find its wave length in water, given that the index of refraction from air to water is 1.32. State your answer in cm and in Angstroms.

24. A certain colour of red light has a wave length in air of 7.8×10^{-5} cm. Calculate its wave length in turpentine, given that the index of refraction from air to turpentine is 1.47.

25. The speed of light is 3×10^{10} cm per sec. Calculate the frequency of vibration for the hydrogen red line (wave length 6563 A).

26. The speed of sound in air is 3.4×10^2 m/sec, and frequencies ranging from about 17 to 17000 cycles per second are audible to human beings. Calculate the audible range of wave lengths.

4–16 SUMMARY

A periodic wave is initiated by a vibration; this vibration is transferred through the medium from particle to particle. The particles of the medium vibrate but are not transferred. The length of the recurring pattern in the medium is called the wave length. The wave length, frequency of vibration, and speed of propagation of the wave are related by the equation $v = f\lambda$.

Waves may be reflected. At a free end a pulse is reflected right side up; at a fixed end it is reflected upside down. Partial transmission and reflection may occur at the junction of two media.

When two pulses are superimposed, the resultant displacement is the algebraic sum of the individual displacements. Standing waves may be produced by two equal waves travelling in opposite directions. The distance from node to node is $\frac{1}{2}\lambda$.

For waves, the ratio of *sin i* to *sin R* is a constant, and the value of this constant is $\frac{v_1}{v_2}$, as for light. The wave model may be used to explain reflection and refraction of light, partial transmission and reflection, and dispersion. But waves seem to require a medium, and light does not.

late the frequency of vibration
).
and frequencies ranging from
le to human beings. Calculate

two pulses are superimposed, the
t displacement is the algebraic
the individual displacements.
waves may be produced by two
aves travelling in opposite direc-
he distance from node to node

aves, the ratio of *sin i* to *sin R*
stant, and the value of this con-
$\frac{v_1}{v_2}$, as for light. The wave model
used to explain reflection and
n of light, partial transmission
ction, and dispersion. But waves
require a medium, and light
.

Chapter 5

WAVE PHENOMENA: Diffraction and Interference

5–1 INTRODUCTION

A particle theory for light gives us, at best, awkward explanations for partial transmission and reflection, for dispersion, and for the different reflecting and absorbing properties of different surfaces. In spite of these shortcomings of the particle theory, the wave theory which Huygens proposed in 1678 gained little immediate support, mainly because waves seem to require a medium and light does not. The most convincing evidence for the wave theory was discovered in the years between 1860 and 1890, when measurements of the speed of light in various media made it apparent that the particle theory predicted the wrong change of speed during refraction. But even before 1860, support for Huygen's theory had been increasing. Diffraction and interference of light had been discovered early in the nineteenth century.

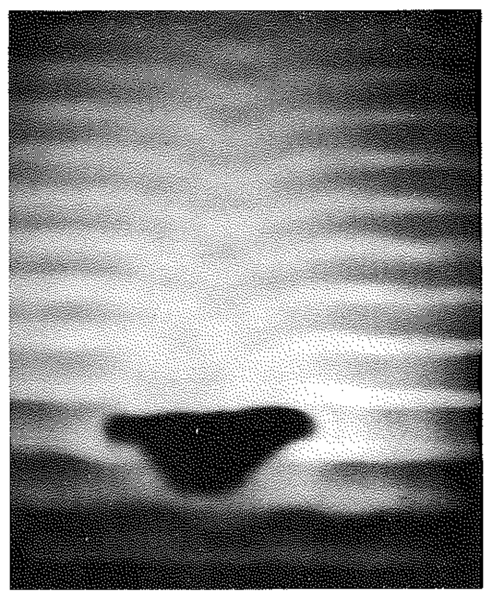


Fig. 5.1. Diffraction of water waves around an obstacle.

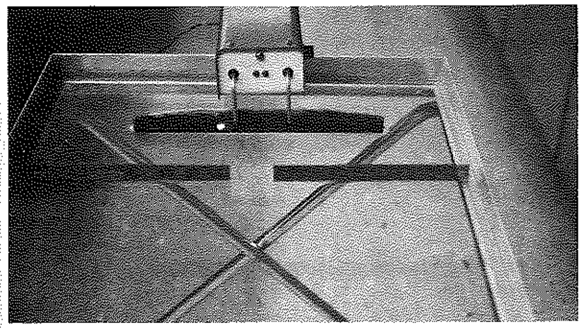


Fig. 5.2. Ripple tank arrangement for illustrating diffraction from an aperture.

These phenomena are wave phenomena. Let us study them first in connection with water waves.

5-2 DIFFRACTION OF WATER WAVES

If an obstacle is placed in the path of a wave front, the disturbance spreads into the region behind the obstacle (Fig. 5.1). This "spreading around corners", called diffraction, also takes place when a wave is incident on a barrier in which there is an aperture (Fig. 5.2). The resulting diffraction pattern is shown in Figure 5.3. The amount of diffraction depends both on the wave length λ and on the width d of the opening (Fig. 5.4); the amount of diffraction decreases as the ratio $\frac{\lambda}{d}$ decreases. If the wave length is very small compared to the width of the opening, there is very little diffraction and an almost geometric shadow is produced. What significance has this result in connection with light?

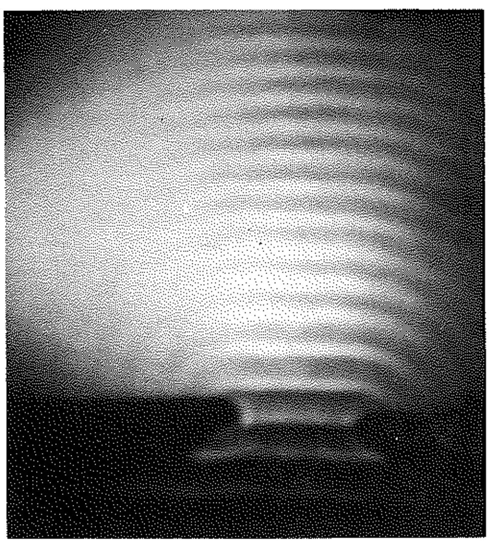


Fig. 5.3. Pattern produced when water waves are diffracted from an aperture.

5-3 INTERFERENCE OF WATER WAVES

If two point sources, each producing circular wave fronts, are used in the ripple tank, the effects of the two wave trains on one another may be studied. In general, the superposition principles discussed in Section 4-8 seem to apply, for, at some points in the area affected by the two wave trains, crests or troughs higher or deeper than usual can be seen for short periods of time, whereas other points are momentarily calm under the action of the two wave trains. (Hereafter these high crests and low troughs are called double-crests and double-troughs.)

The interaction of two wave fronts to reinforce one another is called constructive interference and results in either a double-crest or a double-trough. The interaction of two wave fronts to cancel one another is called destructive interference and results in a calm area.

Perhaps the easiest interference pattern to attain, and the most instructive, is the region behind the obstacle (Fig. his "spreading around corners", liffraction, also takes place when is incident on a barrier in which an aperture (Fig. 5.2). The re- diffraction pattern is shown in 5.3. The amount of diffraction de- oth on the wave length λ and on th d of the opening (Fig. 5.4); the of diffraction decreases as the decreases. If the wave length is all compared to the width of the , there is very little diffraction almost geometric shadow is pro- What significance has this result ection with light?

ITERFERENCE OF 'ATER WAVES

o point sources, each producing wave fronts, are used in the ripple e effects of the two wave trains nother may be studied. In general, erposition principles discussed in 4-8 seem to apply, for, at some n the area affected by the two ains, crests or troughs higher or than usual can be seen for short of time, whereas other points are arily calm under the action of wave trains. (Hereafter these high nd low troughs are called double- nd double-troughs.)

nteraction of two wave fronts to e one another is called construc- erference and results in either a crest or a double-trough. The in- n of two wave fronts to cancel other is called destructive inter- and results in a calm area.

ps the easiest interference pattern n, and the most instructive, is the

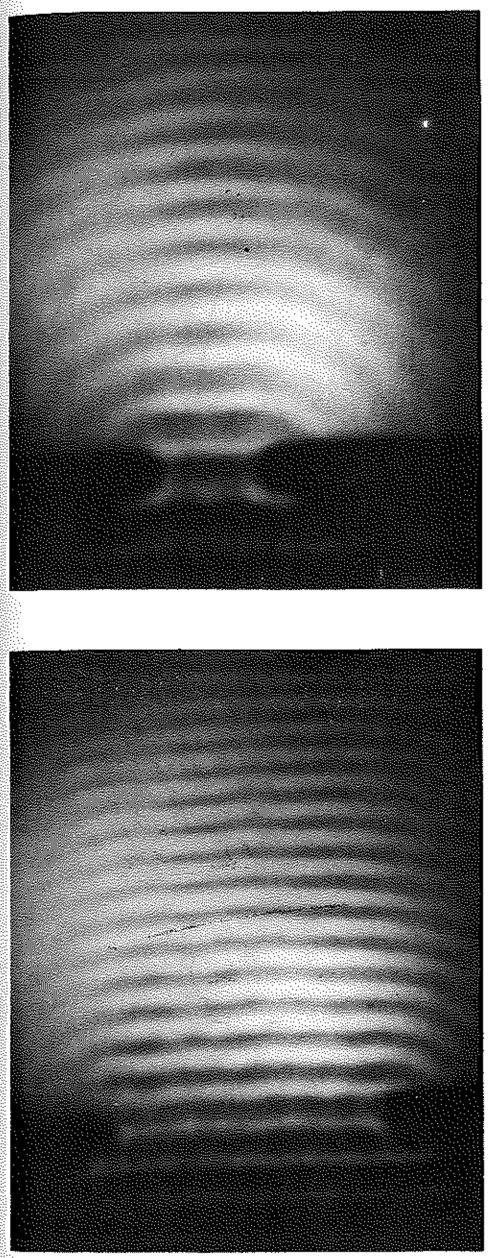


Fig. 5.4. Two cases of diffraction from an aperture. In the lower photograph, λ is less and d is greater than in the upper photograph. The amount of diffraction is less in the lower photograph.

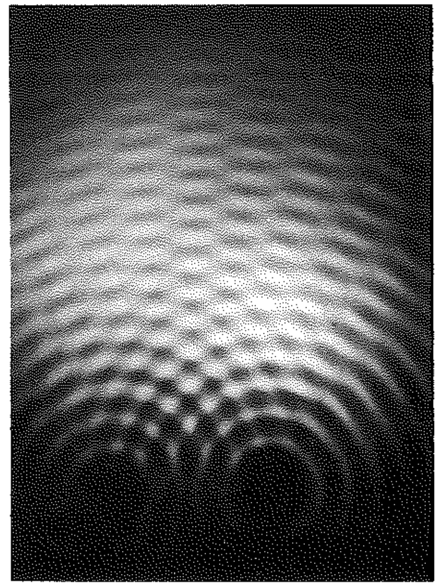


Fig. 5.5. The interference pattern produced by the waves from two point sources in phase. The nodal lines are clearly visible.

pattern which is an extension, into the whole horizontal plane, of the standing wave pattern on a string. Two vibrating sources, A and B, having the same frequency and vibrating in phase, are used in the ripple tank. A standing water wave pattern (Fig. 5.5) is produced. The "lines" visible in this photograph are nodal lines —hyperbolae having A and B as foci. At any point on these lines there is continuous destructive interference. At points in between these lines, there are alternating double-crests and double-troughs, i.e., loops occur.

Figure 5.6 shows the arrangement of crests and troughs from the two sources at a particular moment. The crests are drawn as solid lines; the troughs as broken lines. Several conclusions may be drawn:

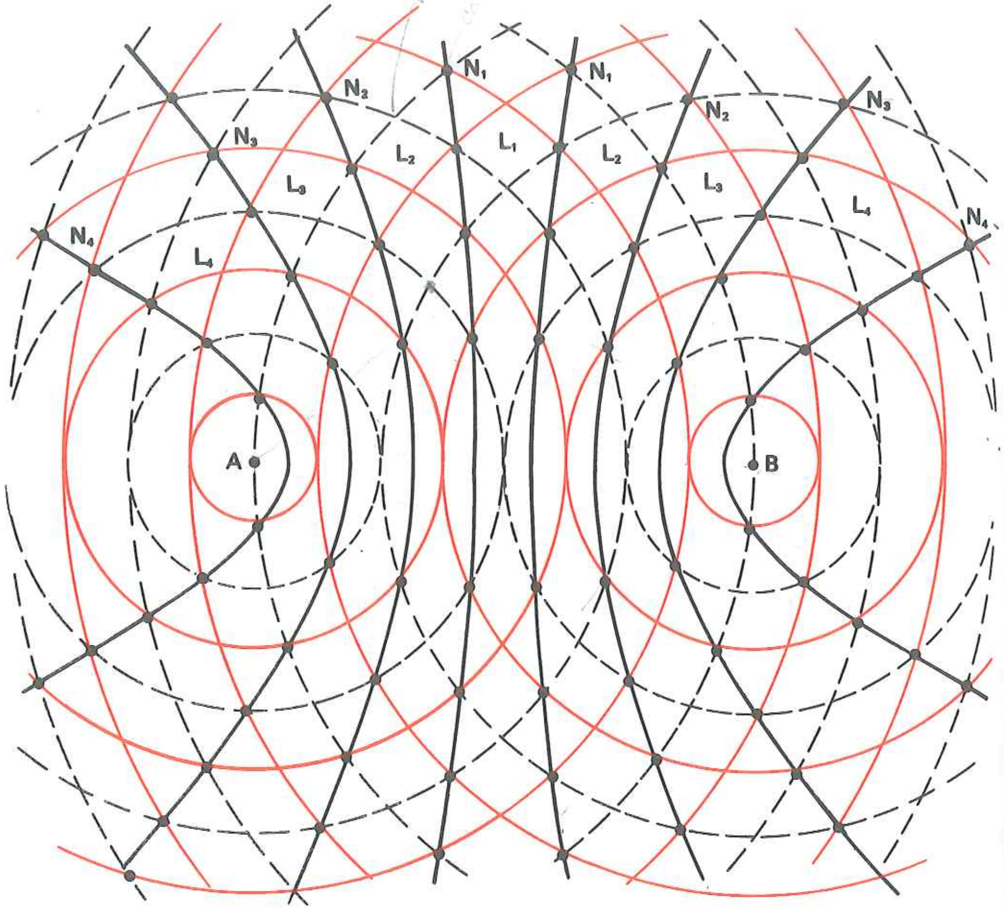
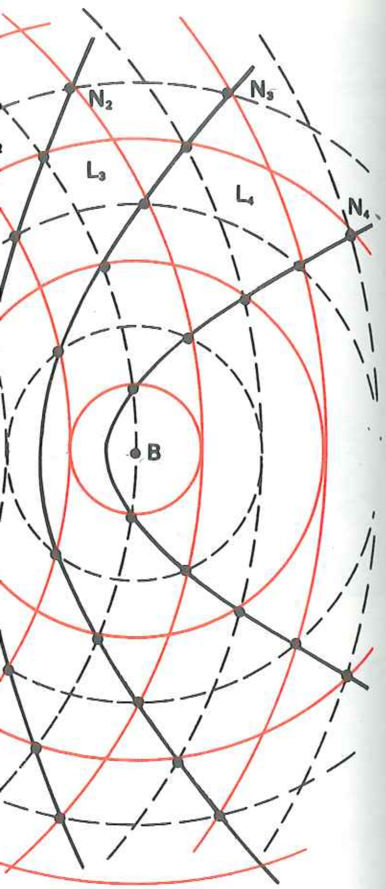


Fig. 5.6. This diagram shows how the interference of two wave trains produces nodal lines.

(*a*) The two waves reach any point P in the water by two paths PA and PB. The interference effect produced depends on $|PA - PB|$, the absolute value of the path difference. If the path difference is $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc., corresponding to the nodal lines N_1, N_2, N_3, etc., the two interfering waves are at all times in opposite phase and destructive interference occurs. Conversely, the path difference for any point on the n^{th} nodal line, where n may have the values 1, 2, 3, 4, etc., is $(n - \frac{1}{2})\lambda$. If the path difference is zero, λ, 2λ, 3λ, etc., corresponding to the intermediate regions L_1, L_2, L_3, L_4, etc., the two interfering waves are continually in the same phase and loops exist in those areas. Double-crests and double-troughs occur in these regions.



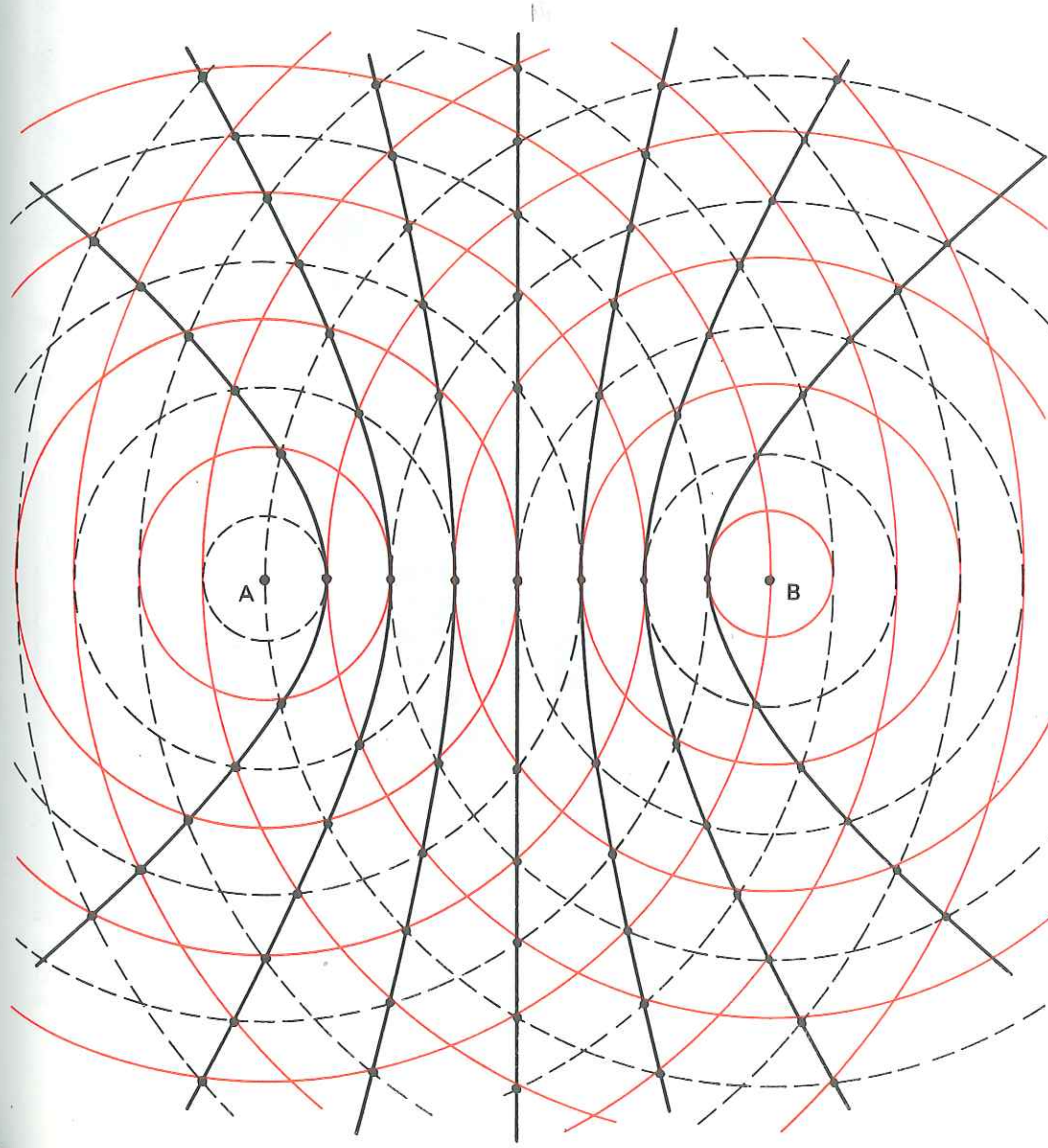
wave trains produces nodal lines.

the nth nodal line, where n may
values 1, 2, 3, 4, etc., is $(n - \frac{1}{2})\lambda$.
path difference is zero, λ, 2λ, 3λ,
rresponding to the intermediate
L_1, L_2, L_3, L_4, etc., the two inter-
vaves are continually in the same
nd loops exist in those areas.
-crests and double-troughs occur
regions.

(*b*) Between adjacent nodal lines, or between adjacent lines of maximum vibration, the path difference changes by one wave length.

(*c*) As time goes on, the double-crests and double-troughs follow one another out in the directions L_1, L_2, L_3, etc., because the interfering crests and troughs which produce them move out. This fact may be verified readily by observing the pattern on the ripple tank, or by redrawing Figure 5.6 showing the patterns one-half period later. At this time the first circle about each of A and B as centre will be a broken line.

Fig. 5.7. If the two sources A and B are not in phase, the interference pattern is displaced to one side.



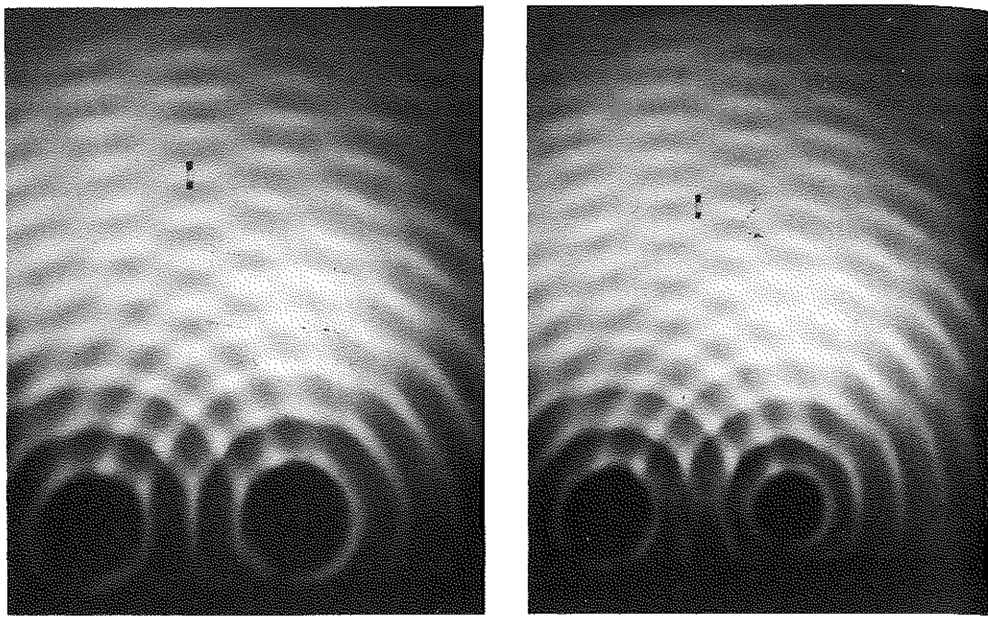


Fig. 5.8. Two interference patterns, produced by two point sources. The sources were in the same phase for the left photograph, and in opposite phase for the right photograph. The marker, which was in the same position for the two photographs, shows how the nodal lines shifted.

(*d*) The wave length of these water waves in the ripple tank may be found by measuring the paths PA and PB for any point on the n^{th} nodal line and equating $|PA - PB|$ to $(n - \frac{1}{2})\lambda$. Suppose, for example, that for a point on the third nodal line $PA = 50$ cm and $PB = 45$ cm. Then $\frac{5}{2}\lambda = 5$ cm, and $\lambda = 2$ cm.

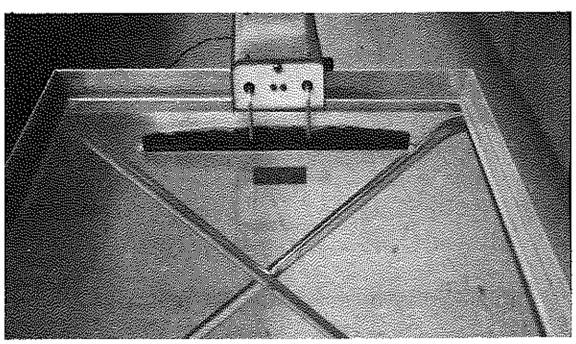
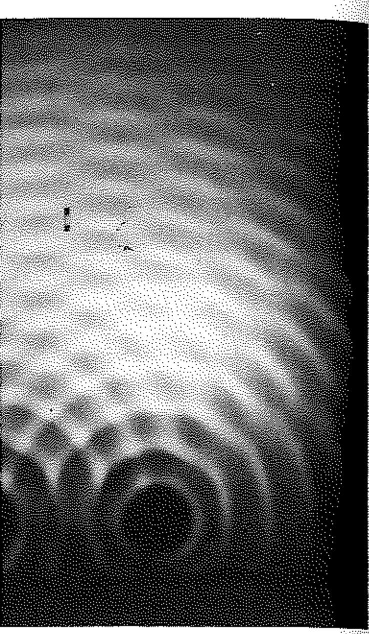


Fig. 5.9. A single small barrier in a ripple tank.

We must be sure to remember that the patterns and mathematical relationships which we have been discussing in this section apply only when the two sources, A and B, are in phase. If the two sources are not in phase, the crests and troughs originate from one of the sources a little later than they would if the sources were in phase (Fig. 5.7). This diagram would lead us to predict that the resulting interference pattern would be exactly the same as before, except that the whole pattern has been displaced to one side. This prediction is verified by the photographs in Figure 5.8.



urces. The sources were in the same
ght photograph. The marker, which
the nodal lines shifted.

ple, that for a point on the third
PA = 50 cm and PB = 45 cm.
= 5 cm, and λ = 2 cm.
st be sure to remember that the
and mathematical relationships
have been discussing in this
pply only when the two sources,
are in phase. If the two sources
n phase, the crests and troughs
from one of the sources a little
n they would if the sources were
(Fig. 5.7). This diagram would
predict that the resulting inter-
attern would be exactly the same
, except that the whole pattern
displaced to one side. This pre-
verified by the photographs in
8.

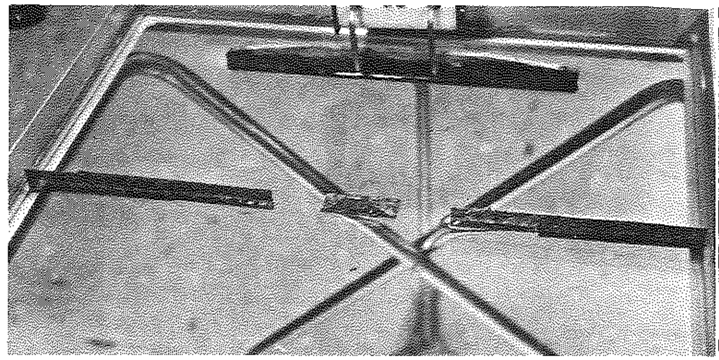


Fig. 5.10. Three barriers forming two apertures in a ripple tank.

5–4 LABORATORY EXERCISE: DIFFRACTION AND INTERFERENCE

You should now use the ripple tank to verify the statements made in Sections 5–2 and 5–3, and to investigate several other situations.

(*a*) Place two barriers (see Fig. 5.2) separated by a small aperture, in the ripple tank. Generate straight waves, and observe the diffraction at the aperture. What is the effect of using (1) longer waves, (2) shorter waves, (3) a wider opening, (4) a narrower opening?

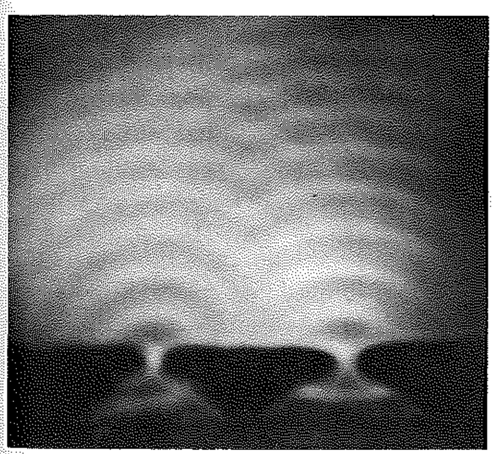


Fig. 5.11. Diffraction from each of two apertures. The diffracted wave trains interfere and produce several faint nodal lines.

(*b*) Remove the two barriers. Place a single barrier (Fig. 5.9) in the path of a set of straight waves. Observe the diffraction which takes place at the corners of the barrier. What is the effect of using (1) longer waves, (2) shorter waves, (3) a longer barrier, (4) a shorter barrier?

(*c*) Use a motor-driven generator fitted with two ripplers to generate two sets of circular waves. Be sure that the two ripplers are in phase. Observe the interference pattern, noting particularly the number of nodal lines and the positions of the nodal lines. What is the effect of using (1) longer waves, (2) shorter waves, (3) a greater distance between the sources, (4) a smaller distance between the sources?

(*d*) Now place three barriers in the ripple tank as shown in Figure 5.10. Generate straight waves and observe the two sets of waves produced as a result of the diffraction at the two openings. Do these two sets of waves interfere with one another? Can you see any nodal lines? Is the pattern the same as that produced by the two point sources? Compare the pattern you obtain with that shown in Figure 5.11.

(*e*) Repeat part (*a*). Are there any nodal lines in the diffraction pattern for a single opening? Compare the pattern you obtain with that shown in Figure 5.12.

5–5 DIFFRACTION AND INTERFERENCE COMBINED

Figure 5.11 shows diffraction taking place from each of two openings. Each opening acts as if it were a source of waves. The two sets of waves, as you might expect, interfere and produce an interference pattern similar to that produced by two point sources (Fig. 5.5).

Figure 5.12 shows diffraction taking place at a single opening, and here again, nodal lines appear. The fact that diffraction at a single opening is accompanied by interference is unexpected, for there seems to be only one set of waves. In order to explain this interference, we must assume that each point across the opening (that is, each particle of water in the opening) acts as a source of waves. These many sets of waves interfere to produce the interference pattern. We may test the validity of this assumption experimentally by allowing plane waves to be incident on a barrier (Fig. 5.13) in which there are many openings close together. The resulting interference pattern (Fig. 5.14) resembles closely that produced by the diffraction at a single opening.

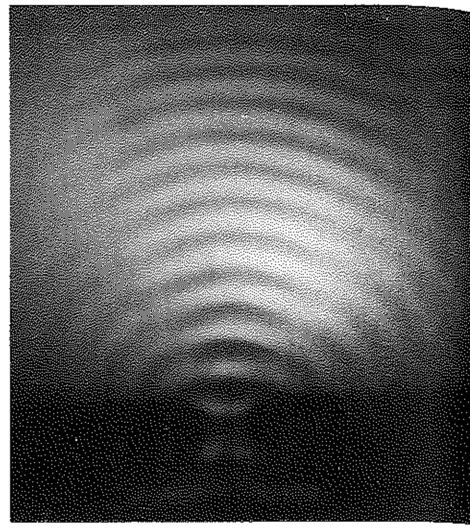


Fig. 5.12. Photograph of interference pattern accompanying diffraction from a single aperture.

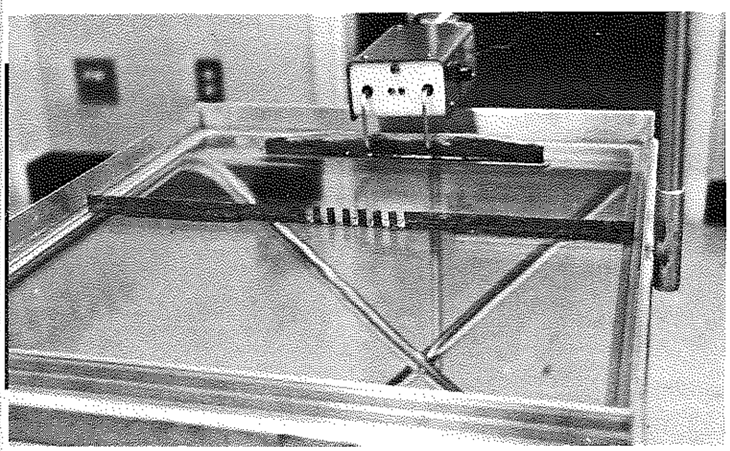
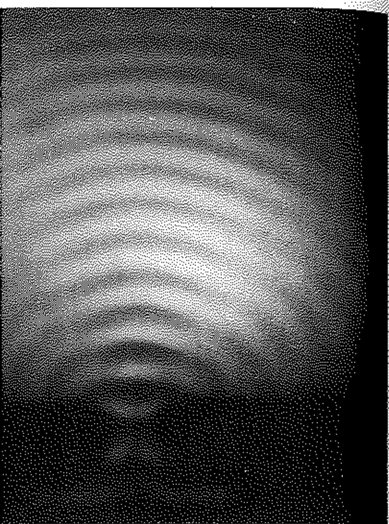


Fig. 5.13. A barrier with several apertures, in a ripple tank.



2. Photograph of interference pattern ac-
ing diffraction from a single aperture.

er (Fig. 5.13) in which there are
openings close together. The re-
interference pattern (Fig. 5.14)
les closely that produced by the
ion at a single opening.

Fig. 5.13. A barrier with several apertures, in a ripple tank.

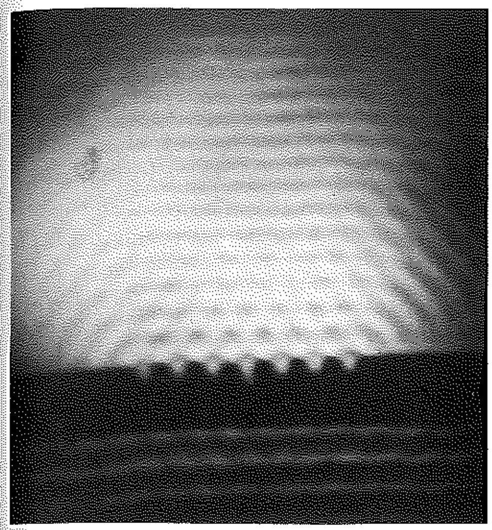


Fig. 5.14. Compare this multiple-aperture diffraction pattern with the single-aperture pattern shown in Figure 5.12.

5-6 CONDITIONS NECESSARY FOR INTERFERENCE OF LIGHT WAVES

Diffraction and interference seem to be phenomena which are unique to waves; it is difficult to visualize their occurrence in connection with particles. The diffraction which we have already noted in connection with light (Sect. 1–14) leads us to believe that light should exhibit interference phenomena as well.

Suppose that two small light sources (Fig. 5.15) are arranged as were the two point sources in the ripple tank, and that a screen, on which to observe the interference pattern, is set up at some distance from the sources. If light behaves like water waves do, a pattern such as that shown in Figure 5.5 should exist in the space surrounding the sources. At points on the nodal lines the light waves should cancel one another, causing darkness, and in the areas L_1, L_2, etc., they should reinforce one another and cause "double-brightness." We should not expect to see this pattern in space, but it should be visible on the screen. There should be dark lines where the nodal lines cut the screen, and bright areas in between.

With the arrangement shown in Figure 5.15, no interference pattern is ever observed. However, we would be unwise to conclude, as a result of this one experiment, that light does not exhibit interference phenomena. Perhaps the sources should be smaller, closer together, and relatively further from the screen. We already know that white light is composed of many colours, and we suspect that the colours are associated with different wave lengths. If the sources we use emit white light, the interference pattern for one colour may overlap and obscure the interference pattern for another colour. Another possibility is that the frequencies of the two sources are not equal, or that, if they are equal, the sources, even if they begin in phase, do not remain in phase.

In 1801 Thomas Young, an English physicist, devised a method which showed

Fig. 5.15. This arrangement of apparatus should demonstrate interference of light, but it does not.

that light waves do interfere with one another. In Young's method, a single source is used. The light waves from this source are divided into two parts. These parts travel by two different paths before being brought together again. Young obtained interference patterns even with white light, but the results are less complex if monochromatic (single colour) light is used.

5–7 YOUNG'S DOUBLE-SLIT EXPERIMENT

The laboratory exercise outlined in Section 5–10 describes how you may repeat Young's experiment using a modification of his original method. You should prepare the double-slit now and make some preliminary qualitative observations.

Coat a microscope slide with graphite paint, and draw two narrow, straight, parallel slits in the paint using two razor blades side by side. Use the long narrow filament of a show-case light bulb as the light source. You can obtain approximately monochromatic light by wrapping the bulb in a coloured cellophane filter. Hold the double slit near the eye, with the slits parallel to the filament, or allow the light passing through the slits to fall on a screen (Fig. 5.16). The resulting interference pattern is shown in Figure 5.17. The bright lines shown in Figure 5.17, and marked B, B_1, B_2, in Figure 5.16, are red if a red filter is used, blue if a blue filter is used, etc. The dark lines are dark in all cases.

At the centre of the pattern is a bright fringe B where the disturbances are in phase because the distances FB and EB are equal. On either side of B is a dark fringe D_1 where the disturbances are in opposite phase because FD_1 and ED_1 differ by one-half wave length. For the next dark fringe D_2 the path difference is $\frac{3}{2}$ wave lengths. Thus, successive dark fringes occur for path differences of $\frac{1}{2}\lambda$,

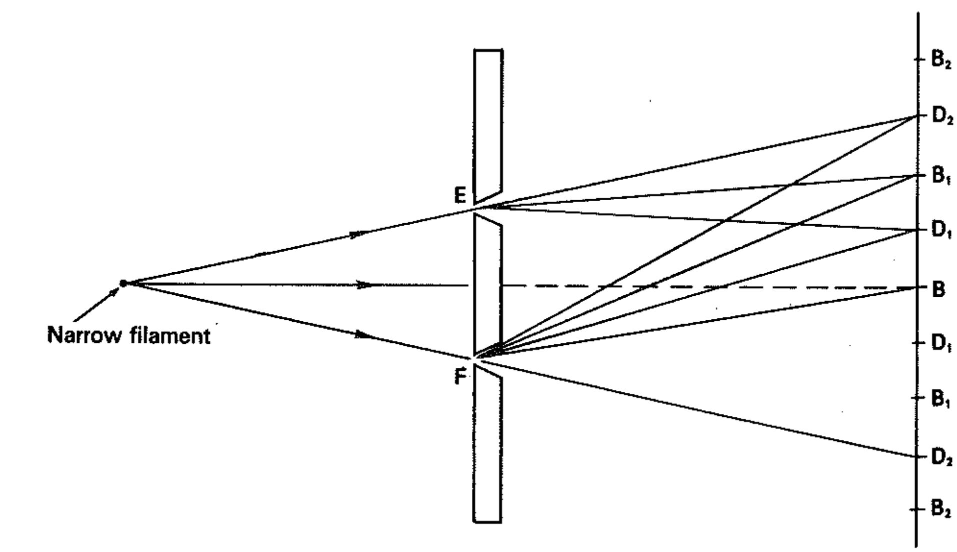
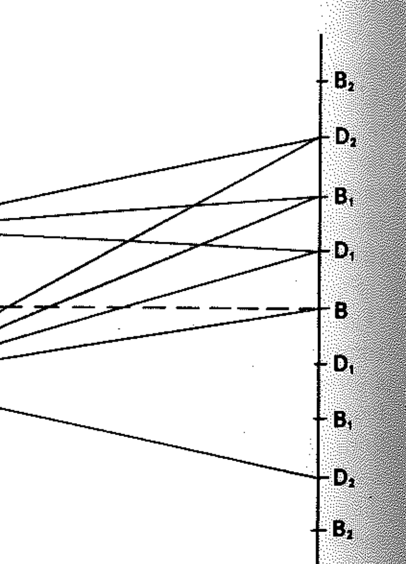


Fig. 5.16. Light from a narrow filament passes through a double slit to form interference fringes.

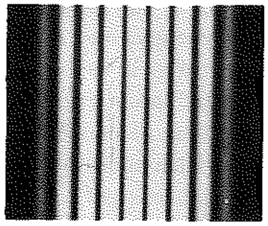
urce. You can obtain approximate-
ochromatic light by wrapping the
a coloured cellophane filter. Hold
uble slit near the eye, with the
rallel to the filament, or allow the
assing through the slits to fall on
n (Fig. 5.16). The resulting inter-
pattern is shown in Figure 5.17.
ight lines shown in Figure 5.17,
rked B, B_1, B_2, in Figure 5.16, are
a red filter is used, blue if a blue
used, etc. The dark lines are dark
ases.

ne centre of the pattern is a bright
B where the disturbances are in
because the distances FB and EB
al. On either side of B is a dark
D_1 where the disturbances are in
e phase because FD_1 and ED_1 dif-
ne-half wave length. For the next
inge D_2 the path difference is $\frac{3}{2}$
engths. Thus, successive dark
occur for path differences of $\frac{1}{2}\lambda$,



louble slit to form interference fringes.

$\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc., and bright fringes occur for path differences of 0, λ, 2λ, 3λ, etc. The dark lines are further apart for red light than for blue light. Apparently the wave lengths for red light are longer than those for blue light.



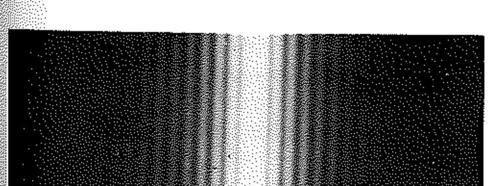
Physics Department, University of Western Ontario

Fig. 5.17. A photograph of interference fringes produced by a double slit.

5–8 SINGLE-SLIT DIFFRACTION AND INTERFERENCE

Diffraction and interference effects are produced when light passes through a single slit. Use the tip of a small screw-driver to rule a single narrow slit on a microscope slide coated with graphite paint. (A razor blade produces too narrow a slit). Look at a narrow source of mono-chromatic light through the single slit held close to the eye.

A typical single slit diffraction pattern is shown in Figure 5.18. There is a broad,



Physics Department, University of Western Ontario

Fig. 5.18. A photograph of diffraction and inter-ference effects produced by a single slit.

central, bright band flanked by alternate dark and bright lines. The intensity of the bright bands decreases rapidly as the distance from the centre increases. You should compare this pattern with the similar water wave pattern shown in Figure 5.12.

5–9 ANALYSIS OF DOUBLE-SLIT DIFFRACTION AND INTERFERENCE

Diffraction and interference effects seem to indicate that light has wave properties. If light has wave properties, what are the wave lengths? We have guessed already that the wave lengths are very short, much too short to be measured directly. However, they can be calculated from measurements made in connection with interference experiments.

It is not possible to calculate the wave length by measuring the path difference for a point on the n^{th} nodal line as we did for water waves in the ripple tank. The light sources are too close together and the paths are very nearly equal. However, λ can be calculated from other measurements. Consider Figure 5.19. E and F represent the slits in Young's ex-periment. P is the point where the n^{th} nodal line cuts the screen. AB is the right bisector of EF, making $EA = AF$. PC is perpendicular to EF produced and is equal to AB. Also, $BP = AC$.

In $\triangle EPC$, $\angle C = 90°$

$$\begin{aligned} PE^2 &= EC^2 + PC^2 \\ &= (AC + EA)^2 + AB^2 \\ &= AC^2 + 2AC \cdot EA + EA^2 + AB^2 \end{aligned}$$

In $\triangle FPC$, $\angle C = 90°$

$$\begin{aligned} PF^2 &= FC^2 + PC^2 \\ &= (AC - AF)^2 + AB^2 \\ &= AC^2 - 2AC \cdot AF + AF^2 + AB^2 \\ &= AC^2 - 2AC \cdot EA + EA^2 + AB^2 \end{aligned}$$

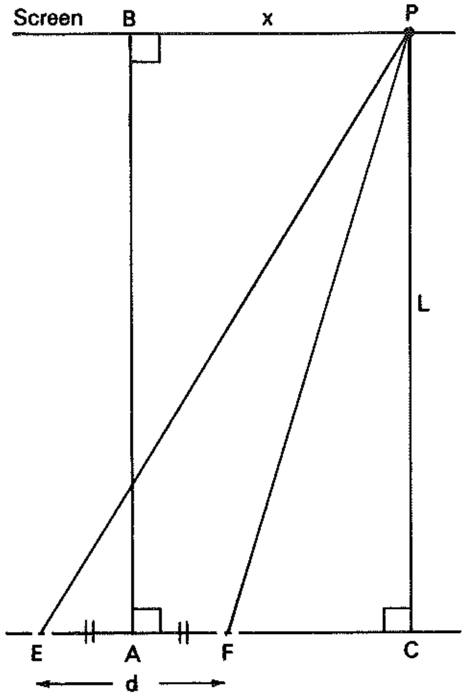


Fig. 5.19. Diagram for mathematical analysis of double-slit interference.

Subtracting,

$$\begin{aligned} PE^2 - PF^2 &= 4AC \cdot EA \\ &= 4AC \cdot \frac{EF}{2} \\ &= 2AC \cdot EF \\ &= 2BP \cdot EF \end{aligned}$$

Factoring,

$$(PE - PF)(PE + PF) = 2BP \cdot EF \text{ (1)}$$

Now $PE - PF$ is the path difference and is equal to $(n - \frac{1}{2})\lambda$. Also, if AB is large, relative to PB, both PE and PF are, for practical purposes, equal to AB. Equation (1) then becomes

$$\lambda(n - \tfrac{1}{2})(AB + AB) = 2BP \cdot EF$$

$$\lambda = \frac{BP \cdot EF}{AB(n - \frac{1}{2})}$$

Using the symbols shown in Figure 5.19,

$$\lambda = \frac{x \cdot d}{L(n - \frac{1}{2})}$$

where x is the distance from the centre of the pattern to the n^{th} dark line, d is the slit separation, and L is the distance from the slits to the screen.

The distance L can be measured with reasonable accuracy, and it may be possible to measure x. If the slits are drawn with two razor blades side by side, then d is the thickness of one blade and may be measured with a micrometer. The order of magnitude, at least, of the wave length can then be calculated. Suppose that $d = 0.01$ cm, L is 100 cm, and $x = 2.25$ cm for the fifth dark band. Then

$$\begin{aligned} \lambda &= \frac{2.25 \times 0.01}{100(5 - \frac{1}{2})} \\ &= 5 \times 10^{-5} \end{aligned}$$

The wave length is of the order of 5×10^{-5} cm, or, since 1 Angstrom unit $= 10^{-8}$ cm, the wave length is approximately 5000A.

The value of x is the distance from the centre of the pattern to the n[th] dark line. We may have difficulty in measuring x because of the difficulty in locating the centre of the interference pattern. However, further mathematical analysis reveals a way of avoiding this difficulty.

Rearranging the formula,

$$\lambda = \frac{x \cdot d}{L(n - \frac{1}{2})}$$

we obtain $x = (n - \frac{1}{2})\dfrac{\lambda L}{d}$

Thus the value of x for the 25th dark line is $24\frac{1}{2} \times \dfrac{\lambda L}{d}$ and for the 24th dark line is $23\frac{1}{2} \times \dfrac{\lambda L}{d}$.

The difference between these two values of x, which we shall call Δx, is $\dfrac{\lambda L}{d}$,

$$\lambda = \frac{x \cdot d}{L(n - \frac{1}{2})}$$

is the distance from the centre
attern to the n^{th} dark line, d is
eparation, and L is the distance
slits to the screen.
stance L can be measured with
le accuracy, and it may be pos-
neasure x. If the slits are drawn
razor blades side by side, then
thickness of one blade and may
red with a micrometer. The order
tude, at least, of the wave length
n be calculated. Suppose that
. cm, L is 100 cm, and $x = 2.25$
ie fifth dark band. Then

$$\lambda = \frac{2.25 \times 0.01}{100(5 - \frac{1}{2})}$$
$$= 5 \times 10^{-5}$$

e length is of the order of 5×10^{-5}
nce 1 Angstrom unit $= 10^{-8}$ cm,
length is approximately 5000A.
lue of x is the distance from the
the pattern to the n^{th} dark line.
have difficulty in measuring x
of the difficulty in locating the
the interference pattern. How-
ther mathematical analysis re-
ay of avoiding this difficulty.
nging the formula,

$$\lambda = \frac{x \cdot d}{L(n - \frac{1}{2})}$$

ain $x = (n - \frac{1}{2})\frac{\lambda L}{d}$

value of x for the 25th dark line
$\frac{\lambda L}{d}$ and for the 24th dark line is
$\frac{L}{d}$.

rence between these two values
ch we shall call Δx, is $\frac{\lambda L}{d}$,

X —>

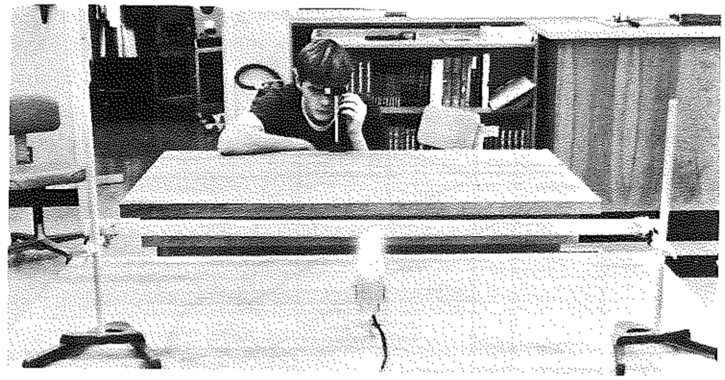


Fig. 5.20. The light source is viewed through a double slit. The spacing of the bright or dark lines in the interference pattern can be measured on the ruler.

$$\text{i.e., } \Delta x = \frac{\lambda L}{d}$$

$$\text{or} \quad \lambda = \frac{d \cdot \Delta x}{L}$$

where Δx is the distance between any two successive dark lines, or the average spacing of the dark lines.

5–10 LABORATORY EXERCISE: MEASURING λ

Continue Young's double-slit experiment (Sect. 5–7). Place a ruler above the monochromatic light source (Fig. 5.20) so that you will be able to measure the spacing of the dark lines, and scratch a "window" in the paint, across the double slit, so that you are able to see the ruler as well as the interference pattern. Measure the distance, L, (about two or three metres) from the double slit to the source. Count the number of dark lines which you observe between two markers placed on the ruler, and calculate Δx. Then calculate λ from the relationship $\lambda = \frac{d \cdot \Delta x}{L}$.

5–11 COLOUR AND WAVE LENGTH

We have already noted that the dark band spacing varies for different colours. That is, if d and L are kept constant, the value of Δx is not the same for red light as for blue, and therefore the wave lengths differ. Experiments similar to the one above indicate that the wave lengths range from about 4000A for the violet end of the visible spectrum to about 7800A for the red end. (See the top photograph in the colour plate opposite page 22.)

5–12 THIN FILMS

The beautiful colours produced in soap bubbles and in oil films on water are familiar to everyone. If a wire frame is dipped in a soap solution to which a little glycerine has been added, a fairly durable thin film is produced. This film consists of water held in place by soap "membranes" on either side. If the wire frame is held in a vertical position, the film is wider at the bottom than at the top because the force of gravity pulls many particles of water towards the lower edge. When viewed by reflected white light, the surface of the film is seen to be covered with a series of horizontal spectral bands in brilliant colours. If a red glass filter is placed in the path of the light, the spectral bands are replaced by alternate dark and red bands.

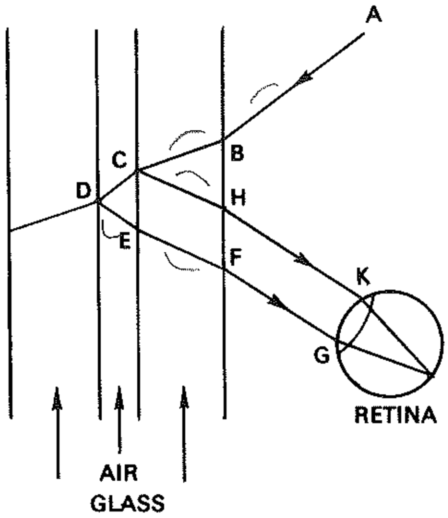


Fig. 5.21. Reflection at the two surfaces of a thin film produces interference.

The same type of interference pattern occurs when there is a thin wedge-shaped film of air between two glass plates (Fig. 5.21). We shall explain the interference pattern with reference to this air wedge. A ray of monochromatic light from a source, A, enters the glass plate on the right at B. On reaching C part of the light is reflected and part of it is refracted. The reflected ray returns to the surface of the glass following the path CHK to the eye. The part of the light which is refracted at C is both reflected and refracted at D; the reflected portion returns to the eye along the path $DEFG$. The parallel rays HK and FG are focused on the retina. Thus, the conditions necessary for interference are fulfilled. Light waves coming from a single source, A, are divided at C, they travel two different paths, CHK and $CDEFG$, and they are brought together again on the retina. The path difference for the two portions of the ray is CDE.

If the path difference CDE is λ, 2λ, 3λ, etc., the two disturbances would be expected to reinforce each other on the retina. Similarly if CDE is $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc., destructive interference would be expected. Actually, the opposite is the case. The reflection at C is at the surface of a less dense medium, and the beam is reflected right side up. The reflection at D is at the surface of a denser medium, and the beam is reflected upside down. In determining the nature of the interference, this difference between the reflections at C and D is equivalent to a path difference of half a wave length.

In a wedge shaped film where the thickness of the film increases steadily, cancellation and reinforcement will occur at a succession of positions and alternate dark and bright bands will be seen. If the path difference CDE produces reinforcement, so does $CDE + \lambda$, and $CDE + 2\lambda$, etc. If the incident light is perpendicular to the film, CDE equals twice the thickness of the film. Therefore, for successive dark fringes the film must increase in thickness by one-half wave length.

The study of interference effects in thin films provides another method of measuring the wave length of light.

5–13 THE AIR-WEDGE EXPERIMENT

A wedge-shaped air film may be formed between two sheets of plane glass about 10 cm long, touching at one end and separated by a sheet of thin paper at the other. If light from a monochromatic source is reflected to the eye by the two surfaces of this air film, alternate, parallel dark and bright fringes cover the whole area of the glass (Fig. 5.22), and, if the glass

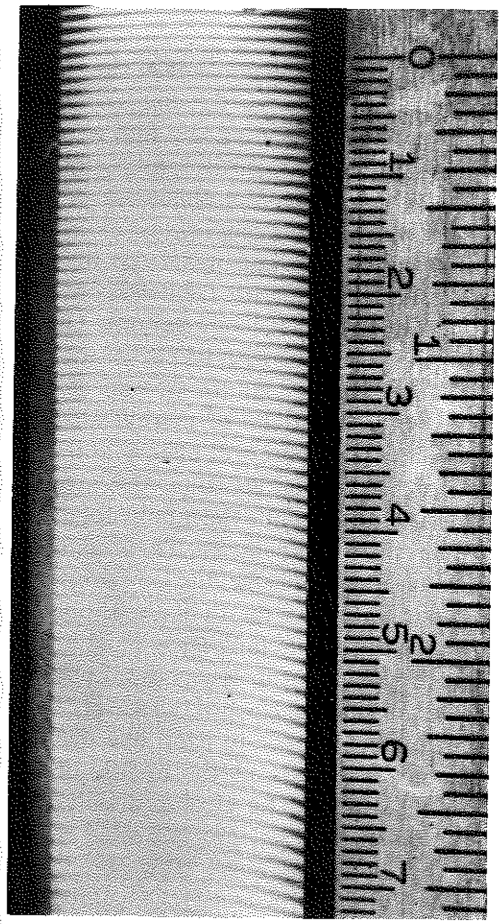
for the two portions of the ray
path difference CDE is λ, 2λ, 3λ,
two disturbances would be ex-
o reinforce each other on the
imilarly if CDE is $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$,
tructive interference would be
Actually, the opposite is the
reflection at C is at the surface
dense medium, and the beam is
right side up. The reflection at
he surface of a denser medium,
beam is reflected upside down.
mining the nature of the inter-
this difference between the re-
at C and D is equivalent to a
erence of half a wave length.
dge shaped film where the thick-
he film increases steadily, can-
and reinforcement will occur at
ion of positions and alternate
bright bands will be seen. If the
erence CDE produces reinforce-
does $CDE + \lambda$, and $CDE + 2\lambda$,
e incident light is perpendicular
m, CDE equals twice the thick-
e film. Therefore, for successive
ges the film must increase in
by one-half wave length.
dy of interference effects in thin
vides another method of meas-
wave length of light.

HE AIR-WEDGE XPERIMENT

e-shaped air film may be formed
two sheets of plane glass about
g, touching at one end and sepa-
a sheet of thin paper at the other.
rom a monochromatic source is
to the eye by the two surfaces
ir film, alternate, parallel dark
nt fringes cover the whole area
ass (Fig. 5.22), and, if the glass

is optically plane, the fringes are straight. To facilitate measurement of the fringes, a centimetre scale may be glued over a portion of the glass surface which faces the source of light.

In Figure 5.23, PF represents a beam of light perpendicular to the surface of



Courtesy L. G. Mitchell

Fig. 5.22. A photograph of interference fringes produced by a wedge-shaped air film.

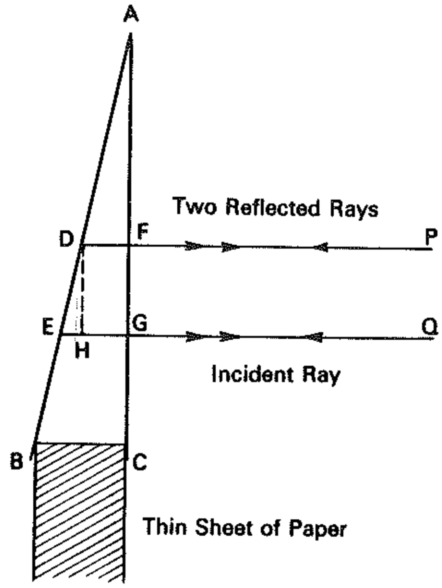


Fig. 5.23. If D and E represent adjacent dark (or bright) fringes, then $EH = \frac{1}{2}\lambda$.

the wedge-shaped air film. Since the film is very thin, both the light reflected at F and the light reflected at D will travel back along, or very close to, the path of incidence. The path difference $2DF$ determines whether there will be cancellation or reinforcement. For a similar ray QG, the path difference $2EG$ determines the nature of the interference. If D and E represent the apparent positions of two adjacent dark fringes, $2EG - 2DF$ equals one wave length, and

$$EG - DF = EH = \frac{\lambda}{2}$$

Since the distance EH is too small to be measured directly, it must be computed using the similar triangles ABC and DEH. Measurements of BC, AB, and DE are now taken as carefully as possible. The distance BC, the thickness of the paper used to separate the plates, can be obtained by measuring with a micrometer the thickness of several sheets of the paper and dividing by the number of sheets. The length AB can be measured with a ruler. DE is the average distance between adjacent bright or dark fringes and is obtained by measuring across several fringes and dividing by the number of fringes. Suppose that $BC = 0.002$ cm, $AB = 10.0$ cm, and $DE = 0.14$ cm.

In triangles ABC and DEH, $\frac{EH}{DE} = \frac{BC}{AB}$

$$\frac{\frac{\lambda}{2}}{0.14} = \frac{0.002}{10.0}$$

$$\lambda = \frac{0.002 \times 0.14 \times 2}{10.0}$$

$$= 5.6 \times 10^{-5}$$

Thus, the wave length is 5.6×10^{-5} cm, or approximately 5600 Angstrom units.

5-14 AN INVESTIGATION OF THE RELATIVE SIZES OF LIGHT WAVES AND MOLECULES

The wave lengths of light are obviously very short. How do they compare with the dimensions of molecules? Undoubtedly the size of a molecule varies from substance to substance. However, if we knew even the order of magnitude of molecular dimensions, we could make a rough comparison of the sizes of light waves and molecules.

One method, which may be used to determine the size of a molecule, depends on the fact that many liquids spread out to form a thin film on the surface of water. The liquid must be insoluble in water; oil is an obvious example.

Suppose that a drop of oil, whose volume is 0.1 cm^3, spreads out on the surface of water to form a circular film of radius 5 cm. How thick is the film? The film is in the form of a cylinder whose volume, V, is given by the formula $V = Ah$, where A is the surface area of the film and h is its thickness.

Then $h = \frac{V}{A}$

Here, $V = 0.1 \text{ cm}^3$

and $A = \pi r^2 = 3.14 \times 5^2 \text{ cm}^2$

$= 79 \text{ cm}^2$

Then $h = \frac{0.1}{79}$ cm

Thus the order of magnitude of the thickness of the oil film is 10^{-3} cm.

If the liquid forming the film spreads out sufficiently far, it will form a molecular layer; that is, the thickness of the film will be the thickness of one molecule. Oleic acid will form a molecular layer if you use a small quantity of it and provide enough space for it to spread out. Dilute 5 cm^3 of oleic acid with 95 cm^3 of methyl alcohol, and then dilute 10 cm^3 of this solution with 90 cm^3 of methyl alcohol. Calculate the volume of oleic acid in one cm^3 of this second solution. Use an eyedropper to determine the number of drops in one cm^3 of this solution. Then calculate the volume of oleic acid in one drop.

Pour water to a depth of about one cm into a clean ripple tank tray. Dust a thin film of chalk dust or lycopodium powder evenly over the surface of the water. Then use the eyedropper to drop one drop of oleic acid solution at the middle of the water surface. (The alcohol dissolves in the water; the film that is visible is due to the oleic acid only. How could you show that this is true?) Calculate the area and hence the thickness of the film. How does the thickness of this molecular layer compare with the average wave length of visible light?

ow thick is the film? The film is rm of a cylinder whose volume, ven by the formula $V = Ah$, is the surface area of the film its thickness.

$$h = \frac{V}{A}$$

$$V = 0.1\ \text{cm}^3$$

$$A = \pi r^2 = 3.14 \times 5^2\ \text{cm}^2 = 79\ \text{cm}^2$$

$$h = \frac{0.1}{79}\ \text{cm}$$

order of magnitude of the thick- he oil film is 10^{-3} cm.

liquid forming the film spreads iently far, it will form a molecu- ; that is, the thickness of the be the thickness of one molecule. d will form a molecular layer if small quantity of it and provide pace for it to spread out. Dilute oleic acid with 95 cm^3 of methyl and then dilute 10 cm^3 of this with 90 cm^3 of methyl alcohol. e the volume of oleic acid in one his second solution. Use an eye- o determine the number of drops ^{3} of this solution. Then calculate me of oleic acid in one drop. water to a depth of about one cm ean ripple tank tray. Dust a thin halk dust or lycopodium powder ver the surface of the water. Then eyedropper to drop one drop of d solution at the middle of the rface. (The alcohol dissolves in r; the film that is visible is due leic acid only. How could you at this is true?) Calculate the hence the thickness of the film. es the thickness of this molecular mpare with the average wave- f visible light?

5–15 PROBLEMS

1. A point source of light casts very "sharp" shadows of surrounding objects. What information does this fact give concerning light?
2. Draw an interference pattern, similar to that shown in Figure 5.6, for two sources of the same frequency and in phase, the distance between the sources being 3λ.
3. For a point on the first nodal line N_1 (see Fig. 5.6) the path difference is found to be 3.0 cm. What is the path difference for (*a*) a point on the line N_2, (*b*) a point on the line N_3, (*c*) a point mid-way between N_1 and N_2, and (*d*) a point mid-way between N_2 and N_3?
4. If the common frequency of vibration of the two sources in Question 3 is 7 cps, what is the speed of the waves?
5. A standing wave pattern is set up by two sources of the same frequency and in phase. A point on the first nodal line is 10 inches from one source and 8 inches from the other; the speed of the waves is 0.75 ft/sec. Calculate the common frequency of the two sources.
6. Continuing destructive interference is observed at a point for which the path difference is 16 cm. The sources have the same frequency and are in phase. What are the two largest possible values of the wave length? What are the two lowest possible values for the common frequency of the two sources, if the speed of the waves is 20 cm/sec?
7. In a ripple tank interference experiment, you measure the distances from sources A and B to a point P on a nodal line. What further information must you have in order to calculate the wave length?
8. In a ripple tank experiment to demonstrate interference, two point sources having a common frequency of 6.0 c/s are used. The sources are 5.0 cm apart and vibrate in phase. A metre stick is placed in the water parallel to the line joining the sources. The central axis of the pattern crosses the metre stick at the 50 cm mark. The first nodal lines cross the metre stick at the 40 cm and 60 cm marks. Each of these points is 50 cm from the mid-point of the line joining the sources. (*a*) Calculate the wave length. (*b*) Calculate the speed of the waves.
9. Consider the relationship

$$\lambda = \frac{d \cdot \Delta x}{L}$$

(*a*) What is the effect on Δx of doubling λ? (*b*) If λ changes by a factor of 1.5, by what factor does Δx change? (*c*) What is the effect on Δx of changing L by a factor of 3? (*d*) Interpret your answers to (*a*), (*b*) and (*c*) in terms of the laboratory exercise described in Section 5.10.

10. (*a*) For two slit sources 0.30 mm apart, emitting light of wave length 6000A, calculate the separation of adjacent dark bands observed at a distance of 2.0 metres. (*b*) State the change in band separation that would be observed if blue light were used instead of yellow.

11. A student stands 4.0 m from a monochromatic light source and observes the light through a pair of narrow parallel slits 2.0×10^{-2} cm apart. He determines that the distance from the first node to the eighth node in the interference pattern is 8.0 cm. Calculate the wave length of the light.
12. Two parallel slits 3.0×10^{-5} m apart are illuminated by parallel rays of monochromatic light of wave length 4.5×10^{-7} m. The interference fringes are observed on a screen placed 90 cm from the slits. Find the distance from the mid-point of the central maximum to the mid-point of the second maximum to the right.
13. If θ represents the angle BAP in Figure 5.19, prove that
$$\sin\theta = \frac{\lambda(n - \frac{1}{2})}{d}$$
14. Two point sources A and B, 4.5 cm apart in a ripple tank, vibrate in phase at a common frequency of 9.0 c/s. The angle between the right bisector of AB and the second nodal line is 40°. Calculate (*a*) the wave length, (*b*) the speed of the waves.
15. Why is interference from two rows of fluorescent lights in a classroom not noticeable?
16. Calculate the frequency associated with light of each of the following wave lengths: (*a*) 4000A, (*b*) 5000A, (*c*) 6000A.
17. Two plane glass plates 10 cm long are touching at one end and are separated at the other end by a strip of paper 0.0015 cm thick. When the plates are illuminated by monochromatic light the average distance between consecutive dark fringes is 0.2 cm. Calculate the wave length of the light in Angstrom units.
18. Two plane glass plates 12 cm long, touching at one end and separated at the other end by a strip of paper, are illuminated by light of wave length 0.000063 cm. A count of the fringes gives an average of 8 dark fringes per cm. Calculate the thickness of the paper.
19. An air-wedge is formed between two glass plates 15.6 cm long by placing them in contact at one end and separating them at the other end by a thin strip of paper. The wedge is illuminated by light of wave length 546×10^{-7} cm and the interference pattern in the reflected light is observed. The average distance between two dark bands in the pattern is found to be 1.2 mm. Calculate the thickness of the paper strip separating the plates at the large end of the wedge.
20. The diameter of a fine straight fibre is determined by an interference experiment, as follows. The fibre is placed on the surface of a plane glass plate, parallel to and very close to one edge of the plate. The plate is 10 cm square. Another identical plate is placed on top of the first, and the plates are illuminated by sodium light (wave length 5890 Angstrom units) which falls perpendicularly on them. It is observed that a series of bright and dark lines cross the plates, and 12 bright and 12 dark lines are counted. Calculate the diameter of the fibre.
21. Two strips of optically plane glass plate are held together at one end and separated at the other end by a thin strip of metal foil 0.022 mm thick,

forming an air wedge 12 cm long. When sodium yellow light illuminates the plates, bright and dark fringes are observed with an average distance of 1.6 mm between consecutive dark fringes. (*a*) Calculate the wave length of the light. (*b*) Indicate the frequency of this wave motion. (Speed of light = 3×10^{10} cm per sec.)

22. (*a*) If sound is transmitted by means of waves, what should be the effect of (*i*) destructive interference (*ii*) constructive interference, in sound? (*b*) Do these phenomena occur for sound? (*c*) What does your answer to (*b*) indicate about how sound is transmitted?
23. Two identical tuning forks are placed 10 metres apart. At a point on the line joining them, 4 metres from one of the forks, constructive interference is obesrved. If the forks are in phase, calculate two possible values for their common frequency. Assume that the speed of sound in air is 340 m/sec.
24. In order to make a comparison of the wave and particle theories, prepare the following lists. (*a*) List all of the phenomena associated with light which can be explained in terms of waves. (*b*) List the phenomena which cannot be explained in terms of waves. (*c*) List the phenomena which can be explained in terms of particles. (*d*) List the phenomena which cannot be explained in terms of particles.
25. Wheat flour contains 1% oil by weight, and the oil has a density of 0.9 gm/cm³. Chalk dust is spread uniformly over the surface of water in a tray, and 0.003 gm of flour is dropped into the centre of the chalk dust. The oil spreads out on the surface of the water, pushing the chalk dust and flour ahead of it, and forming a clear circular area of diameter 0.2 metres. Calculate (*a*) the mass, (*b*) the volume, (*c*) the thickness of the oil film.
26. Oleic acid is dissolved in alcohol, the oleic acid concentration being 0.5% by volume. One cm³ of the solution contains 30 drops. One drop of the solution spreads out on water to form a molecular layer of average diameter 32 cm. What is the thickness of the film?

5–16 SUMMARY

Diffraction is a property of waves; the amount of diffraction at an aperture is proportional to $\frac{\lambda}{d}$. Interference is also a property of waves. For two point sources A and B, of the same frequency and in phase, continuous destructive interference of the two wave trains occurs at a point P in the surrounding medium when $|PA - PB| = (n - \frac{1}{2})\lambda$. From this relationship, if PA and PB are both very large relative to AB, the following relationships may be developed:

$$\lambda = \frac{x \cdot d}{L(n - \frac{1}{2})}$$

$$\text{and } \lambda = \frac{d \cdot \Delta x}{L}$$

Diffraction and interference occur for light as they do for waves. If we assume a wave model for light, we may then calculate a wave length for each colour. Two methods, double slit diffraction and thin film interference, may be used. The two methods yield identical results. The range of wave lengths in the visible spectrum is from 4000A for violet light to 7800A for red light.

ANSWERS

Chapter 1—Section 1-15, page 11

2\. 20°   5. (*a*) 10^{13} (*b*) 10^{13} 6. (*a*) 10^{7} (*b*) 10^{-6}

7\. 4.0×10^{8} 8. 10^{-2} 9. 10^{4}

10\. (*a*) 3×10^{11} (*b*) 5×10^{5} (*c*) 1.95×10^{6}
(*d*) 6×10^{-5} (*e*) 4×10^{-6} (*f*) 1.6

12\. 48 in/ft; $P = 48L$

13\. (*a*) (*i*) 2.5 cm   (*ii*) 1 cm   (*iii*) 3 cm
(*b*) (*i*) 0.8 kg   (*ii*) 1.6 kg   (*iii*) 0.2 kg

14\. (*i*) 0.4; 2.8; 4   (*ii*) 1.5; 4.5; 13.5

15\. 9 ft²/yd²; $A = 9L^2$ 16. π; 100π

19\. (*a*) 1.67×10^{-10} sec
(*b*) (*i*) 1.11×10^{-10} sec   (*ii*) 1.21×10^{-10} sec
(*iii*) 1.99×10^{-10} sec   (*iv*) 4.86×10^{-10} sec

20\. (*a*) 20; 2; 0.25   (*b*) 0.5; 5; 100

21\. (*a*) E changes by a factor of (*i*) 2   (*ii*) 4   (*iii*) $\frac{3}{4}$
(*b*) (*i*) 2   (*ii*) $\sqrt{2}$

22\. (*a*) F changes by a factor of (*i*) 2   (*ii*) 8   (*iii*) 16   (*iv*) 256
(*b*) (*i*) 3   (*ii*) 3   (*iii*) $\frac{1}{\sqrt{3}}$

24\. 36 foot-candles   25. 3.2 ft approximately

Chapter 2—Section 2-16, page 25

2\. (*a*) 0.5000   (*b*) 0.3371   (*c*) 0.6769   (*d*) 0.9820
(*e*) 0.9999   (*f*) 0.0035

3\. (*a*) 39.4°   (*b*) 45.0°   (*c*) 4.5°   (*d*) 49.2°
(*e*) 59.0°   (*f*) 69.1°   (*g*) 74.3°   (*h*) 19.2°

5\. 19.5°; 79.8°; 20.4°; 22.3°; 1.41; 0.94

7\. 1.52   9. (*a*) 47°   (*b*) 37°   (*c*) 24°

10\. approximately 39°

Chapter 3—Section 3-9, page 33

6\. 1.82×10^{8} m/sec; 4.95×10^{8} m/sec

7\. 2.26×10^{8} m/sec; 3.99×10^{8} m/sec

8\. 1.14

Chapter 4—Section 4-15, page 51

1. (*a*) 0.005 sec (*b*) $\frac{1}{7}$ sec (*c*) $\frac{1}{30}$ sec
(*d*) 10 sec (*e*) 2×10^{-6} sec (*f*) 5×10^{-9} sec
2. (*a*) 5 c/s (*b*) 2.5 c/s (*c*) $\frac{1}{8}$ c/s
(*d*) 25 kc/s (*e*) 20 mc/s
4. (*a*) 40 revs/sec (*b*) $\frac{1}{40}$ sec
5. 1.5 c/s
6. 30 revs/sec or some integral multiple thereof
7. 32 revs/sec or some integral multiple thereof
8. 8 revs/sec or some integral multiple thereof
12. 52.5
13. 10^6 cps
14. 300 cm/sec
15. 2 sec
16. 20 cm; 2400 cm/sec
17. 4096 cm/sec
18. 20 cm/sec
19. 10 cps
20. 0.72 in
21. approximately 24°
22. 6.6 m/sec
23. 0.000042 cm; 4200A
24. 5300A
25. 4.57×10^{14} cps
26. 20m to 2×10^{-2} m

Chapter 5—Section 5-15, page 71

3. (*a*) 9.0 cm (*b*) 15.0 cm (*c*) 6.0 cm (*d*) 12.0 cm
4. 42 cm/sec
5. $2\frac{1}{4}$ cps
6. 32 cm; $10\frac{2}{3}$ cm; $\frac{5}{8}$ cps; $1\frac{7}{8}$ cps
8. (*a*) 2.0 cm (*b*) 12 cm/sec
9. (*a*) Δx is doubled
(*b*) Δx changes by a factor of 1.5
(*c*) Δx changes by a factor of 3
10. (*a*) 0.40 cm
11. 5.7×10^{-5} cm
12. 2.7 cm
14. (*a*) 1.9 cm (*b*) 17 cm/sec
16. (*a*) 7.5×10^{14} c/s (*b*) 6.0×10^{14} c/s (*c*) 5.0×10^{14} c/s
17. 6000
18. 0.003 cm
19. 3.54×10^{-3} cm
20. 3.5×10^{-4} cm
21. (*a*) 5867A (*b*) 5.1×10^{14} cps
23. 170 c/s; 340 c/s
25. (*a*) 3×10^{-5} gm (*b*) 3.3×10^{-5} cm^3 (*c*) 10^{-7} cm
26. 20A