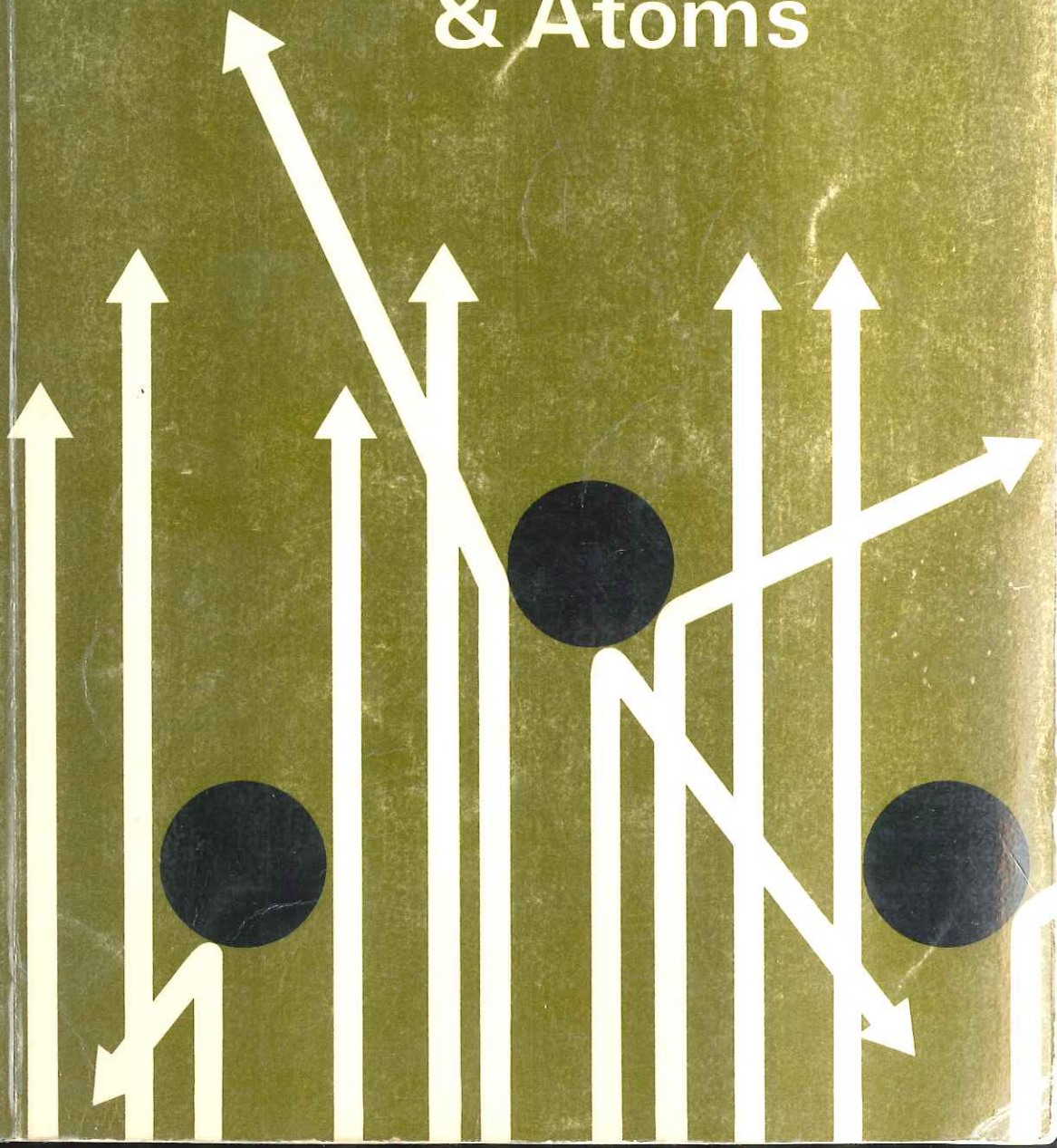
J. S. WRIGHT

Radiation and Matter
BOOK 3

Electrons & Atoms



Chapter 1

Electrostatics

1–1 INTRODUCTION

The importance of electricity to business, to industry, indeed to all facets of twentieth century living, is so obvious as to need no comment here. All of us use, and are served by, a multiplicity of electrical devices every day. In many cases we do not know how or why the devices work. Nor will it be our aim in this book to describe such devices and their operation. Rather, we will concentrate on that portion of the story of electricity which led not only to an understanding of the nature of electricity, but also to an understanding of atomic structure.

Most electrical devices are operated by electricity which flows through them, that is, by current electricity. Yet, until early in the nineteenth century, current electricity was unknown. Before that time, knowledge of electricity was limited to cases where the electricity remained essentially stationary or static, and practical applications of electricity were unknown. We know now, however, that the principles and relationships established in a study of static electricity are vital to an understanding of all electrical devices, and of molecular and atomic interactions.

1–2 OCCURRENCE OF STATIC ELECTRICITY

Examples of static electricity are common. A person walks across a thick-piled rug and touches some metal object. There is a spark and the person feels a tingle at his finger tip. A young lady combs her hair vigorously. There is a crackle and the hair flies up and clings to the comb. A child at a party rubs a balloon on his sleeve and then "sticks" the balloon to the wall. A man removes his sweater and, particularly if the sweater is of some synthetic material, finds that it tends to cling to his shirt. He may find also that he has difficulty in folding the sweater, for

any two parts of it seem to repel one another.

All of the effects described in the preceding paragraph occur because of static electricity.

1–3 STATIC ELECTRICITY IN THE LABORATORY

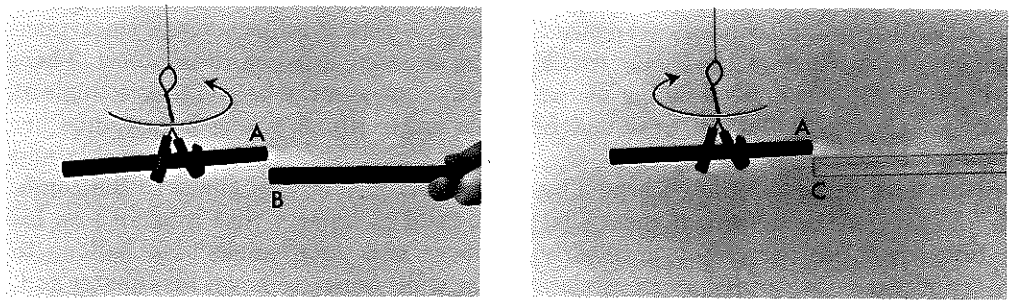
In the laboratory, we usually produce static electricity by rubbing materials together. The method dates back about 2500 years to the Greek physicist-astronomer, Thales, who found that rubbed amber became electrified and would attract small grains of sand. In fact the name, electricity, is derived from the Greek word, elektron, meaning amber.

The materials used most frequently in the laboratory are rods of lucite (or glass) and ebonite. If a lucite rod is rubbed with silk, nylon, or polyethylene, or if an ebonite rod is rubbed with fur, the rod acquires the ability to pick up sawdust, small pieces of paper, and other small objects. Since these particles are attracted to the rod and adhere to it, some force must be acting. This force is called an electric force, and the rod is said to bear an electric charge. Materials which exert no electric force are said to be uncharged or neutral.

1–4 TWO KINDS OF ELECTRIC CHARGE

Though the charge on the lucite rod and the charge on the ebonite rod, described above, are produced in similar ways, they are not necessarily alike. The phenomena may be investigated further as in the following experiment.

An ebonite rod A rubbed with dry fur or flannel, is placed in a glass or rubber-covered support which is suspended by a string so that the rod is free to turn in a horizontal plane (Fig. 1.1*a*). A second ebonite rod B similarly prepared, is then brought close to A. Rod A turns away from B, indicating the existence of a repelling force between the two. If now a lucite (or glass) rod C rubbed with dry silk or nylon, is brought near A (Fig. 1.1*b*), A draws closer to C and will follow it as C is moved, thus indicating the existence of an attracting force between



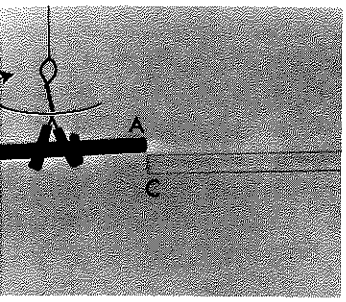
(a) (b)

Fig. 1.1. Demonstrating the two kinds of electric charge.

ce, and the rod is said to bear
charge. Materials which exert
force are said to be uncharged

KINDS OF ELECTRIC RGE

the charge on the lucite rod
arge on the ebonite rod, de-
ove, are produced in similar
are not necessarily alike. The
may be investigated further
llowing experiment.
te rod A rubbed with dry fur
s placed in a glass or rubber-
pport which is suspended by
that the rod is free to turn in
l plane (Fig. 1.1*a*). A second
B similarly prepared, is then
se to A. Rod A turns away
dicating the existence of a re-
e between the two. If now a
lass) rod C rubbed with dry
on, is brought near A (Fig.
ws closer to C and will follow
moved, thus indicating the
an attracting force between



(b)

ectric charge.

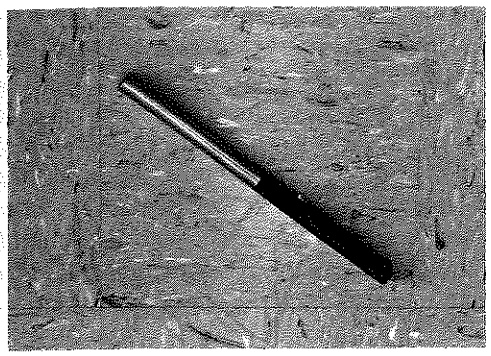


Fig. 1.2. A conducting rod with an insulating handle.

A and C. A second lucite rod D rubbed with silk, will repel the lucite rod C if the latter is suspended. The different effects of the charged rods B and C on the same rod A indicate that the charges on the lucite and ebonite are not of the same kind.

Experiments with a great many objects have revealed that when they are charged, they act either like the charged lucite or the charged ebonite. Thus, there would seem to be two and only two kinds of electric charge. To distinguish between the two known kinds, the great American statesman, publisher, and scientist, Benjamin Franklin (1706-1790), suggested in 1747 that charges similar to those produced on glass (or lucite) rubbed with silk be called positive charges, and that charges similar to those produced on ebonite rubbed with fur be called negative charges.

The experiment just described shows that two positive charges or two negative charges (two like charges) repel one another, whereas one positive and one negative charge (two unlike charges) attract one another. This is the law of charges.

Almost all substances can be given electric charges; the most effective method is to rub the substance with some other material. However, electrification occurs to a lesser extent even if the two substances are merely brought into close contact and then separated. Furthermore, the type of charge which a given substance receives depends upon the other substance with which it is brought into contact. For example, ebonite becomes negatively charged if it is rubbed with fur, but positively charged if it is rubbed with sulphur. Also, the strength of the charge depends on the nature of the two substances.

1–5 CONDUCTORS AND INSULATORS

An ebonite rod, held in the hand as it is rubbed with fur, becomes charged; a rod of steel or copper, similarly held and rubbed, does not become charged. This should not be interpreted as proof that steel or copper cannot be charged by friction methods. A length of copper tubing fitted with an ebonite handle, as shown in Figure 1.2, becomes charged, if the rod is held by its ebonite handle and if the copper tube is rubbed with fur. The copper will then attract small bits of paper and sawdust. However, if the copper tube, after being rubbed with fur, is touched, no matter how briefly, with the finger, or is touched to a water pipe, its ability to attract small bits of paper or to repel another charged object disappears. In other words, its charge is lost and the copper tube is said to have been neutralized. This reaction of the charged copper tube to a brief touch with the finger is

in sharp contrast to that of a charged ebonite rod. In the latter case, almost the whole surface area of the rod must be wiped by hand before all of the charge is removed.

Since, in the above experiment, charge flows readily from all portions of the copper to the point where it is touched, copper is said to be a good conductor of electricity. And since, in the case of the ebonite, charge does not flow to the contact point, ebonite is said to be a good insulator. Substances differ greatly in their ability to conduct electricity. A classification into three groups is presented in the following table.

GOOD CONDUCTORS	FAIR CONDUCTORS	GOOD INSULATORS
silver	wood	mica
copper	tap water	glass
aluminum	the human body	silk
nickel	earth	sulphur
iron	germanium	amber
platinum	carbon	rubber
mercury	solutions of acids, bases, salts	shellac
		lucite
		ebonite

In the first of the above groups are copper and aluminum, the materials most often used as the conductors in electric circuits. Materials which are only fair conductors are used to limit and control the rate of flow of charge in such circuits; they are called resistors.

1–6 THE ELEMENTARY CHARGE

Many different theories have been proposed to explain electric charges. One common assumption has been that all matter contains small particles of at least two types—one positive and the other negative. As you no doubt know, this has been found to be the case. The sub-atomic particle bearing the basic or elementary negative charge is the electron, and we shall, therefore, call the magnitude of this charge the elementary charge. In the atom, the nucleus around which the electrons travel is positively charged; the elementary positive charge is borne by protons in the nucleus. The number of elementary positive charges in the nucleus is normally equal to the number of elementary negative charges borne by the electrons. That is, the number of protons is equal to the number of electrons, since each bears an elementary charge.

We shall have to wait until later chapters to see how a knowledge of electrons and protons (and other sub-atomic particles) was developed, but we shall not hesitate to use some of this knowledge in the meantime. Since it is normally the electrons rather than the protons which move, we explain the basic electrostatic phenomena in terms of electrons.

1–7 POSITIVE AND NEGATIVE CHARGES EXPLAINED

When an ebonite rod is rubbed with fur, electrons from atoms in the fur transfer to atoms at the surface of the ebonite. Thus the ebonite rod obtains a surplus of electrons and becomes negatively charged. Negatively charged objects possess more than the normal number of electrons.

When a lucite rod is rubbed with silk, electrons transfer from the lucite to the silk. Thus the lucite rod (or any positively charged object) has some atoms with fewer electrons than protons, i.e., there is a deficit of electrons.

As a corollary, the fur should become positively charged and the silk negatively charged. The fur may be observed to repel a positively charged lucite rod; thus

bearing the basic or elementary
charge is the electron, and we
erefore, call the magnitude of this
he elementary charge. In the
e nucleus around which the elec-
avel is positively charged; the
ry positive charge is borne by
in the nucleus. The number of
ry positive charges in the nucleus
lly equal to the number of ele-
negative charges borne by the
. That is, the number of protons
to the number of electrons, since
rs an elementary charge.
all have to wait until later chap-
ee how a knowledge of electrons
ons (and other sub-atomic par-
as developed, but we shall not
to use some of this knowledge in
ntime. Since it is normally the
rather than the protons which
e explain the basic electrostatic
na in terms of electrons.

SITIVE AND NEGATIVE HARGES EXPLAINED

an ebonite rod is rubbed with
rons from atoms in the fur trans-
ms at the surface of the ebonite.
ebonite rod obtains a surplus of
and becomes negatively charged.
ly charged objects possess more
normal number of electrons.
a lucite rod is rubbed with silk,
transfer from the lucite to the
s the lucite rod (or any positively
object) has some atoms with
ctrons than protons, i.e., there
it of electrons.
orollary, the fur should become
y charged and the silk negatively
The fur may be observed to
sitively charged lucite rod; thus

the predicted presence of a positive charge on the fur is verified. Similarly, the silk will repel a negatively charged ebonite rod and so is shown to be negatively charged. Furthermore, the strength of the negative charge on the ebonite should be equal to that of the positive charge on the fur, since the numerical value of the electron surplus for the ebonite will equal the numerical value of the electron deficit for the fur.

1–8 CONDUCTORS AND INSULATORS EXPLAINED

A conductor was previously defined as a substance through which charge can flow easily. A flow of charge in a solid conductor is due to the motion of electrons through it. The electrons move from atom to atom. In order for this flow of electrons to occur, the atoms of the conductor must be such that some of their electrons are loosely bound to the nuclei. For example, copper is a good conductor because in each atom of the copper one of the electrons is easily detached and drifts along from atom to atom. These drifting electrons are called free electrons. Conversely, insulators are materials in which all of the electrons are tightly bound to their nuclei; there are no free electrons.

1–9 ATTRACTION OF NEUTRAL OBJECTS EXPLAINED

The electrons in a neutral conductor, since they are negatively charged, will be affected by nearby charges, for example, by a charged ebonite rod (Fig. 1.3). Some electrons are repelled from the surface of the conductor near the negatively charged rod to the surface farthest from the rod. Thus, on the average, the attracting force

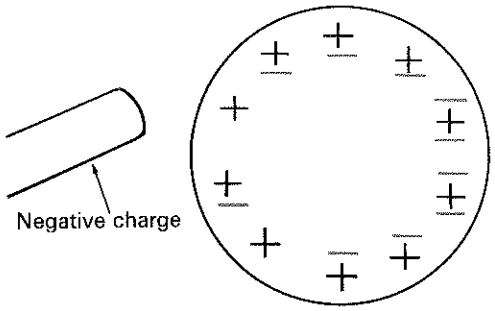


Fig. 1.3. The charge on the rod alters the distribution of electrons in the neutral object.

between the rod and the protons in the conductor is greater than the repelling force between the rod and the electrons in the conductor. Therefore, there is a resultant force of attraction between the charged rod and the neutral conductor.

The charged ebonite rod, or any charged object, also attracts neutral insulators such as pieces of paper, sawdust, etc. The explanation for this attraction is similar to that given in the above paragraph for a neutral conductor. However, in the case of the insulator, it is likely that the displacement of the electrons occurs only within the atoms themselves.

1–10 THE METAL LEAF ELECTROSCOPE

Small bits of paper or pith, or a suspended rod, may be used as detectors of electrostatic charges, but the method is not very convenient and the agents are not very sensitive. A much better detector is the metal leaf electroscope, an instrument which can be used not only to detect the presence of charge but also to determine its kind and to compare the intensities of charges.

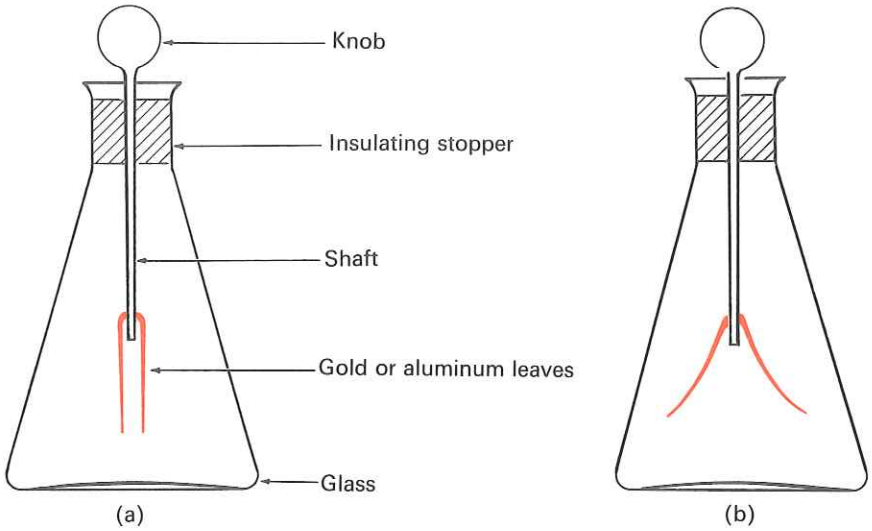


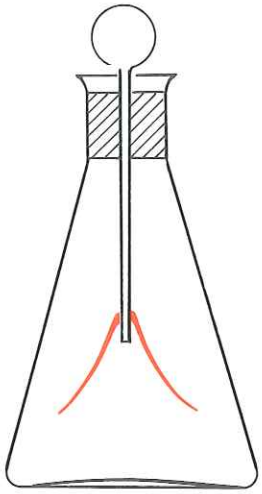
Fig. 1.4. The metal leaf electroscope.

The electroscope (Fig. 1.4*a*) consists essentially of a metal rod or shaft which is a good conductor. Near one end of the shaft are attached two narrow and very thin strips of metal leaf, gold or aluminum; at the other end is a metal knob. Since any slight air current would disturb the leaves, the unit is enclosed in a glass container, or in a metal container with glass windows front and back. In the latter case, the shaft must be insulated from the case by passing it through a cork or rubber stopper.

1–11 DETECTION, IDENTIFICATION, AND COMPARISON OF CHARGES

With the electroscope knob, shaft, and leaves electrically neutral, the leaves will hang vertically under the action of the force of gravity. If the knob is touched with a charged ebonite rod, some of the charge (electrons) will transfer from the rod to the knob. This charge will distribute itself throughout the electroscope since the shaft and leaves are conductors. Thus, the two leaves will bear the same kind of charge and a repelling force will exist between them. If this repelling force is great enough, it will overcome the force of gravity acting on the leaves and they will rise or deflect as shown in Figure 1.4(*b*). In a similar manner, a charged lucite rod may be used to place a positive charge on the electroscope and cause a deflection of the leaves.

To prevent the transfer of too great a quantity of charge, the charged rod should not be touched to the knob of a sensitive electroscope. Instead, a proof plane, which consists of a small metal disc with an ebonite handle, may be used. The proof plane is first touched to the conductor under test and is then touched to the



(b)

roscope.

(electrons) will transfer from the
ne knob. This charge will distribute
nroughout the electroscope since
't and leaves are conductors. Thus,
) leaves will bear the same kind
ge and a repelling force will exist
n them. If this repelling force is
nough, it will overcome the force
ity acting on the leaves and they
e or deflect as shown in Figure
In a similar manner, a charged
od may be used to place a positive
on the electroscope and cause a
on of the leaves.

revent the transfer of too great a
y of charge, the charged rod should
ouched to the knob of a sensitive
cope. Instead, a proof plane, which
of a small metal disc with an
handle, may be used. The proof
s first touched to the conductor
est and is then touched to the

knob of the electroscope. If the leaves diverge, the conductor bears a charge.

The electroscope can also be used to determine the kind of charge on an insulated conductor. Suppose the electroscope is charged negatively and a sample charge from the conductor under test is added. If the leaves show further divergence, the conductor bears a negative charge; if the leaves at first collapse, partially or completely, the conductor bears a positive charge.

Finally, the instrument can be used to compare the intensities of charges on different insulated conductors or on different parts of the surface of the same conductor. The more intense charge will cause the greater divergence of the leaves.

In the methods described above, the electroscope was charged by contact, i.e., the charged conductor, or the proof plane which had been in contact with the conductor, was placed in contact with the knob of the electroscope. If the conductor is negatively charged, the proof plane receives some of the surplus electrons and passes them on to the electroscope. Hence the electroscope, including the leaves, bears the same kind of charge as the conductor under test.

1–12 TEMPORARY INDUCED CHARGES

The leaves of an electroscope can be made to diverge by bringing a charged ebonite rod near the knob. If the rod is then removed from the vicinity of the knob, the leaves collapse. The divergence of the leaves indicates that, for a brief time at least, they bore a charge. Electrons did not transfer from the rod to the knob since these did not touch and since air is a poor conductor. This kind of charge, produced when there is no direct contact, is called an induced charge.

Electrostatic induction can be explained readily in terms of electron motion. Electrons are miniature negatively charged particles. The ebonite rod is negatively charged. Since like charges repel, free electrons in the knob of the electroscope are forced down to the leaves. Hence the leaves temporarily become negatively charged. The knob, having lost some of its electrons, should be positively charged. This reasoning can be verified as follows.

Two similar metal spheres, A and B, each on an insulating stand, are placed in contact (Fig. 1.5). A charged ebonite rod is then brought near the sphere A. If the theory holds, the negatively charged rod should repel electrons from sphere A to sphere B so that A will be charged positively and B negatively.

With the ebonite rod still in position, sphere B can be moved away from A. If the spheres, A and B, are now tested on

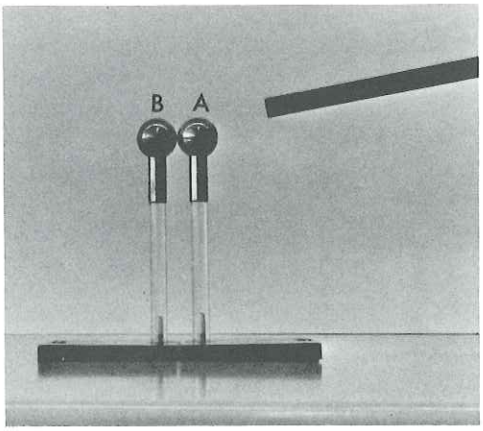


Fig. 1.5. The charged ebonite rod induces equal and opposite charges on the conductors *A* and *B*.

the electroscope, A is found to be charged positively and B negatively. If A is now allowed to touch B, testing shows that both are neutral. Thus the two temporary induced charges are equal in size but opposite in sign. This conclusion confirms the assumption made in Section 1–9 to explain the attraction of an uncharged object by a charged object.

1–13 PERMANENT INDUCED CHARGES

Except for the case when the two spheres A and B (Fig. 1.5) are separated while the charged rod is held near A, the induced charges described above have been temporary. In other words, when the charged rod is removed, the electrons in the conductors return to their normal positions. However, even in a single conductor like AB in Figure 1.6, the charge induced by the ebonite rod can be made permanent. If a grounded wire is connected to the end B of the conductor and then disconnected, and if the ebonite rod is then removed, a test on the electroscope will show that the conductor is charged positively over the whole of its surface. Apparently, while the brief contact was made with the earth, some or all of the electrons which the ebonite rod repelled to end B are further repelled to earth. When the contact with the earth is broken, the return of these electrons is cut off and the insulated conductor now has a permanent positive charge.

If a charged lucite rod is used in place of the ebonite, the conductor can be given a permanent negative charge. Thus, the permanent charge produced on an insulated conductor by induction and grounding, is opposite in kind to that of the charging agent.

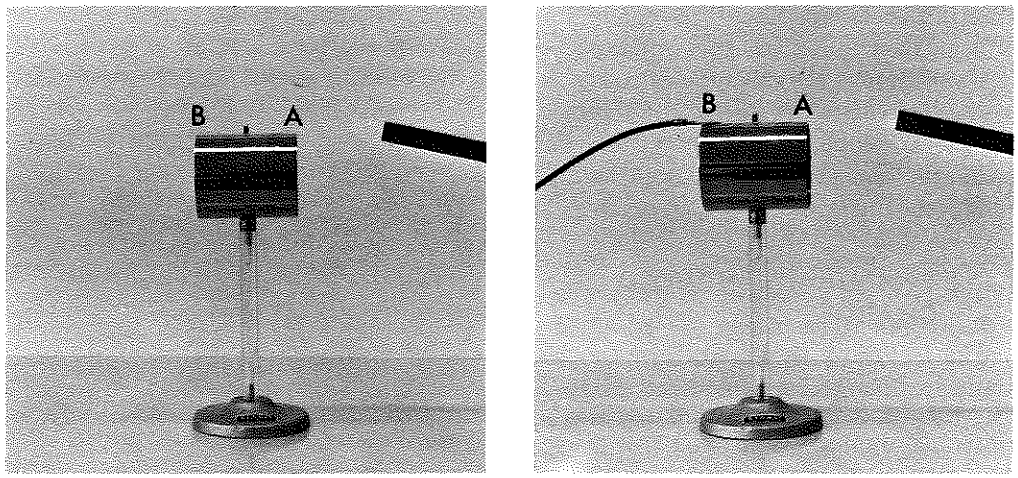
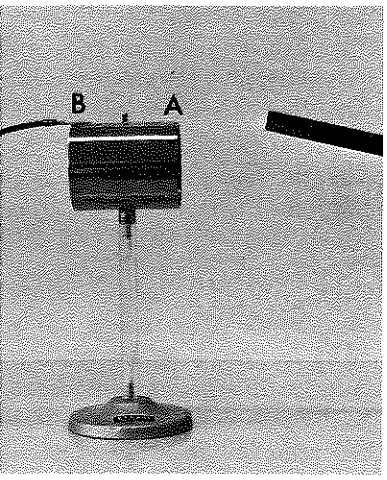


Fig. 1.6. Charging a conductor by induction and grounding. A charged rod is brought near the conductor as in *(a)*. The conductor is then grounded while the rod is in place, as in *(b)*. The ground connection must be removed before the rod is removed.

by the ebonite rod can be made
ent. If a grounded wire is con-
o the end B of the conductor and
connected, and if the ebonite rod
removed, a test on the electro-
ill show that the conductor is
positively over the whole of its
Apparently, while the brief con-
s made with the earth, some or
e electrons which the ebonite rod
to end B are further repelled to
hen the contact with the earth is
the return of these electrons is
nd the insulated conductor now
rmanent positive charge.
harged lucite rod is used in place
onite, the conductor can be given
nent negative charge. Thus, the
nt charge produced on an insu-
nductor by induction and ground-
pposite in kind to that of the
agent.



(b)

. A charged rod is brought near the
the rod is in place, as in *(b)*. The
ed.

1–14 PROBLEMS

1. Suppose that a rod of material X, when rubbed with a material Y, attracted an ebonite rod which had been rubbed with fur, attracted a lucite rod which had been rubbed with polyethylene, and repelled another rod of material X, rubbed with material Y. What conclusion would have to be drawn?

2. A suspended pith ball is attracted by a lucite rod which has been rubbed with polyethylene. What can be said about the state of charge of the ball?

3. Five rods have the following effects on one another. D attracts E and repels A. B attracts E and has no effect on C. A negatively charged ebonite rod attracts both D and C. What are the states of charge of the rods?

4. Describe several methods by means of which an insulated conductor can be charged.

5. Two identical conducting spheres, A and B, are placed some distance apart and are connected by a wire. They are then charged. A grounded conducting sphere C is brought near to A. The connecting wire is removed, and then C is removed. Will the charges on A and B now be equal? Explain.

6. A copper rod is suspended by an insulating fibre so that it is free to rotate in a horizontal plane. An ebonite rod is rubbed with fur and then brought close to the side of one end of the copper rod. (*a*) Describe what is observed. (*b*) Explain the observation in terms of electron motion. (*c*) How would the observation be changed if the ebonite rod were replaced by a glass rod rubbed with silk?

7. Explain why an initial repulsion is followed by an attraction when a small conductor bearing a large quantity of charge is brought slowly up to a large conductor bearing a small quantity of charge of the same sign.

8. Describe how an electroscope might be used to test the conducting properties of (*a*) a dry cotton thread, and (*b*) a wet cotton thread.

9. Why do electrostatic experiments usually work poorly on humid days?

10. You are given three insulated conductors A, B, and C. A is negatively charged, B and C are neutral. How can you charge B positively and C negatively, without altering the charge on A?

11. When an insulated conductor is charged permanently by induction and grounding, does it matter where the ground wire is connected to the conductor?

12. Describe how you would give an insulated conductor a permanent negative charge by induction and grounding. Explain the charging action in terms of electron motion.

13. Two insulated conducting spheres, A and B, are placed near, but not in contact with, one another. B is connected by a wire to an electroscope. State and explain the behaviour of the leaves of the electroscope when (a) A is given a positive charge, (b) the wire is then removed and A is removed, (c) the electroscope is touched momentarily, (d) the wire connection from B to the electroscope is restored.

14. A lucite rod with a strong positive charge is brought near the knob of a negatively charged electroscope. As the rod approaches the knob, the leaves collapse and then diverge again. Account for these observations in terms of electron motion. What is the final charge on the leaves?

1–15 SUMMARY

An object may become electrically charged when rubbed with a dissimilar material. Charged objects attract neutral objects.

There are two kinds of charge: positive and negative. Like charges repel each other, unlike charges attract each other.

Conductors transfer charge readily; insulators do not transfer charge. A resistor is a conductor, but not a good one. Most metals are good conductors.

Electrostatic phenomena are explained in terms of the motions of electrons in conductors. The electron carries one negative elementary charge.

The metal leaf electroscope may be used to detect, identify, and compare charges.

If a charged object is held near an insulated conductor, temporary equal and opposite charges are induced on the conductor. The charge on that portion of the conductor nearest the charged object is opposite in sign to the charge on the object.

An insulated conductor may be given a permanent charge by induction. The conductor is connected to ground, then disconnected, while the charged object is held nearby. When the charged object is removed, the conductor has a permanent charge, opposite in sign to the charge on the object.

are placed near, but not in
a wire to an electroscope.
es of the electroscope when
e is then removed and A is
mentarily, (d) the wire con-

brought near the knob of a
proaches the knob, the leaves
ese observations in terms of
ne leaves?

etal leaf electroscope may be
detect, identify, and compare

harged object is held near an
conductor, temporary equal and
charges are induced on the con-
he charge on that portion of the
r nearest the charged object is
n sign to the charge on the object.
ulated conductor may be given
ent charge by induction. The
r is connected to ground, then
ted, while the charged object is
by. When the charged object is
the conductor has a permanent
pposite in sign to the charge on
t.

Chapter 2

Electric Charge, Force, and Potential

2–1 THE CONSERVATION OF CHARGE

Quantitative measurements in electricity begin with measurement of quantities of charge. Basic to these measurements is the assumption—implied or expressed—that charge is conserved. If two insulated conductors, one charged and one neutral, for example, are connected by a conductor, the charged conductor will share its charge with the neutral conductor. However, in the transfer, charge is not created or destroyed; the total quantity of charge is the same after the transfer as before.

Such transfers of charge are the result of the motions of electrons carrying their elementary charges, and we automatically assume that electrons do not appear or disappear in the process. Many theories other than the electron theory have been advanced, but all have implied charge conservation and thus made clear what was meant by quantity of charge. The first question, then, is how to measure quantities of charge, and in what units. Since charged objects exert forces on one another, forces which depend on the quantities of charge on the objects, it is natural to attempt to measure charge in terms of these forces.

2–2 THE WORK OF COULOMB

The repelling force between two charged ebonite rods obviously depends on how strongly each is charged, and upon the distance between the rods. The greater the quantity of charge on either of the rods, or the closer the rods are to one another, the greater is the repelling force. The French physicist, Charles Coulomb (1736-1806) investigated the relationship between the force F, the charges Q and q, and the distance r. The basic portion of

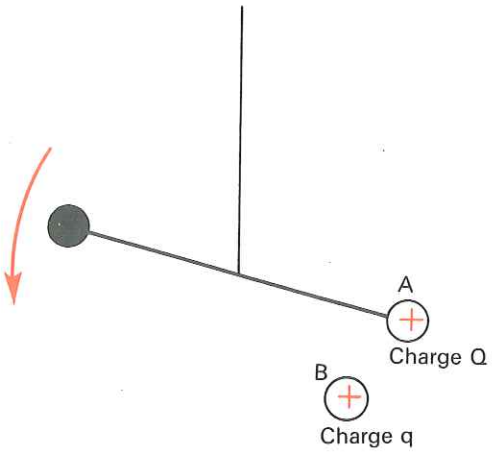


Fig. 2.1. The basic parts of Coulomb's torsion balance.

the apparatus he used is shown in Figure 2.1. He suspended a rod, on each end of which was a small sphere, by a long fibre. Such an arrangement is called a torsion balance, and Coulomb calibrated this balance so that he knew the force necessary to twist the fibre through any angle. The sphere A was then charged with a quantity of charge which we shall call Q. Another sphere B was given a like charge which we shall call q. Keeping q and Q constant, Coulomb determined the force of repulsion for various values of the distance r between the centres of A and B, and found that $F \propto \frac{1}{r^2}$.

Keeping r constant, Coulomb then allowed B to share its charge with an identical sphere, which was then removed. Thereby q was halved, and Coulomb found that F was halved also. Other variations of this procedure led to the conclusion that $F \propto q$, and also, by having A share its charge, that $F \propto Q$. Therefore the force of repulsion (or attraction) between two charged objects is directly proportional to each of their charges, and inversely proportional to the square of the distance between them. (If the spheres bearing the charges are not of negligible size compared to the distance between them, this distance must be measured from the centre of one to the centre of the other.) In formula form

$$F = \frac{kqQ}{r^2}$$

where k is a variation constant whose value depends on the units used for F, q, Q, and r, and also on the medium between the two charges.

This formula for electric force is called Coulomb's law; its similarity to Newton's law of gravitation is obvious. In fact Coulomb, aware of the law of gravitation, probably expected the law for electric forces to have this form. Since Coulomb's time, the validity of the relationship $F \propto \frac{1}{r^2}$ has been confirmed by experiments whose possible error is less than 1 part in 10^9.

2–3 UNITS OF CHARGE

As a result of Coulomb's law, various units of charge may be defined, depending on the units of force and distance used. The statcoulomb (electrostatic unit or esu) of charge is defined as that charge which at a distance of 1 cm from an identical charge in a vacuum repels it with a force of one dyne. For various reasons the coulomb, rather than the statcoulomb, is the most frequently used unit of charge. The coulomb is defined in terms of the magnetic effect which is produced by charge in motion; the exact definition cannot be given here.

arged objects is directly proportional to each of their charges, and inversely proportional to the square of the e between them. (If the spheres the charges are not of negligible mpared to the distance between his distance must be measured e centre of one to the centre of the In formula form

$$F = \frac{kqQ}{r^2}$$

r is a variation constant whose epends on the units used for F, nd r, and also on the medium the two charges.

formula for electric force is called b's law; its similarity to Newton's gravitation is obvious. In fact b, aware of the law of gravitation, y expected the law for electric o have this form. Since Coulomb's he validity of the relationship has been confirmed by experi-

vhose possible error is less than n 10^9.

NITS OF CHARGE

result of Coulomb's law, various charge may be defined, depending the units of force and distance ne statcoulomb (electrostatic unit of charge is defined as that charge at a distance of 1 cm from an l charge in a vacuum repels it force of one dyne. For various the coulomb, rather than the omb, is the most frequently used charge. The coulomb is defined in the magnetic effect which is pro- by charge in motion; the exact on cannot be given here.

1 coulomb = 3.0×10^9 statcoulombs, approximately.

The definition of the statcoulomb is such that the magnitude of the variation constant k in Coulomb's law is 1. The units for k may be found by solving the Coulomb law formula for k.

$$k = \frac{Fr^2}{qQ}$$

$$\therefore k = 1\frac{\text{dyne-(cm)}^2}{\text{(statcoulomb)}^2}$$

But 1 dyne = 10^{-5} newtons

1 cm = 10^{-2} m

and 1 statcoulomb = $\frac{1}{3.0 \times 10^9}$ coulombs, approximately.

$$\therefore k = \frac{(10^{-5}\text{ newt})\ (10^{-2}\text{ m})^2 \times 9.0 \times 10^{18}}{\text{(coulomb)}^2}$$

$$= 9.0 \times 10^9 \frac{\text{newton-(metre)}^2}{\text{(coulomb)}^2}.$$

That is, two charges, each of magnitude 1 coulomb, and 1 metre apart in a vacuum, repel each other with a force of 9.0×10^9 newtons.

2–4 ELECTRIC FIELDS OF FORCE

Any charged object exerts a force on other charges near it. The region in which this force is felt is called the electric field of force of the object. Since the force is inversely proportional to the square of the distance from the object, we may plot the force field by drawing force vectors as in Figure 2.2. Experimentally, the nature of electric force fields may be investigated by a method similar to the procedure in which iron filings are used to investigate magnetic fields. The charged objects are placed at the surface of a non-conducting liquid in an insulating container. Grass seeds sprinkled on the surface of the liquid reveal the nature of the field. Figure 2.3 shows photographs of five electric fields. Let us consider each in turn.

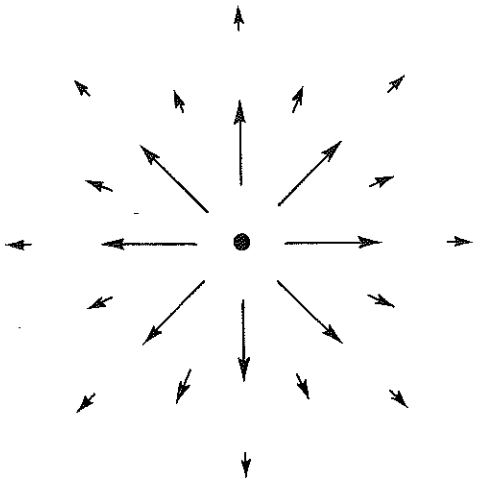


Fig. 2.2. An electric force field. The force is inversely proportional to the square of the distance.

Figure 2.3(a) shows the electric field around a vertical charged rod. The grass seeds align themselves in lines called electric lines of force. The direction of a line of force at any point in the field is the direction of the force vector at that point. This photograph verifies at least qualitatively the predictions made in Figure 2.2. A rough estimate of the magnitude of the electric force at any point in the field may be made by observing the concentration of the grass seeds in the vicinity of that point.

Figures 2.3(b) and (c) show, respectively, the fields due to the combined action of equal unlike and equal like charges. The force at any point in either of these fields is the resultant of two forces, one exerted by each of these

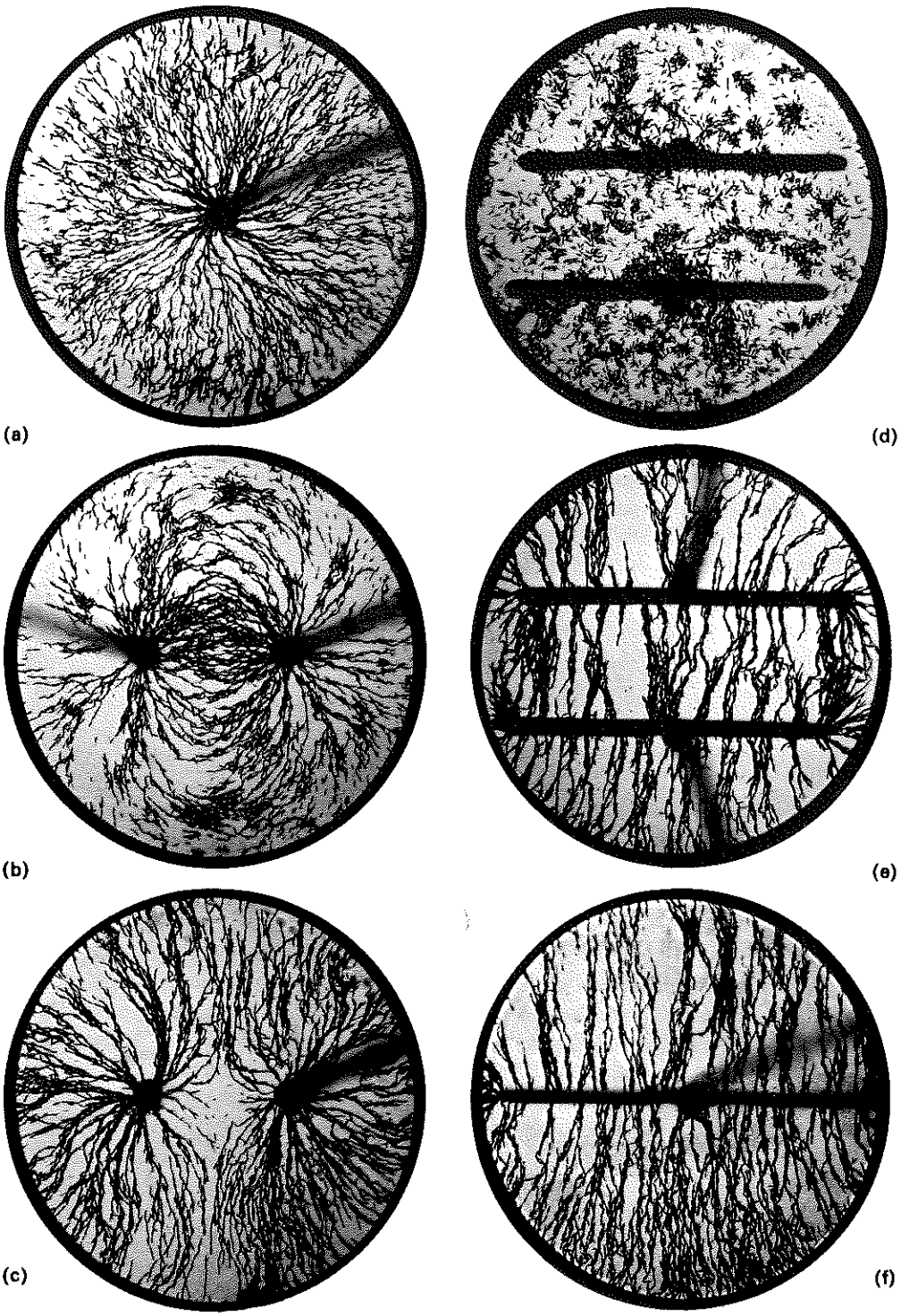
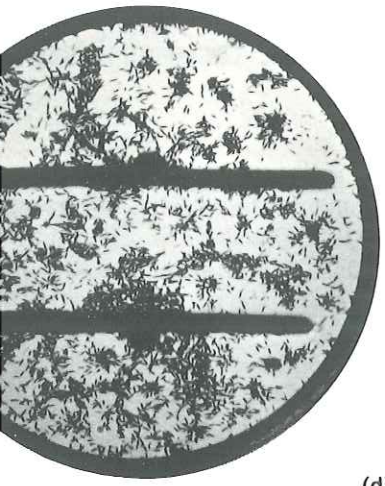
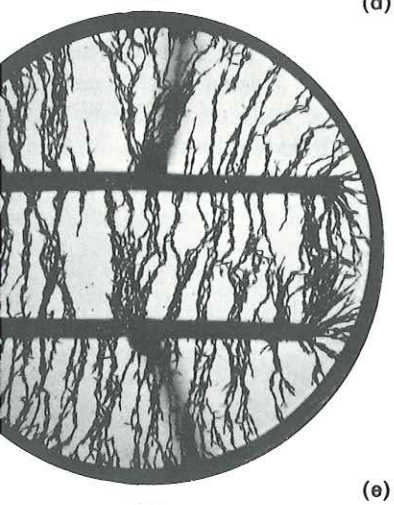


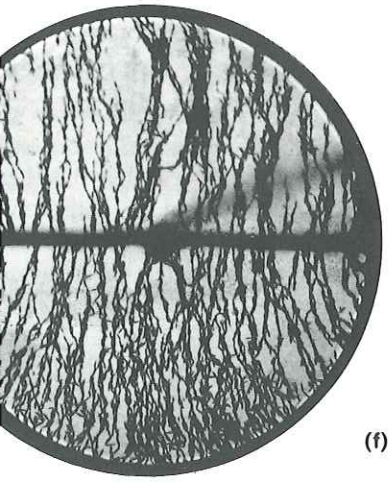
Fig. 2.3. Electric fields *(a)* about a point charge, *(b)* between two unlike point charges, *(c)* between two like point charges, *(d)* between uncharged parallel plates, *(e)* between parallel plates with unlike charges, *(f)* about a single charged plate.



(d)



(e)



(f)

tween two unlike point charges, *(c)* parallel plates, *(e)* between parallel

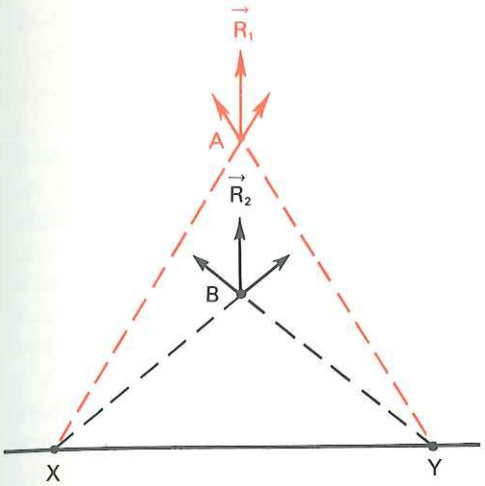


Fig. 2.4. $\vec{R}_1$ and $\vec{R}_2$ are very nearly equal.

charges. The nature of each of these fields might have been predicted from the law of charges.

Next we consider a very important field, that between two parallel charged plates. Figure 2.3(*d*) shows the positions of the grass seeds before the plates are charged; Figure 2.3(*e*) shows the positions of the grass seeds after the plates are charged. An examination of this latter photograph leads us to the conclusion that the resultant of the forces exerted by the two plates on a charge placed between them is uniform in the space between the plates, except near their edges. This might be expected, for, if a charge is moved from a position near the top plate toward the lower plate, the force exerted on it by the upper plate decreases, but the force exerted by the lower plate increases. Hence the magnitude of the resultant force remains constant. It is evident from the photograph that the direction of the resultant is perpendicular to the plates.

Figure 2.3(*f*) shows the field of force around a single charged plate. Again the lines of force are perpendicular to the plate except near the edges of the plate, and the magnitude of the electric force seems to be independent of distance from the plate. This conclusion does not seem correct at first glance, but consider Figure 2.4. Here we have simplified the situation by assuming that the charge on the plate may be assumed to be concentrated equally at two points X and Y on it. If a charge is moved from A to B toward the plate, the forces exerted by X and Y both increase, but the angle between them increases as well. Hence their resultant may very well remain constant, as Figure 2.3(*f*) seems to indicate.

2–5 A TEST OF COULOMB'S LAW

Figure 2.5 shows the electric field of charge placed on a hollow cylinder. Although the grass seeds outside the cylinder are affected by the field, those inside the

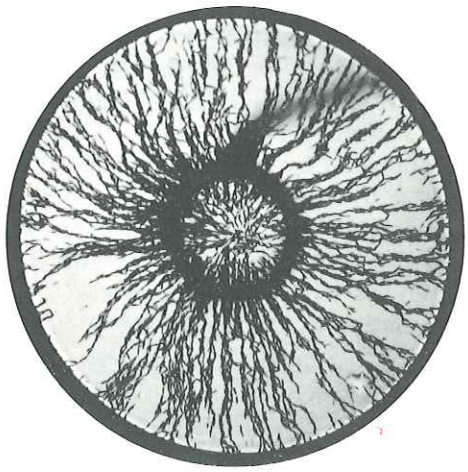


Fig. 2.5. The charge on the cylinder has no effect on the grass seeds inside the cylinder.

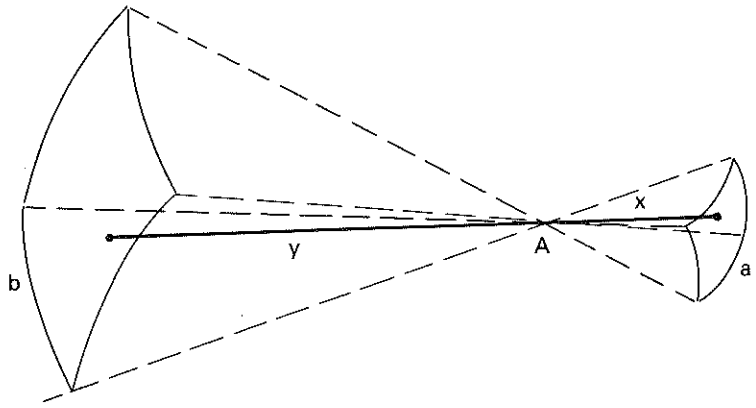


Fig. 2.6. Diagram for the analysis of the electric field inside a charged conductor.

cylinder do not appear to be affected at all. Apparently the resultant electric force acting at any point inside the cylinder is zero. This fact can be predicted with the help of Coulomb's law, and the correctness of the prediction indicates the validity of an inverse square law for electric force.

Consider a charge at a point A inside the cylinder (Fig. 2.6). Consider first the force exerted on it by the charge on a square of length and width a on the surface of the cylinder. This charge exerts on A a force F_1 to the left. However, F_1 is opposed by a force F_2 to the right, exerted by the charge on a square of length and width b, positioned on the opposite side of A as shown in Figure 2.6. Are F_1 and F_2 equal in magnitude?

According to Coulomb's law,

$$F_1 = k\frac{Qq_1}{x^2} \text{ and } F_2 = k\frac{Qq_2}{y^2}$$

where Q is the charge at A, q_1 and q_2 are the charges on the squares of sides a and b, respectively, and x and y are the distances from A to these squares, as shown in Figure 2.6.

$$\therefore \frac{F_1}{F_2} = \frac{q_1 y^2}{q_2 x^2}$$

If we assume that the charge is distributed uniformly over the surface of the cylinder, then

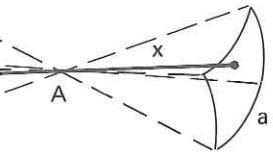
$$\frac{q_1}{q_2} = \frac{a^2}{b^2}$$

From similar triangles, $\frac{a}{b} = \frac{x}{y}$

and therefore $\frac{a^2}{b^2} = \frac{x^2}{y^2}$

Hence $\frac{q_1}{q_2} = \frac{x^2}{y^2}$ and $F_1 = F_2$

The above proof is true for any pair of opposite portions of the surface; in all cases F_1 and F_2 are equal in magnitude and opposite in direction. Therefore there is no electric field inside a hollow charged conductor. Because of this fact, the charge on a conductor may be shielded from the effects of other charges by surrounding the conductor with a metallic shield. Moreover, as we have noted above, the fact that the field inside a conductor is zero is convincing evidence of the validity of Coulomb's law.



ld inside a charged conductor.

$$\therefore \frac{F_1}{F_2} = \frac{q_1 y^2}{q_2 x^2}$$

e assume that the charge is dis-
d uniformly over the surface of
inder, then

$$\frac{q_1}{q_2} = \frac{a^2}{b^2}$$

similar triangles, $\frac{a}{b} = \frac{x}{y}$

and therefore $\frac{a^2}{b^2} = \frac{x^2}{y^2}$

Hence $\frac{q_1}{q_2} = \frac{x^2}{y^2}$ and $F_1 = F_2$

above proof is true for any pair of
ie portions of the surface; in all
F_1 and F_2 are equal in magnitude
posite in direction. Therefore there
ectric field inside a hollow charged
tor. Because of this fact, the charge
nductor may be shielded from the
of other charges by surrounding
nductor with a metallic shield.
ver, as we have noted above, the
at the field inside a conductor is
convincing evidence of the validity
lomb's law.

2–6 ELECTRIC FIELD INTENSITY

The Coulomb law formula

$$F = k\frac{qQ}{r^2}$$

enables us to compute the force F exerted on a charge q at a distance r from an object bearing a charge Q. The magnitude of F depends on the magnitude of the charge q. In drawing fields of force it is convenient to have a vector whose magnitude is independent of q, and for this purpose field intensity vectors (also called field strength vectors) are used. The electric field intensity at a point is defined as the force per unit positive charge exerted on a small charge at that point, that is,

$$\text{Field intensity} = \frac{F}{q} = \frac{kqQ}{qr^2} = \frac{kQ}{r^2}$$

Field intensity is a vector quantity; its direction is the same as that of the force vector. Field intensity is measured in units of newtons per coulomb.

2–7 POTENTIAL DIFFERENCE

Suppose that an insulated conductor bears a positive charge Q_1 (Fig. 2.7), and that a small quantity Q_2, say 4×10^{-6} coulombs, of positive charge is at A. To simplify the problem assume that Q_1 and Q_2 are point charges and that they are isolated from the effects of all other charged objects. Since the two charges are alike, there will be a force of repulsion between Q_1 and Q_2. As a result, work must be done to move Q_2 closer to Q_1, say to a position B, 80 cm from A. The work done can be shown to be independent of the path taken from A to B, just as the work necessary to raise an object from one elevation to another is independent of the path taken. The force of repulsion between Q_1 and Q_2 will of course increase as Q_2 moves closer to Q_1. Suppose that the average value of the force is 10^{-3} newtons. Then the work done in moving the 4×10^{-6} coulombs of positive charge from A to B is 10^{-3} newtons $\times$ 0.8 metres or 8×10^{-4} joules. In other words, work is done at the rate of $\frac{8 \times 10^{-4}}{4 \times 10^{-6}} = 200$ joules per coulomb.

As a result of the work done in transferring Q_2 from A to B (Fig. 2.7), Q_2 possesses 8×10^{-4} joules more electric potential energy at B than at A. The difference in electric potential energy, or in short, the potential difference between points A and B is always calculated per unit charge. Thus the potential difference between A and B in this example is 200 joules per coulomb. One joule per coulomb is called one volt; therefore, the potential difference (P.D.) between A and B is 200 volts.

The following definitions are implied in the above discussion:

(a) The P.D. between two points X and Y in an electric field is the work done

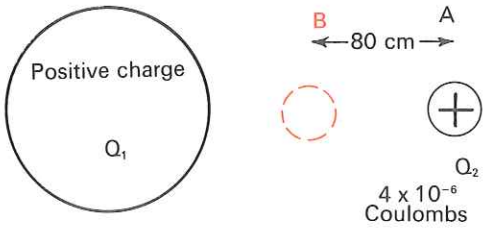


Fig. 2.7. Work must be done to move Q_2 from A to B.

per unit charge in moving positive charge from Y to X, or it is the electric potential energy gained per unit charge when positive charge is transferred from Y to X.

(b) One volt = 1 joule per coulomb. The P.D. between two points X and Y is 1 volt if work is done or energy is transformed at the rate of 1 joule per coulomb when positive charge is transferred from Y to X.

(c) If W joules of work is required to move a positive charge of Q coulombs from Y to X, then the work required per unit charge is $\frac{W}{Q}$, and thus

$$V = \frac{W}{Q} = \frac{E}{Q} \text{ and hence } E = W = QV$$

where V is the P.D. in volts between X and Y, and E is the number of joules of energy transformed.

2–8 WORKED EXAMPLE

The P.D. between two points in an electric field is 300 volts. How much work is required to move 5×10^{-9} coulombs of charge between these points? What average force is required if the points are 0.5 metres apart?

SOLUTION

$$\begin{aligned} W &= QV \\ &= 5 \times 10^{-9} \times 300 \text{ joules} \\ &= 1.5 \times 10^{-6} \text{ joules} \\ F &= \frac{W}{s} = \frac{1.5 \times 10^{-6}}{0.5} \text{ newtons} \\ &= 3 \times 10^{-6} \text{ newtons} \end{aligned}$$

2–9 ZERO OF POTENTIAL

The electric potential energy possessed by Q_2 (Fig. 2.7) was calculated to be 8×10^{-4} joules greater at B than at A. This statement in no way implies that the electric potential energy of Q_2 was zero at point A. Indeed, work must have been done at some time to bring Q_2 to point A. At what point would Q_2 have zero potential energy? A reasonable assumption, and one frequently made, is that its potential energy would be zero at an infinite distance from Q_1, for at this distance the repelling force exerted by Q_1 on it would be zero.

A somewhat similar argument provides a more practical choice for zero of electric potential energy. No electric forces are exerted by an isolated uncharged object; hence, an uncharged object may be considered to have zero electric potential energy. The earth may be regarded as a huge spherical conductor, so large that the addition to it of any likely quantity of charge leaves it still essentially uncharged, and at the same potential. Thus the earth or any conductor connected to earth may be reasonably assumed to be at zero potential.

Whether either of these choices of zero electric potential energy is correct is in practice immaterial. What matters to a man climbing a mountain is that the difference in elevation between the bottom and the top is 10,000 ft, even though a map may indicate that the respective elevations are 1000 ft and 11,000 ft above sea level. In electricity the similar vital information is that the difference in potential between two points is 10,000 volts, even though the respective potentials are 1000 volts and 11,000 volts above some arbitrarily chosen zero of potential, usually the earth.

The potential at a point A is defined to be the potential difference between that point and the point of zero potential, i.e., the work per unit charge necessary to bring positive charge to A from the point of zero potential.

done at some time to bring Q_2 to A. At what point would Q_2 have otential energy? A reasonable as-ion, and one frequently made, is ts potential energy would be zero nfinite distance from Q_1, for at this ce the repelling force exerted by it would be zero.

mewhat similar argument provides e practical choice for zero of electric tial energy. No electric forces are d by an isolated uncharged object; an uncharged object may be con-d to have zero electric potential . The earth may be regarded as e spherical conductor, so large that ddition to it of any likely quantity arge leaves it still essentially un-ed, and at the same potential. Thus rth or any conductor connected to may be reasonably assumed to be o potential.

ether either of these choices of zero c potential energy is correct is in ce immaterial. What matters to a limbing a mountain is that the dif-e in elevation between the bottom he top is 10,000 ft, even though a may indicate that the respective ions are 1000 ft and 11,000 ft above vel. In electricity the similar vital nation is that the difference in poten-etween two points is 10,000 volts, hough the respective potentials are volts and 11,000 volts above some rarily chosen zero of potential, y the earth.

e potential at a point A is defined the potential difference between oint and the point of zero potential, he work per unit charge necessary ng positive charge to A from the of zero potential.

Since work must be done to move positive charge towards a positively charged conductor, the conductor is said to be at a positive potential. Since work would have to be done to move positive charge away from a negatively charged conductor, this conductor is said to be at a negative potential.

2–10 DIRECTION OF MOTION OF CHARGE

Potential is traditionally and arbitrarily defined as above, i.e., in terms of positive charge. On such a basis, the discussion concerning Figure 2.7 has shown that the potential increases as the distance from the positive charge Q_1 decreases.

Suppose that a positive charge is released near a positively charged object (Fig. 2.8). It will be repelled by and move away from the positively charged object. A positive charge, if free to move in an electric field, will move towards lower potential. However, if a negative charge (Fig. 2.8) is released near a positively charged object, it will be attracted by and move towards the positively charged object. A negative charge, if free to move in an electric field, will move towards higher potential.

Since the charges which are free to move in conductors are most frequently the negative charges borne by free electrons, the following facts are worthy of note:

An electron flow will occur in a conductor AB and will be from B toward A if any of the following conditions exist:

(a) B is at a negative potential, while A is positive.

(b) B is at zero potential, while A is positive.

(c) B is at a negative potential while A is at zero potential.

(d) B is at a negative potential while A is at a higher negative potential (nearer zero).

(e) B is at a positive potential while A is at a higher positive potential.

In fact, an electron flow will occur in the conductor AB provided that a potential difference exists between A and B. Moreover, if there is no P.D., there will be no flow of charge. Conversely, if the charges in a conductor are at rest, all parts of the conductor must be at the same potential.

2–11 GRAVITATIONAL ANALOGY

The behaviour of electric charge in the electric field near a charged object is

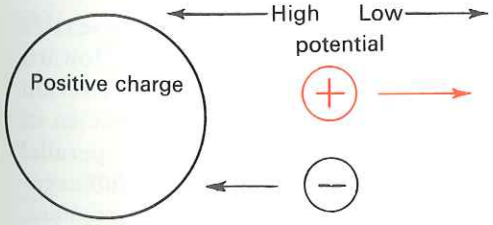


Fig. 2.8. Direction of motion of charge in an electric field.

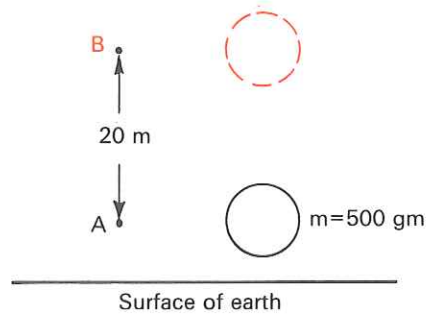


Fig. 2.9. Diagram to illustrate the similarities between a gravitational field and an electric field.

similar to the behaviour of matter in the gravitational field of the earth. Consider a 500-gm object at a position A above the surface of the earth (Fig. 2.9). The weight of the object constitutes a downward force acting on the object. The gravitational force, like the electric force, is inversely proportional to the square of the distance.

In the gravitational field, the gravitational force per unit mass is called the gravitational field strength and is given the symbol g. At the surface of the earth, the value of g is approximately 9.8 newtons/kg, and hence the acceleration due to gravity is approximately 9.8 m/sec^2. Similarly, in the electric field the electric force per unit charge, measured in newtons/coulomb, for example, is called the electric field strength (Sect. 2–6).

If AB (Fig. 2.9) is 20 m, and if $g = 9.8$ newtons/kg, the work required to raise the 500-gm mass from A to B is 98 joules. As a result, the gravitational potential energy of the mass is 98 joules greater at B than at A. This statement in no way implies that the gravitational potential energy of the object was zero at A. For the sake of convenience, the gravitational potential energy may be taken as zero at the surface of the earth. It is also common to consider the gravitational potential energy of an object to be zero at an infinite distance from the earth. Similar arbitrary choices of zero of potential are made in the electric field (Sect. 2–9).

In the gravitational field, the change in gravitational potential energy per unit mass, in joules per kg for example, is rarely calculated. However, in the electric field, the change in electric potential energy per unit charge, in joules per coulomb for example, is frequently calculated and is called potential difference.

If the 500-gm mass is released at B (Fig. 2.9), it will fall towards the earth, i.e., it will move towards a position of lower gravitational potential energy. The motion of mass which is free to move in the gravitational field is in the same direction as the motion of positive charge which is free to move in the electric field.

As the mass falls, its gravitational potential energy is converted to kinetic energy, and the law of conservation of energy applies. Similarly, electric potential energy may be converted to kinetic energy.

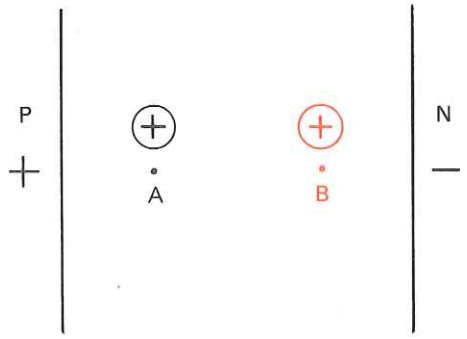
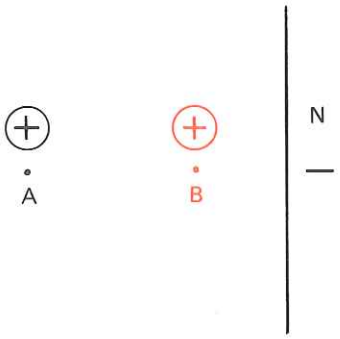


Fig. 2.10. If a positive charge is released at *A*, it accelerates as it moves toward *B*.

2–12 ACCELERATION OF CHARGE

When an object is released in the earth's gravitational field, it accelerates toward the earth. Let us consider the acceleration of charge in an electric field as shown in Figure 2.10. P and N are two parallel metal plates charged positively and negatively, respectively. A small light sphere, positively charged, is held at a position A between the plates. As we have already

nd is called potential difference.
ne 500-gm mass is released at B
.9), it will fall towards the earth,
will move towards a position of
ravitational potential energy. The
of mass which is free to move in
avitational field is in the same
on as the motion of positive charge
s free to move in the electric field.
he mass falls, its gravitational
al energy is converted to kinetic
, and the law of conservation of
applies. Similarly, electric poten-
ergy may be converted to kinetic



0. If a positive charge is released at A, it
tes as it moves toward B.

ACCELERATION OF CHARGE

n an object is released in the earth's
tional field, it accelerates toward
th. Let us consider the acceleration
ge in an electric field as shown in
2.10. P and N are two parallel
plates charged positively and nega-
respectively. A small light sphere,
ely charged, is held at a position A
n the plates. As we have already

noted, the force exerted on this positively charged sphere by the plates remains constant when the sphere is permitted to move. As a result the sphere, when released, moves with uniform acceleration toward N, gaining speed and kinetic energy. We may predict its speed at any other point B as in the following example.

2–13 WORKED EXAMPLE

The charge on the sphere (Fig. 2.10) is 2×10^{-8} coulombs, the electric force exerted on it is 8×10^{-5} newtons, the mass of the sphere is 0.5 gm, and $AB = 4$ cm. Calculate (a) the work done by the electric force on the sphere as the sphere moves from A to B, (b) the P.D. between A and B, (c) the horizontal component of the velocity of the sphere when it reaches B. Assume that there is no frictional resistance to the motion; i.e., that the work done by the force on the sphere (the electric potential energy lost by the charge) is equal to the kinetic energy gained by the sphere.

SOLUTION

(a)

$$W = Fs = 8 \times 10^{-5} \times 4 \times 10^{-2} \text{ joules}$$
$$= 32 \times 10^{-7} \text{ joules}$$

(b)

$$V = \frac{W}{Q} = \frac{32 \times 10^{-7}}{2 \times 10^{-8}} \text{ volts} = 160 \text{ volts}$$

(c)

$$E_K \text{ at } B = 32 \times 10^{-7} \text{ joules}$$
$$\text{i.e., } \tfrac{1}{2}mv^2 = 32 \times 10^{-7}$$
$$\tfrac{1}{2} \times 0.5 \times 10^{-3}\, v^2 = 32 \times 10^{-7}$$
$$v^2 = 128 \times 10^{-4}$$
$$\text{and } v = 11.3 \times 10^{-2} \text{ approx.}$$

Therefore the horizontal component of the velocity at B will be approximately 11.3×10^{-2} m/sec, or 11.3 cm/sec.

2–14 THE PROBLEM OF MEASURING Q AND V

So far in this chapter we have discussed the concepts of quantity of charge and potential difference; represented by the symbols Q and V. We have defined units, namely the coulomb and the volt, in which to measure Q and V, though admittedly the coulomb was only vaguely defined, and we shall have to leave it that way. However, in all of this, we have given no indication of how either Q or V might be measured.

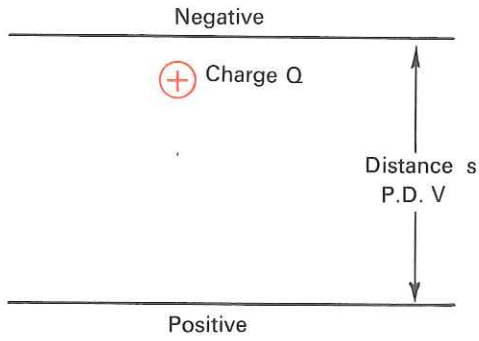


Fig. 2.11. V can be calculated if the force required to move Q from the negative plate to the positive plate can be measured.

The problem is not as complicated as it seems at first glance, for if Q can be measured, V can be calculated, and vice versa. Consider the situation shown in Figure 2.11, assuming that the effect of gravity is negligible. The small positive charge Q may be moved from the vicinity of the negative plate to the positive plate by a force F just sufficient to balance the electric force. The work W done by the force can be calculated from the formula $W = Fs$. But if V is the potential difference between the plates,

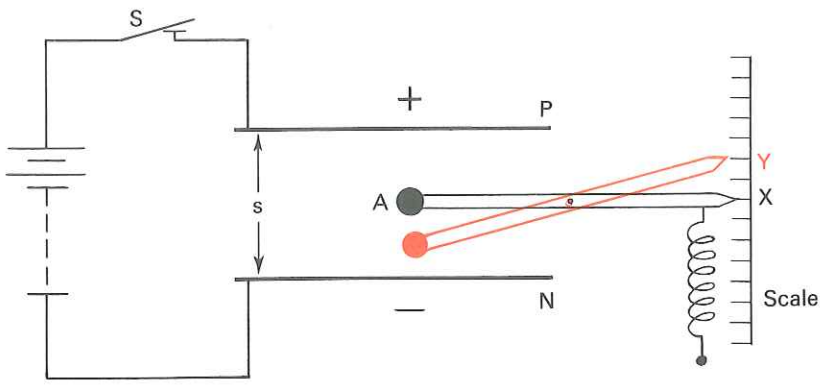


Fig. 2.12. Diagram of the main parts of an electrical balance.

$$W = QV$$
$$\therefore QV = Fs$$
$$\text{and } V = \frac{Fs}{Q}$$

F and s can be measured; then if we can measure Q we can calculate V.

The Coulomb's law formula

$$F = \frac{9.0 \times 10^{-9} qQ}{r^2}$$

may be applied in a measurement of quantity of charge. The two charges q and Q may be made equal by charge sharing as outlined in Section 2–2. Then a Coulomb type torsion balance may be used to measure F and r. The common magnitude of q and Q can then be calculated. If one of these charges is then placed in the apparatus whose very simple outline is shown in Figure 2.12, F and s may be measured and V calculated. Let us consider this apparatus in greater detail.

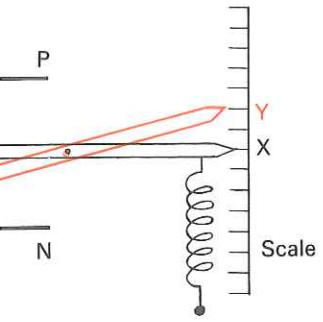
2–15 THE ELECTRIC BALANCE

The apparatus shown in Figure 2.12 is called an electrical balance. A battery, the positive and negative terminals of which are connected to P and N respectively, charges the plates P and N and establishes the electric field between the plates. As a result there is a potential difference V between them when the switch S is closed. If the sphere A is uncharged, the opposite end of its insulating arm is at X. Now a measured positive charge Q is placed on A. A moves down and the opposite end of the arm moves up to Y. The force necessary to produce the extension XY of the spring can be found readily, the distance s between the plates can be measured, Q is known and V can be calculated.

The use of a Coulomb torsion balance and then of an electrical balance for the determination of V may seem a laborious procedure, and it is. However, once such work is done, direct-reading voltmeters may be designed and calibrated. Further measurements are then very much simpler; in fact the electrical balance can then be used for determining Q, since V may be measured directly. We shall discuss this procedure in Chapter 4.

2–16 POTENTIAL AT A POINT

In Section 2–4 we investigated several electric fields qualitatively. Among them was the field about a point charge (Fig.



an electrical balance.

ishes the electric field between the . As a result there is a potential ence V between them when the n S is closed. If the sphere A is rged, the opposite end of its in- ng arm is at X. Now a measured ve charge Q is placed on A. A moves and the opposite end of the arm s up to Y. The force necessary to ce the extension XY of the spring e found readily, the distance s en the plates can be measured, Q is n and V can be calculated.

e use of a Coulomb torsion balance hen of an electrical balance for the mination of V may seem a laborious dure, and it is. However, once such is done, direct-reading voltmeters e designed and calibrated. Further urements are then very much sim- in fact the electrical balance can be used for determining Q, since V be measured directly. We shall dis- his procedure in Chapter 4.

POTENTIAL AT A POINT

Section 2–4 we investigated several ic fields qualitatively. Among them he field about a point charge (Fig.

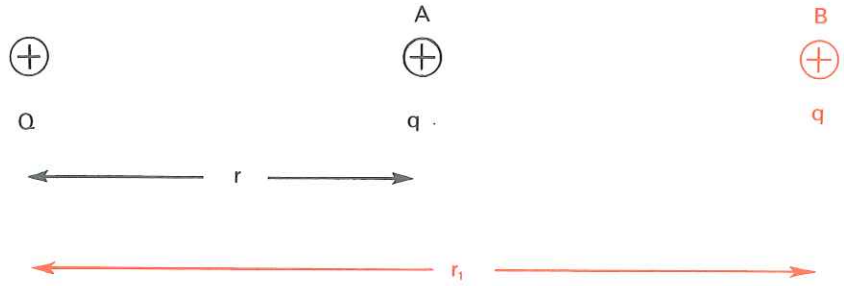


Fig. 2.13. Work must be done on the charge q in order to move it from A to B.

2.3*a*). Let us now analyse this field mathematically.

Suppose that a small positive charge q (Fig. 2.13) is placed at A, at a distance r from a positive charge Q. The charge Q exerts on q a force to the right, and if q is released at A, this force causes q to move to the right, losing electric potential energy as it moves. Suppose that its final position is B, at a distance r_1 from Q.

The potential energy lost by q as it moves from A to B is equal to the work done on q by the electric force. At A, the magnitude of this force is $\frac{kqQ}{r^2}$. If this force remains constant as q moves from A to B, then the work done by the electric force is given by

$$W = Fs$$
$$= \frac{kqQ}{r^2}(r_1 - r)$$

But the force does not remain constant, and r^2 is not the correct denominator to use here, nor is r_1^2. By means of mathematics beyond the scope of this book, it may be shown that the denominator should be rr_1.

$$\therefore W = \frac{kqQ(r_1 - r)}{rr_1}$$
$$= kqQ\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

If r_1 is infinitely large,

$$W = \frac{kqQ}{r}$$
$$\text{or } \Delta E = \frac{kqQ}{r}$$

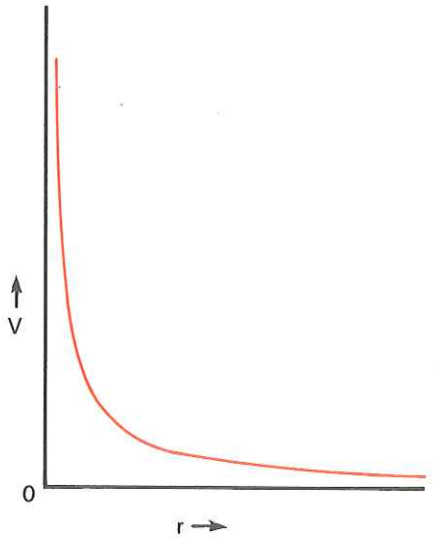


Fig. 2.14. Graph of the inverse relationship between V and r.

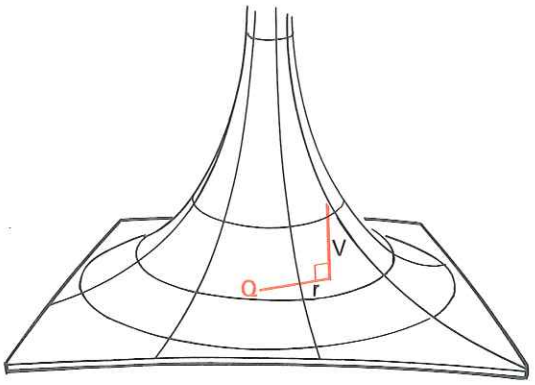


Fig. 2.15. If the graph in Figure 2.14 is rotated about its vertical axis, a potential hill is formed.

That is, the change in the electric potential energy of the charge q, as q is moved from a distance r to a point at infinite distance from Q, is $\frac{kqQ}{r}$. If we consider that electric potential energy is zero at infinite distance from Q (Sect. 2–9), then the potential energy at A (Fig. 2.13) must have been

$$E = \frac{kqQ}{r} \qquad (1)$$

We may go one step further and calculate for the charge q its energy per unit charge, that is, its potential V. In order to do this, we divide equation (1) by q, and obtain

$$V = \frac{kQ}{r} \qquad (2)$$

Equation (2) tells us that the potential at a point at a distance r from a charged object is inversely proportional to r. Figure 2.14 shows the graph of this inverse relationship between V and r. We shall make considerable use of this relationship between V and r in Chapter 5, particularly in the form shown in Figure 2.15. Here the graph has been rotated about its vertical (potential) axis to form what is called a potential hill. Figure 2.16 is a photograph of a model of a potential hill.

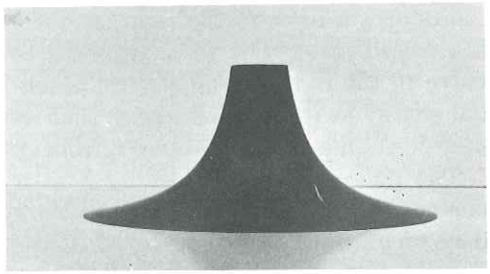


Fig. 2.16. A photograph of a model of a potential hill.

2–17 PROBLEMS

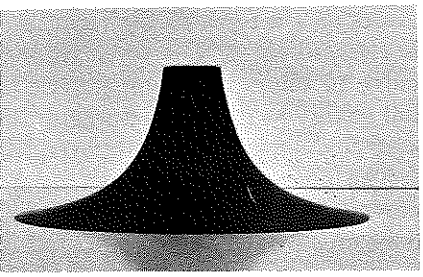
Where necessary, assume that
(a) in a vacuum, the constant k in Coulomb's law
$= 9.0 \times 10^9 \frac{\text{newton-(metre)}^2}{\text{(coulomb)}^2}$
(b) the charge on the electron $= 1.60 \times 10^{-19}$ coulombs
(c) the mass of the electron $= 9.11 \times 10^{-31}$ kg.

1. Consider the relationship $F = \frac{kqQ}{r^2}$. What is the effect on F of (a) changing q

e, that is, its potential V. In order this, we divide equation (1) by q, btain

$$V = \frac{kQ}{r} \qquad (2)$$

uation (2) tells us that the potential oint at a distance r from a charged t is inversely proportional to r. e 2.14 shows the graph of this in- relationship between V and r. We make considerable use of this re- ship between V and r in Chapter 5, ularly in the form shown in Figure Here the graph has been rotated its vertical (potential) axis to form is called a potential hill. Figure 2.16 hotograph of a model of a potential



.16. A photograph of a model of a potential

Coulomb's law

$.60 \times 10^{-19}$ coulombs
1×10^{-31} kg.

s the effect on F of (*a*) changing q

by a factor of 3, (*b*) changing Q by a factor of $\frac{1}{2}$, (*c*) changing r by a factor of 3, (*d*) making all of the above changes?

2. The force of repulsion between two small, positively charged objects, A and B, is 3.6×10^{-5} newtons when $AB = 0.10$ m. What is the force of repulsion if AB is increased to (*a*) 0.20 m, (*b*) 0.25 m, (*c*) 0.30 m?

3. Two charged objects, 2.0 cm apart, repel each other with a force of 3.0×10^{-5} newtons. What does the force become if (*a*) one of the charges is halved, (*b*) both of the charges are doubled, (*c*) the separation is increased to $2\sqrt{3}$ cm?

4. If each of two charges is tripled, while the distance between the charges remains the same, by what factor does the force between them change?

5. By what factor must the distance separating two charges be changed if the force between them is to be changed by a factor of 3?

6. Two small objects, each carrying a positive charge of 4.0×10^{-9} coulombs, are placed 2.0 cm apart in a vacuum. What is the magnitude of the force that each exerts on the other?

7. Calculate the force between charges of 5.0×10^{-8} coulombs and 1.0×10^{-7} coulombs, if they are 5.0 cm apart in a vacuum.

8. Calculate the force between charges of 4.0 microcoulombs and 6.0 microcoulombs when they are 8.0 cm apart in a vacuum.

9. Calculate the magnitude of the charge which will repel an equal charge with a force of 10 newtons when the charges are 10 cm apart in a vacuum.

10. At a certain point in an electric field, the magnitude of the field strength vector is 15 newtons/coulomb. Calculate the magnitude of the electric force exerted on a point charge of 3.0×10^{-7} coulombs placed at this point in the field.

11. A pithball bearing a negative charge of 0.50 microcoulombs is placed at a point in an electric field and is subject to an electric force of 1.0×10^{-4} newtons. What is the magnitude of the electric field intensity at that point?

12. At what distance from a charge of 6.0×10^{-6} coulombs is the field intensity equal to 3.0 newtons/coulomb?

13. The electric field strength mid-way between a pair of oppositely charged parallel plates is 3.0×10^{3} newtons/coulomb. What is the field strength half-way between this point and the positively charged plate?

14. When 4.0×10^{-3} coulombs of charge is transferred from A to B, 0.80 joules of work is done. What is the P.D. between A and B?

15. The potential difference between two plates is 1.2×10^{3} volts. If 0.12 joules of work is done in moving a charge from one plate to the other, what is the magnitude of the charge?

16. How much work is done when a charge of 0.005 coulombs is moved through a potential difference of 300 volts? If the distance the charge moves is 30 cm, calculate the average force required.

17. In a uniform electric field, the P.D. between two points 5.0 cm apart is 50 volts. Calculate the magnitude of the electric field strength.

18. Calculate the field intensity between 2 parallel plates 2.5 cm apart, if the P.D. between the plates is 200 volts.

19. Under normal atmospheric conditions the potential in the atmosphere near the earth's surface rises 80 volts for every metre increase in elevation. (*a*) In what direction will free electrons in the atmosphere move? (*b*) Calculate the loss in electric potential energy of a free electron which moves vertically through 10 metres in the atmosphere.

20. Two large, vertical, insulated metal plates, A and B, are placed 5.0 cm apart. A pith ball carrying a positive charge of 1.0×10^{-9} coulombs hangs on the end of a long silk fibre and is placed between the plates. The ball moves toward plate A under a force of 6.0×10^{-5} newtons. Calculate the P.D. in volts between the plates, and state which is at the higher potential.

21. If 5×10^{-2} joules of work is done in moving 0.01 coulombs of positive charge from earth to a positively charged conductor, what is the potential of the conductor?

22. A pith ball weighing 1.0×10^{-2} gm and bearing a positive charge of 4.0×10^{-7} coulombs is moved 50 cm from A to B through a potential difference of 8.0×10^{2} volts. The pith ball is released at B and "falls" back to A. Calculate (*a*) the work done in moving the pith ball from A to B, (*b*) the average force required, (*c*) the kinetic energy of the ball when it reaches A on returning from B, and (*d*) its speed at A.

23. A proton has a mass of 1.67×10^{-24} gm and a charge of 1.6×10^{-19} coulombs. If a proton, initially at rest, is acted upon by an electric field and moved through a potential difference of 10^{6} volts, what kinetic energy is imparted to the proton and what is its final speed?

24. An electron is accelerated from rest to a speed of 5.4×10^{8} cm/sec in a distance of 2.0 cm in a vacuum tube. Assuming that its mass remains constant, (*a*) what is the acceleration of the electron, (*b*) what average force is necessary to produce this acceleration, and (*c*) what is the potential difference through which the electron moves?

25. If the potential difference between an anode and a cathode is 8000 volts, what speed will free electrons, supplied at rest at the cathode, acquire in moving to the anode?

26. What must be the potential difference between a cathode and an anode to accelerate free electrons from rest to a speed of 3.0×10^{8} cm/sec?

.005 coulombs is moved through
distance the charge moves is 30

veen two points 5.0 cm apart is
lectric field strength.

rallel plates 2.5 cm apart, if the

potential in the atmosphere near
ery metre increase in elevation.
the atmosphere move? (*b*) Cal-
y of a free electron which moves
phere.

A and *B*, are placed 5.0 cm apart.
$.0 \times 10^{-9}$ coulombs hangs on the
ween the plates. The ball moves
$^{-5}$ newtons. Calculate the P.D. in
is at the higher potential.

oving 0.01 coulombs of positive
conductor, what is the potential

nd bearing a positive charge of
rom *A* to *B* through a potential
ball is released at *B* and "falls"
moving the pith ball from *A* to *B*,
inetic energy of the ball when it
s speed at *A*.

gm and a charge of 1.6×10^{-19}
cted upon by an electric field and
10^6 volts, what kinetic energy is
al speed?

speed of 5.4×10^8 cm/sec in a
Assuming that its mass remains
ne electron, (*b*) what average force
n, and (*c*) what is the potential
ves?

ode and a cathode is 8000 volts,
at rest at the cathode, acquire in

tween a cathode and an anode to
eed of 3.0×10^8 cm/sec?

27. Free electrons at rest are supplied at the cathode of an evacuated tube. The anode is at a potential of 240 volts above that of the cathode. Calculate the speed of the electrons as they reach the anode.

28. Consider the relationship $E = \frac{kqQ}{r}$. What is the proportional relationship between E and q, between E and Q, and between E and r?

29. For the relationship $V = \frac{kQ}{r}$, what is the effect on V of (*a*) changing Q by a factor of 10, (*b*) changing r by a factor of 0.2?

30. (*a*) Under what circumstances may the formula $E = \frac{kqQ}{r}$ be used to calculate the electric potential energy of a charge? (*b*) Use this formula to calculate the electric potential energy of a charge of 6×10^{-7} coulombs in the field of a charge of 5×10^{-5} coulombs, if the distance between the charges is (*i*) 10 cm, (*ii*) 20 cm, (*iii*) 30 cm.

31. (*a*) Under what circumstances may the formula $V = \frac{kQ}{r}$ be used to calculate the potential at a point in an electric field? (*b*) Use this formula to calculate the potential in the field of a charge of 8×10^{-7} coulombs, at distances of (*i*) 15 cm, (*ii*) 20 cm, and (*iii*) 30 cm.

2–18 SUMMARY

Charge may be transferred, but it is not created nor destroyed.

Coulomb's law states that the force between two charges is directly proportional to each of the charges, and inversely proportional to the square of the distance between them.

$$F = \frac{kqQ}{r^2}$$

$$k = 9.0 \times 10^9 \frac{\text{newton-(metre)}^2}{\text{(coulomb)}^2}$$

The electric field between parallel, oppositely charged plates is uniform except near the edges of the plates.

Electric field intensity = electric force per unit charge.

The P.D. between two points in an electric field is the work done or energy transformed per unit charge, when charge is transferred from one point to the other.

$$V = \frac{W}{Q} = \frac{E}{Q}$$

1 volt = 1 joule/coulomb

The work done, electric energy transformed, and kinetic energy gained when charged particles are accelerated from rest are connected by the relationships

$$E = W = QV = Fs = \tfrac{1}{2}mv^2$$

If the zero of potential is taken at an infinite distance from a charged object, then (1) the potential energy of a charge in the electric field of the object is given by the formula $E = \frac{kqQ}{r}$, and (2) the potential at a point in the field is given by the formula $V = \frac{kQ}{r}$.

Chapter 3

Charge in Motion

3–1 INTRODUCTION

Until early in the eighteenth century electricity was considered to be a static property of those substances which are now called insulators. The possibility of a transfer of this property (i.e., of charge) was not suspected. Then in 1731 Stephen Gray of England reported that he had succeeded in transferring charge over distances of several hundred feet. Subsequent similar experiments showed that charge could be transferred quite readily through those substances which are now called conductors.

In spite of these discoveries, several decades elapsed before worthwhile advances took place, because no method of producing a continuous flow of charge was known. Then in 1799 the Italian scientist, Alessandro Volta (1745-1827), built a device, later called a voltaic cell, which could maintain a constant potential difference between its terminals and therefore produce a continuous flow of charge in a conductor. In 1820 the Danish physicist, Hans Christian Oersted, discovered that a magnetic field existed about a conductor through which charge was flowing. Scientists then set as their goal the production of a flow of charge in a conductor by the use of a magnet. This proved to be a difficult problem until Michael Faraday reasoned that, since a moving charge produces a magnetic field, perhaps a moving magnet would produce a flowing charge. On October 17, 1831, he verified this deduction and produced a magnetically induced current by moving a magnet into and out of a coil of wire.

3–2 SOURCES OF CHARGE

As a result of the work of Volta and Faraday, batteries and generators are now our most important sources of electricity. However, the charges produced by batteries and generators are exactly the same

re produce a continuous flow of
in a conductor. In 1820 the Danish
st, Hans Christian Oersted, dis-
d that a magnetic field existed
conductor through which charge
wing. Scientists then set as their
e production of a flow of charge
nductor by the use of a magnet.
roved to be a difficult problem
Michael Faraday reasoned that,
moving charge produces a mag-
ield, perhaps a moving magnet
produce a flowing charge. On
r 17, 1831, he verified this deduc-
d produced a magnetically induced
by moving a magnet into and
a coil of wire.

SOURCES OF CHARGE

result of the work of Volta and
y, batteries and generators are now
st important sources of electricity.
er, the charges produced by bat-
nd generators are exactly the same

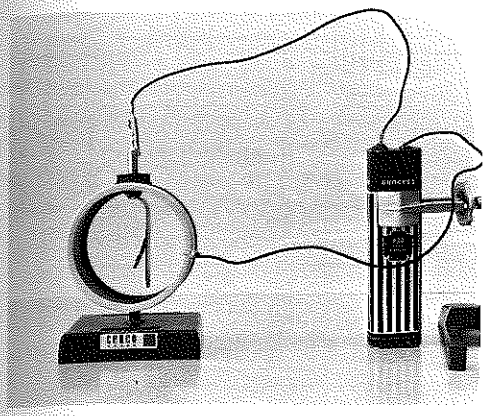


Fig. 3.1. The separation of the leaves of the electroscope indicates that the terminal of the battery bears a static charge.

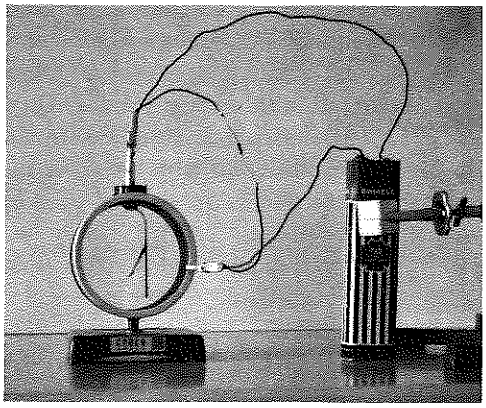


Fig. 3.2. The chemical action in a battery supplies charge continuously to the battery terminals. As a result, the leaves of the electroscope in this photograph remain separated, even though charge may transfer from the leaves to the case through a resistor. The resistor is to the right of the top of the electroscope.

as those previously encountered in electrostatics. If a battery or other source of electricity (of at least 90 volts DC) is connected to a sensitive electroscope as shown in Figure 3.1, the leaves of the electroscope diverge. The charge on the leaves may be identified as outlined in Section 1–11. One of the battery terminals is found to bear a static positive charge, the other a static negative charge.

Apparently a chemical action within the battery separates the charges and causes them to accumulate at the terminals. Moreover, this action is continuous, as we may show by the circuit shown in Figure 3.2. The separation of the leaves of the electroscope remains constant even though charge may transfer slowly from the leaves to the case through the resistor. However, if the battery is disconnected, the electroscope quickly discharges.

3–3 THE P.D. OF A BATTERY

The potential difference between the plates P and N in Figure 2.12 is the P.D. between the terminals of the battery, because the work required to transfer charge from the plates to the terminals through the conducting wires is negligible. A battery is composed of two or more cells, connected usually in series; that is, with the positive terminal of one cell connected to the negative terminal of the next. Experiments with electrical balances (or with voltmeters) indicate that all similar cells have the same P.D.; for example the P.D. of a dry cell is about 1.5 volts. Moreover, the P.D. of a battery of cells connected in series is proportional to the number of cells. Thus the P.D. of a battery of 100 dry cells is 150 volts.

Apparently the chemical action in a cell forces charges to accumulate on the terminals. Thus chemical potential energy is used to increase the electric potential energy of the charges. The statement that

the P.D. of a battery is 12 volts therefore means that each coulomb of charge supplied at the terminals possesses 12 joules of electric potential energy. The P.D. of a battery is the energy supplied by the battery to each unit of charge.

3–4 BASIC ELECTRIC CIRCUIT

We do not intend to give a detailed analysis of electric circuits in this text; we assume that you are sufficiently familiar with some basic facts concerning such circuits. The remainder of this chapter summarizes these facts briefly.

If the terminals of a battery are connected by a solid conductor, electrons flow from the negative terminal to the positive terminal through the conductor. In order to limit the number of electrons transferred in a given length of time, and to exercise some control over this transfer, a resistor (relatively poor conductor) must be included in the circuit (Fig. 3.3). Any electric appliance is a resistor, and therefore the comments which follow apply for any circuit operated by a battery.

3–5 ELECTRIC CURRENT

Just as a current of water, to the hydraulic engineer, means a rate of flow that may be expressed in gallons per second, so current of electricity means the rate of flow of charge. The standard unit of current is the ampere. 1 ampere = 1 coulomb per second. If 10 coulombs of charge are transferred from X to Y (Fig. 3.3) in 5 seconds, the rate of flow is $\frac{10}{5}$ coulombs per second, i.e., 2 coulombs per second or 2 amperes. Since 1 coulomb = 6.25×10^{18} elementary charges, then

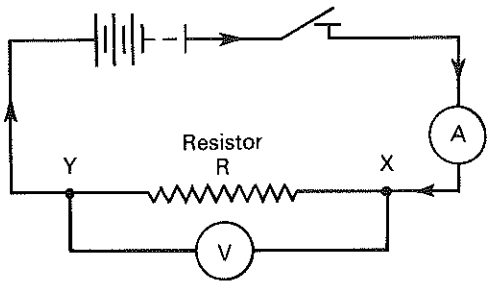


Fig. 3.3. The basic DC circuit.

1 ampere = 6.25×10^{18} elementary charges/second.

The symbol I is used to represent current; the following relationship is obvious: $I = \frac{Q}{t}$, or $Q = It$, when Q is the quantity of charge transferred in time t.

One of the early devices used to measure electric current was a special type of electrolytic cell called a silver voltameter (Fig. 3.4). An electrolytic cell contains two strips of metal, called electrodes, immersed in a liquid. The electrodes are connected to the terminals of a battery; the electrode connected to the positive terminal is called the anode and the electrode connected to the negative terminal is called the cathode. In the silver voltameter the anode is a strip of pure silver, the cathode is a strip of copper, and the liquid is a solution of silver nitrate.

When the switch S (Fig. 3.4) is closed, the ammeter indicates that the solution conducts electricity. One effect of this conduction is to cause a film of silver to be deposited or plated on the surface of the cathode. The mass of silver deposited can be determined by careful weighing of the cathode before and after the transfer of charge. Experiments have shown that

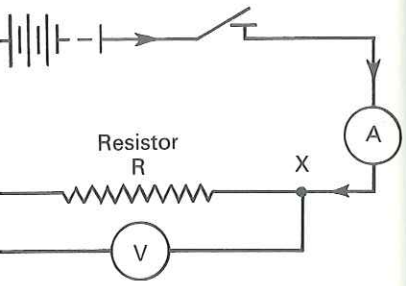


Fig. 3.3. The basic DC circuit.

ere = 6.25×10^{18} elementary
s/second.
symbol I is used to represent cur-
ne following relationship is obvious:

or $Q = It$, when Q is the quantity
ge transferred in time t.
of the early devices used to measure
current was a special type of elec-
c cell called a silver voltameter
.4). An electrolytic cell contains
ips of metal, called electrodes, im-
in a liquid. The electrodes are
ted to the terminals of a battery;
ctrode connected to the positive
al is called the anode and the elec-
connected to the negative terminal
d the cathode. In the silver vol-
r the anode is a strip of pure silver,
hode is a strip of copper, and the
is a solution of silver nitrate.
n the switch S (Fig. 3.4) is closed,
meter indicates that the solution
ts electricity. One effect of this
tion is to cause a film of silver to
osited or plated on the surface of
hode. The mass of silver deposited
determined by careful weighing of
hode before and after the transfer
ge. Experiments have shown that

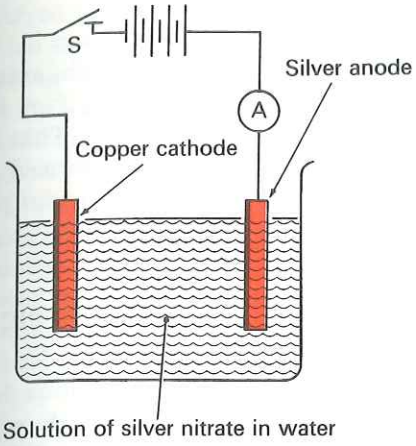


Fig. 3.4. A silver voltameter.

the mass of silver deposited is proportional to the current and also proportional to the time of flow.

An International Electric Congress, meeting first in 1881, and attended by many of the world's leading physicists, made a thorough study of this plating effect and the relationship between the ampere and the mass of silver deposited. Their recommendations were based on theoretical considerations beyond the scope of this book. Revisions were made in later years, and in 1908 the International Ampere was adopted. It was defined as that current which would deposit silver from a silver nitrate solution at the rate of 0.001118 gm per second.

A flow of electrons in a conductor produces a magnetic field in the region surrounding the conductor. This effect has been applied in the moving-coil type of ammeter in common use for measuring current. The silver voltameter may be used to calibrate the moving-coil type of instrument.

An ammeter is shown in Figure 3.3, connected between the negative terminal of the battery and the point X. From the ammeter reading, the quantity of charge transferred in a measured time can be calculated. The relationship $Q = It$ is used. Q is the quantity of charge in coulombs, I is the current in amperes, and t is the time in seconds.

3–6 ENERGY TRANSFORMATIONS

The resistor R (Fig. 3.3), though it conducts charge, offers some opposition to the flow of charge through it due to frictional resistance to electron motion. Two results of this opposition to the flow of charge are discussed below.

(*a*) Some of the electric energy possessed by this charge is expended as the charge flows through the resistor. The law of conservation of energy indicates that this electric energy is transformed to some other form of energy. If the resistor is an element in an electric stove, most of the electric energy is transformed into heat energy. If the resistor is the filament of an incandescent light bulb, both heat and light are produced. If the resistor is an electric motor, the energy produced is mainly in the form of kinetic energy of the armature.

(*b*) Because of this transformation of energy, there is a potential difference between X and Y. This P.D. is defined as the electric energy per unit charge transformed when charge is transferred from X to Y.

3–7 MEASUREMENT OF POTENTIAL DIFFERENCE

The measurement of the P.D. between X and Y (Fig. 3.3) is accomplished by

connecting a voltmeter in parallel with the resistor R. The voltmeter is usually of the moving-coil type and consists essentially of a galvanometer coil in series with a high resistance conductor. To go from X to Y the charge has two possible routes, through the resistor R or through the voltmeter. The high resistance in the voltmeter is usually such that, with a P.D. of 100 volts between X and Y, not more than 1/1000 of an ampere will flow through the voltmeter. Hence, the current in the resistor R is not seriously altered. This small current of 1/1000 of an ampere will, however, deflect the galvanometer coil. The angle of deflection of the coil is approximately proportional to the current through the coil; this current is proportional to the current in the resistor R; and the current in R is proportional to the potential difference between X and Y.

3–8 ELECTRIC ENERGY AND POWER

If the formula $E = QV$ is used to calculate the electric energy transformed, and if Q is in coulombs and V is in volts, E is in joules. Since $Q = It$, the formula $E = QV$ may be written $E = VIt$, the units for E, V, I and t being respectively joules, volts, amperes, and seconds.

For example, referring again to Figure 3.3, if the ammeter reading is 2 amperes and the voltmeter reading is 10 volts, the electric energy transformed in the resistor in a 2 hour period $= E = VIt = 10 \times 2 \times 2 \times 3600$ joules, or 144000 joules.

Power is defined as the rate at which energy is transformed. For electric energy,

$$P = \frac{E}{t} = \frac{QV}{t} = \frac{VIt}{t} = VI$$

The unit of power is the watt.

One watt = one joule/sec.

Then, for the readings given in the example in the preceding paragraph, $P = VI = 10 \times 2$ watts = 20 watts. That is, electric energy is being transformed in the resistor at the rate of 20 joules per second.

3–9 CONDUCTION OF ELECTRIC CHARGES

The list of conductors in Section 1–5 contains only metals, all of which, with the exception of mercury, are solid at normal temperature. Conduction in metals consists of a transfer of electrons from the negative terminal of the source through the circuit to the positive terminal. Thus conduction in a solid metal is a relatively simple process. Conduction in liquids and gases is more complex.

Solutions of acids, bases and salts are listed in Section 1–5 as fair conductors. They do not conduct nearly as well as metals, but the fact that they do conduct is important. Not only is the conduction frequently accompanied by useful chemical changes, but these changes give us considerable insight into the structure of matter. Consider, for example, the silver voltameter circuit shown in Figure 3.4. When the switch is closed the ammeter indicates that the silver nitrate solution is a moderately good conductor. We have already noted that the effect of the conduction is that the mass of the anode decreases, and an equal amount of silver is deposited on the cathode. Not only is charge transferred, but matter (silver) is transferred as well. Apparently the silver nitrate in solution spontaneously dissociates into positively and negatively

unit of power is the watt.

One watt = one joule/sec.

, for the readings given in the exam- the preceding paragraph, $P = VI$ $\times$ 2 watts = 20 watts. That is, ic energy is being transformed in esistor at the rate of 20 joules per d.

CONDUCTION OF ELECTRIC CHARGES

e list of conductors in Section 1–5 ins only metals, all of which, with xception of mercury, are solid at al temperature. Conduction in ls consists of a transfer of electrons the negative terminal of the source gh the circuit to the positive ter- l. Thus conduction in a solid metal latively simple process. Conduction uids and gases is more complex. lutions of acids, bases and salts are in Section 1–5 as fair conductors. do not conduct nearly as well as ls, but the fact that they do conduct portant. Not only is the conduction ently accompanied by useful chemi- hanges, but these changes give us derable insight into the structure of er. Consider, for example, the silver meter circuit shown in Figure 3.4. n the switch is closed the ammeter ates that the silver nitrate solution oderately good conductor. We have dy noted that the effect of the con- on is that the mass of the anode ases, and an equal amount of silver posited on the cathode. Not only is ge transferred, but matter (silver) is ferred as well. Apparently the silver te in solution spontaneously disso- s into positively and negatively

charged particles called ions. Each type of ion is then attracted to the electrode bearing the charge opposite to that on the ion. In general, conduction in a liquid consists of positive ions migrating to the cathode, and negative ions to the anode.

Under normal circumstances, most gases are very poor conductors. An electroscope will retain its charge for several hours or more, even though it is in contact with air. However, if a lighted match is held near the top of a charged electroscope, the charge is lost very quickly. Apparently the effect of the match is to ionize the air. The ions whose charges are opposite to that on the electroscope are attracted to the electroscope and neutralize its charge. An ionized gas, then, conducts electricity. In general, conduction in a gas consists of positive ions travelling in one direction, and negative ions, supplemented perhaps by free electrons, travelling in the other direction. As might be expected, a gas conducts better at reduced pressure, for then the moving ions undergo fewer collisions with gas molecules and with one another. Finally, if the gas is removed entirely, there are no ions present, and the conduction consists solely of a transfer of electrons.

3–10 THE BEGINNINGS OF A CHANGE OF OUTLOOK

In these first three chapters, we have described a number of physical phenomena, mainly in terms of the concepts prevalent at the end of the nineteenth century. These concepts were admirably suited to explaining physics as it was then understood. In fact, the concepts and their accompanying theories and models had been developed just for this purpose. The revolution which caused twentieth century physics to differ radically from nineteenth century physics, was prompted by new discoveries which had to be accompanied by new concepts and a radical change in outlook. Chief among the new concepts, and forcing a change in outlook, was the concept of quantization in all branches of science.

The guiding idea in quantization is that phenomena in science are not to be considered as proceeding smoothly, but as proceeding in steps. In most cases the steps are so small as to give the impression of smoothness, but close examination shows that the steps exist. For example, microscopic examination has shown that living matter is not infinitely divisible, but is composed of building blocks called cells. Moreover, matter in general is not infinitely divisible, but is composed of discrete entities called atoms. Cells and atoms, then, may be called the quanta (singular; quantum) of matter.

In the remaining four chapters of this book we will outline the experimental evidence which led to the conclusion that there is a quantum of electricity, and that there are quanta of radiant energy.

3–11 PROBLEMS

1. The P.D. of a storage cell of the type used in automobile electrical systems is approximately 2 volts. Most modern automobiles use a 12-volt battery. How are the cells in the battery connected? How many cells are there in the battery?

2. If the P.D. of a battery is 90 volts, each coulomb of charge supplied by the battery has 90 joules of electric energy. If the battery is connected in a circuit, charge is transferred through the circuit. What becomes of the energy possessed by the charge?

3. In 1 hour, 900 coulombs of charge are transferred through a circuit. Calculate the current in the circuit.

4. How long must a current of 0.5 amperes flow in a circuit in order to transfer 45 coulombs of charge?

5. A current of 5 amperes flows for 2 minutes. Calculate the quantity of charge transferred (*a*) in coulombs, and (*b*) in elementary charges.

6. Assume that, in a silver voltameter, a current of 1 ampere deposits silver at the rate of 1.12×10^{-3} gm/sec. (*a*) How much charge is transferred along with each 1.12×10^{-3} gm of silver? (*b*) If silver is deposited at the rate of 2.80×10^{-3} gm/sec, what is the current? (*c*) How long must a current of 3 amperes flow to deposit 1.01 gm of silver?

7. The current in a resistor is 3.0 amperes, and the P.D. between the ends of the resistor is 40 volts. (*a*) How much electric energy is transformed by the resistor in 10 minutes? (*b*) What is the power of the resistor? (*c*) What becomes of the energy transformed by the resistor?

8. An electric light bulb is marked: 120 volts, 60 watts. (*a*) What is the current in the filament (*i*) when it is used in a 120-volt circuit, (*ii*) when it is used in a 240-volt circuit? (*b*) What is its power in the 240-volt circuit? (*c*) Account for your answer to (*b*) in terms of the relationship $P = VI$. (*d*) How much electric energy does the bulb transform in 1 hour in the 120-volt circuit? (*e*) What becomes of this energy?

9. The filament of a certain radio tube has a potential difference across it of 6.3 volts, when the current in the filament is 0.34 amps. Calculate the rate, in calories per sec, at which heat is being dissipated by the filament. (The mechanical equivalent of heat is 4.19 joules per calorie.)

10. The accelerating voltage in a TV picture tube is 20,000 volts. Calculate the work done on an electron which is accelerated by this voltage, assuming that the charge on the electron is 1.60×10^{-19} coulombs. If 3.0×10^{14} electrons are accelerated each second, calculate the power necessary.

11. An electric motor, operating in a 120-volt circuit, draws 4.0 amperes of current. It raises a 15-kg mass at a rate of 0.60 m/sec. (*a*) Calculate the rate at which the motor uses electric energy. (*b*) Calculate the rate at which the gravitational potential energy of the 15-kg mass increases. (*c*) Calculate the quantity of charge passing through the motor in 40 sec.

ulomb of charge supplied by the
If the battery is connected in a
e circuit. What becomes of the

sferred through a circuit. Calcu-

w in a circuit in order to transfer

utes. Calculate the quantity of
) in elementary charges.

rent of 1 ampere deposits silver
How much charge is transferred
? (*b*) If silver is deposited at the
rrent? (*c*) How long must a cur-
of silver?

nd the P.D. between the ends of
ric energy is transformed by the
power of the resistor? (*c*) What
resistor?

lts, 60 watts. (*a*) What is the
n a 120-volt circuit, (*ii*) when it
s power in the 240-volt circuit?
ms of the relationship $P = VI$.
o transform in 1 hour in the 120-
rgy?

potential difference across it of
nt is 0.34 amps. Calculate the
eing dissipated by the filament.
) joules per calorie.)

ube is 20,000 volts. Calculate the
rated by this voltage, assuming
$\times 10^{-19}$ coulombs. If 3.0×10^{14}
ulate the power necessary.

t circuit, draws 4.0 amperes of
f 0.60 m/sec. (*a*) Calculate the
. (*b*) Calculate the rate at which
-kg mass increases. (*c*) Calculate
e motor in 40 sec.

3–12 SUMMARY

Batteries and generators are devices for separating charges. The charges are exactly the same as those produced by rubbing objects together. The P.D. of a battery is the energy given by the battery to each unit of charge.

Current is the rate of flow of charge.

$$I = \frac{Q}{t}$$

1 ampere = 1 coulomb/sec

The function of a resistor in an electric circuit is to transform electric energy to some other form. The energy transformed by a resistor is given by the formula $E = VIt$, and the power of a resistor by the formula $P = VI$.

Conduction in solids is accomplished by electron transfer; conduction in liquids is accomplished by the transfer of positive and negative ions; conduction in gases is accomplished by the transfer of positive ions, negative ions, and electrons; conduction in a vacuum is accomplished by electron transfer.

Chapter 4

The Elementary Charge

4–1 INTRODUCTION

In Chapter 2 we discussed certain electrical concepts, units and measurements. We did not place any limits on the magnitudes of the forces, charges or potentials which we were considering but we likely assumed that they were of the order of magnitude of those associated with batteries. The electrical balance we discussed in Section 2–15 is a large one whose field is produced by a large number of cells in series. Now we turn to a consideration of very small charges, particularly the charge which we have already taken the liberty of calling the elementary charge.

4–2 CATHODE RAYS

The existence of atoms—small indivisible building blocks of matter—had been postulated since the days of ancient Greece. By 1890 experiments in chemistry had confirmed the facts that atoms existed and combined with one another to form molecules. Since 1890 many experiments have been performed which confirmed the existence of still smaller (sub-atomic) particles. The first of these particles to be discovered was the electron.

Some of the early evidence for the existence of the electron was obtained from experiments concerning conduction in gases at low pressures. Both Faraday and Coulomb, working independently, began the study of conduction in air at pressures lower than atmospheric. Other research physicists soon joined in the study, and in a period of about forty years (1855-1895) their investigations resulted in the discovery of the electron, the production of X-rays, and the information which is the basis for the amazingly successful electron theory of atomic structure.

The laboratory apparatus now used to illustrate the early studies of Faraday, Coulomb, and Sir William Crookes (1832-1919) is illustrated in Figure 4.1. It is

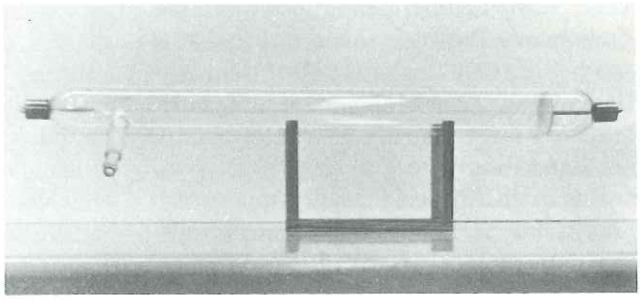


Fig. 4.1. Laboratory form of Crookes' tube.

called a Crookes' tube and consists of a hard glass tube about 15 inches long with one electrode at each end. A short side tube makes it possible to attach an air pump by means of which the air or gas pressure in the main tube can be altered. The electrodes, which may be aluminum, are usually a solid rod and a plane or concave disc. The rod electrode is connected to the positive terminal of a high-voltage induction coil; the disc electrode is connected to the negative terminal of the coil. Thus the rod electrode becomes the anode and the disc the cathode. The terminals of the induction coil should be equipped with needle points, their tips about an inch apart. The side tube is connected to a vacuum pump to which is attached a vacuum gauge.

If the coil is set in operation while the pressure within the tube is atmospheric, a violent discharge occurs between the needle points of the coil. If now the vacuum pump is set in operation and the gas pressure within the tube is steadily lowered, and if the room is darkened, a startlingly beautiful phenomenon is observed. When the pressure reaches about 1 cm of mercury, the discharge between the coil terminals stops. A purple glow appears in the tube on the surface of the cathode and a single sinuous purple streamer extends from the cathode to the anode. This streamer is neither straight nor steady. As the pressure is slowly lowered additional streamers appear until, at a pressure of $\frac{1}{2}$ cm of mercury, the light within the tube becomes diffused and the whole interior becomes luminous. When the pressure is further reduced to about $\frac{1}{2}$ mm of mercury a highly unusual pattern is seen (Fig. 4.2). A thin luminous layer, called the cathode glow, appears on the surface of the cathode. Adjacent to this cathode glow is a sharply defined, dark area which is called Crookes' dark space. Adjacent to this dark space is a diffused luminous area, the negative glow. Beyond this, towards the anode, is another

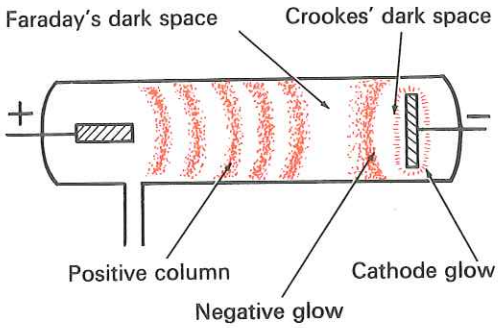


Fig. 4.2. Pattern caused by electrical discharge in a Crookes' tube when the air pressure is 0.5 mm of mercury.

dark, poorly defined area called Faraday's dark space. Between this space and the anode is a luminous area called the positive column. This column is not uniformly luminous but is usually alternately luminous and dark.

It is doubtful that Faraday, due to the limitations of his vacuum pumps, saw the pattern just described. Sir William Crookes continued the study with improved pumps, and as the pressure approached 1/100 mm of mercury he found that Crookes' dark space increased in length until it filled the whole tube. Coincident with this observation he noted that the glass of the tube, particularly in the neighbourhood of the cathode, fluoresced with a greenish colour. The German physicist, Julius Plücker (1801-1868), suggested that this fluorescence of the glass was due to some kind of ray being sent out by the nearby cathode; these rays had sufficient energy, when they struck the glass, to cause fluorescence and heat. Plücker called them cathode rays.

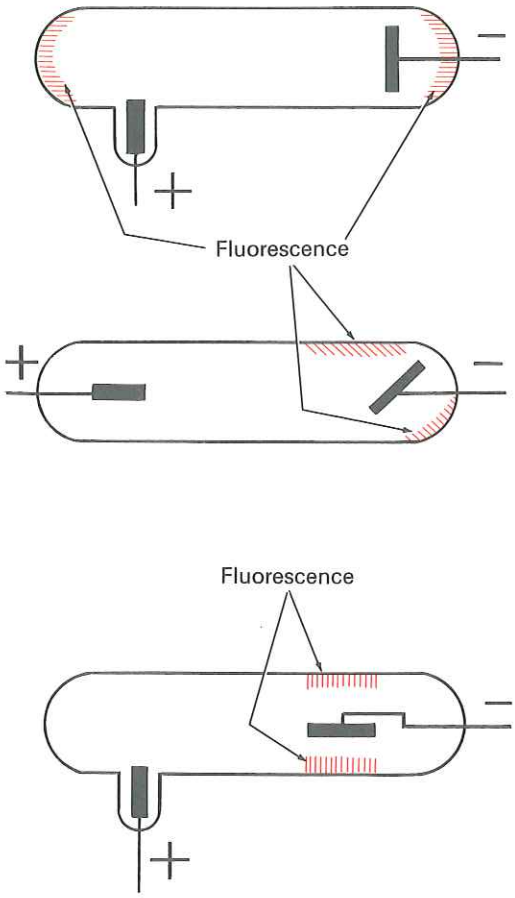


Fig. 4.3. The fluorescent areas of the glass are in directions perpendicular to the surfaces of the cathodes.

4–3 PROPERTIES OF CATHODE RAYS

Intensive investigation soon revealed additional properties of the cathode rays. It was found that the position of the areas of the glass showing the greatest fluorescence depended on the shape and position of the cathode, and that the position or form of the anode was immaterial. A study of Figure 4.3 which shows the fluorescent areas for different cathodes, indicates that the rays are emitted in a direction normal to the surface of the cathode. If a concave cathode is used (Fig. 4.4), and if a piece of tin

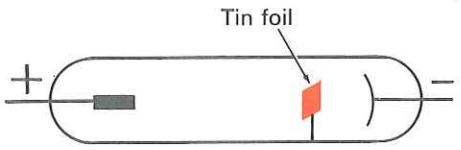
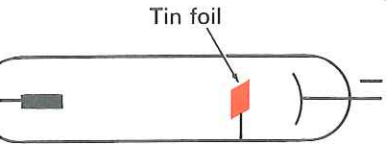


Fig. 4.4. Tube designed to demonstrate the heating effect of cathode rays.

es continued the study with im-
pumps, and as the pressure ap-
ed 1/100 mm of mercury he found
Crookes' dark space increased in
until it filled the whole tube.
dent with this observation he noted
ne glass of the tube, particularly
neighbourhood of the cathode,
ced with a greenish colour. The
n physicist, Julius Plücker (1801-
suggested that this fluorescence of
ass was due to some kind of ray
sent out by the nearby cathode;
rays had sufficient energy, when
ruck the glass, to cause fluorescence
eat. Plücker called them cathode

PROPERTIES OF CATHODE RAYS

nsive investigation soon revealed
nal properties of the cathode rays.
found that the position of the
of the glass showing the greatest
cence depended on the shape and
n of the cathode, and that the
n or form of the anode was im-
al. A study of Figure 4.3 which
the fluorescent areas for different
es, indicates that the rays are
d in a direction normal to the sur-
the cathode. If a concave cathode
(Fig. 4.4), and if a piece of tin



. Tube designed to demonstrate the heat-
t of cathode rays.

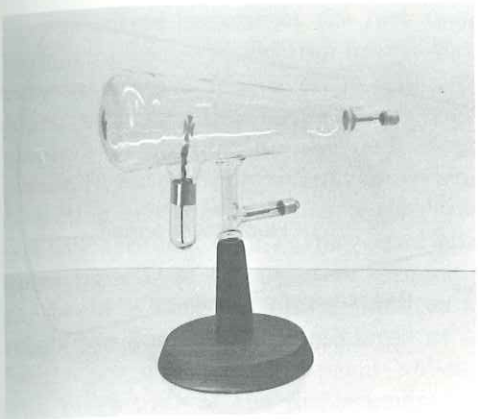


Fig. 4.5. A Crookes' tube used to demonstrate rectilinear propagation of cathode rays and their deflection by magnetic and electric fields.

foil is secured near the focus of the cathode, it is found that the foil becomes incandescent when the tube is operated. This discovery led Crookes to conclude that the rays are a stream of particles and that the heat is produced in the tin foil by the transformation of the kinetic energy of these particles.

Continuing the study of these rays, a tube as shown in Figure 4.5 was built. The cathode was a flat disc mounted at right angles to the central axis of the tube. A small hinged Maltese cross of sheet metal was mounted in the tube so that it could be placed in the path of the beam. Since a sharply defined shadow of the metal cross appeared on the end of the tube it was concluded that the rays travel in straight lines. It was discovered too that, if a permanent magnet was brought close to the tube, the shadow moved in a plane at right angles to the axis of the magnet, an action characteristic of a conductor in which charge is flowing. Also, a metal plate bearing an electrostatic charge deflected the beam towards the plate when it was positively charged and away from the plate when it was negatively charged. From these observations it was concluded that a cathode ray is a stream of negatively charged particles. In 1890 Dr. Johnston Stoney named these particles electrons.

4–4 THE CHARGE-MASS RATIO OF ELECTRONS

In the last decade of the nineteenth century, Sir J. J. Thomson (1856-1940), one of the greatest of English scientists, joined in the study of cathode rays. He fashioned a tube as shown in Figure 4.6. The anode consisted of a cylinder with a hole along its axis so that the electron stream would be a narrow one travelling along the central axis of the tube. The metal plates, A and B, each with a connecting wire, made it possible to place an electrostatic charge inside the tube close to the ray. Thomson hoped to determine the mass of the particles in the cathode ray, and the magnitude of their electric charge. With plates A and B uncharged, Thomson found that the tube face (the end of the tube) fluoresced at P. With plate A positive, the face fluoresced at P_1, and with plate A negative it

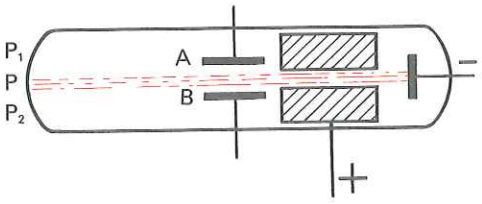


Fig. 4.6. A Crookes' tube of a type used by J. J. Thomson.

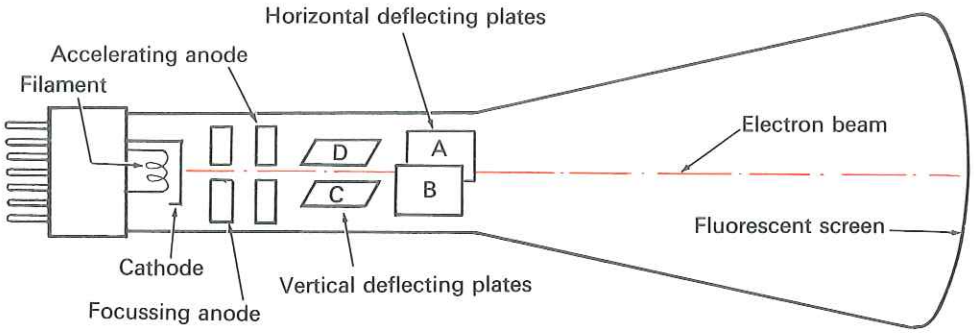


Fig. 4.7. The cathode ray tube in a cathode ray oscilloscope.

fluoresced at P_2, thus confirming Crookes' discovery that the particles bore a negative charge. Then Thomson applied magnetic and electric fields simultaneously, in such a way that the electric field tended to deflect the beam upward, and the magnetic field tended to deflect the beam downward. He adjusted the magnitudes of the two fields until their effects cancelled and the beam was undeflected. From his observations, he concluded that the electron had mass, and he was able to determine that the ratio of the charge to the mass was of the order of 10^{11} coulombs per kilogram. He was unable, however, to determine either the charge or the mass separately.

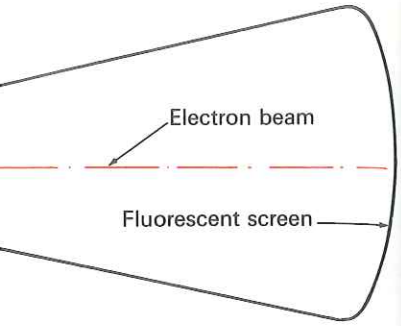
Since all of the observed properties of cathode rays are independent of the kind of metal in either the cathode or the anode, electrons seem to be constituent parts of all atoms.

4–5 THE CATHODE-RAY OSCILLOSCOPE

A Crookes' tube, in the form used by Thomson for his investigations (Fig. 4.6), constitutes a device for firing a stream of high-speed electrons at the end surface of a glass tube. The whole unit is called a cathode-ray tube. The fluorescence produced when the electrons strike the tube face is greatly increased if the inner surface of the tube face is coated with zinc orthosilicate, zinc sulphide, or calcium tungstate. Moreover, the rate of emission of electrons from the cathode is greatly increased if the cathode is heated by a filament. The filament is simply a small electric heating coil. Emission of electrons by a heated cathode is called thermionic emission.

The cathode and the associated anode in such a tube constitute an electron gun. This gun has many uses. It is the basic part of the cathode-ray tubes employed in radar sets, in television cameras and receivers, in the electron microscope, and in the cathode-ray oscilloscope.

In the cathode-ray oscilloscope (Fig. 4.7), the electron gun fires a cathode ray between horizontal and vertical deflecting plates. The path of the cathode ray can be changed quite readily by charges applied to the plates. The oscilloscope is



node ray oscilloscope.

itutes a device for firing a stream
gh-speed electrons at the end surface
glass tube. The whole unit is called
node-ray tube. The fluorescence pro-
d when the electrons strike the tube
s greatly increased if the inner sur-
of the tube face is coated with zinc
silicate, zinc sulphide, or calcium
state. Moreover, the rate of emission
ctrons from the cathode is greatly
ased if the cathode is heated by a
ent. The filament is simply a small
ic heating coil. Emission of electrons
heated cathode is called thermionic
ion.

e cathode and the associated anode
ch a tube constitute an electron gun.
gun has many uses. It is the basic
of the cathode-ray tubes employed
dar sets, in television cameras and
vers, in the electron microscope, and
e cathode-ray oscilloscope.

the cathode-ray oscilloscope (Fig.
the electron gun fires a cathode ray
een horizontal and vertical deflecting
s. The path of the cathode ray can
anged quite readily by charges ap-
to the plates. The oscilloscope is

very sensitive because of the fact that its moving part—an electron beam—has very little mass. Moreover, it responds practically instantaneously to charges or changes in charges on the deflecting plates. If, for example, the upper vertical deflecting plate D is connected to the positive terminal of a battery, and the lower plate C to the negative terminal, the beam is deflected upward and as a result the fluorescence is produced at a higher point on the fluorescent screen. The magnitude of the displacement of the spot depends on the charge on the plates. Thus the oscilloscope may be used to compare charges.

Perhaps the most important use of the oscilloscope is to show how charges change rapidly as time goes on. A circuit in the oscilloscope is used to charge the horizontal deflecting plates, A and B, relatively slowly, then to discharge them quickly. This charging and discharging process is repeated continuously and as a result the fluorescent spot "sweeps" horizontally across the screen and then returns quickly to its starting point. As it moves, changes in the charges applied to the vertical deflecting plates cause the spot to move up or down. Because of persistence of vision on the retina of the eye, and persistence of fluorescence of the spots on the screen, a pattern is traced on the screen (Fig. 4.8). This pattern is in effect a graph of the changes in charge on the vertical deflecting plates against time.

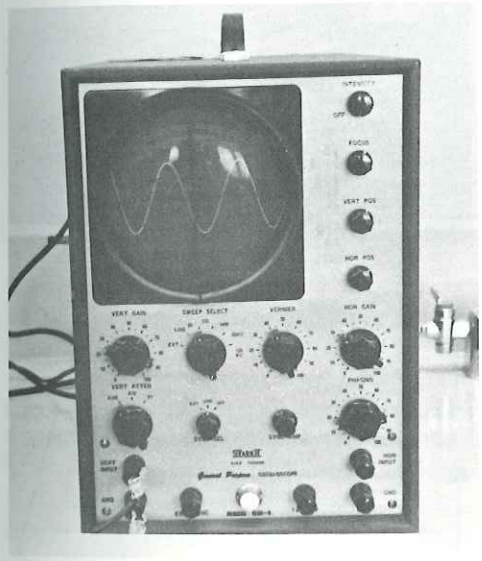


Fig. 4.8. Pattern produced on the screen of a cathode ray oscilloscope when the vertical deflecting plates are connected to 60 cycle alternating current.

4–6 X-RAYS

In November, 1895, Professor Wilhelm Konrad Röntgen (1845-1923), in his laboratory in Germany, was experimenting with a Crookes' tube. With the tube operating at very low gas pressure, he noted that certain minerals in the room fluoresced even when the tube was covered with black paper. To his amazement he discovered that the minerals would fluoresce, as well, in a room adjacent to that in which the tube was operating, even when the door to that room was closed. He concluded that a new and very penetrating kind of radiation was issuing from the tube. Using an algebraic symbol for the unknown, he called this radiation X-rays.

An X-ray tube with a heated cathode is shown in Figure 4.9. Electrons emitted by the cathode strike the tungsten target, which also serves as the anode of the tube. The bombardment of the target by high speed electrons causes X-rays to be radiated from the target.

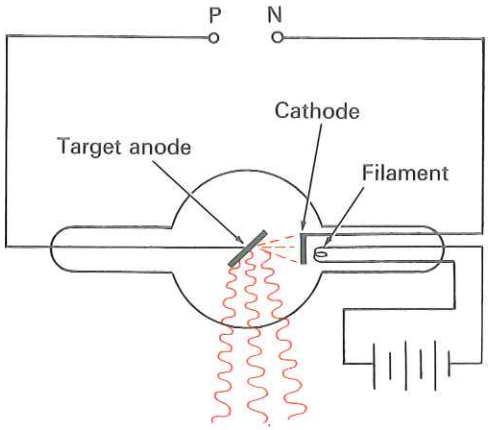


Fig. 4.9. An X-ray tube. *P* and *N* are the positive and negative terminals, respectively, of a DC source.

X-rays possess many of the same properties as does light. Unlike light, they penetrate very well materials composed of elements having low atomic weight, and therefore are commonly used in examining the bones of the body for fractures, and the internal organs of the body for disease or growths.

X-rays differ from light also in their ability to ionize gases through which they pass. This ionizing action may be demonstrated with the apparatus shown in Figure 4.10. A charged electroscope is placed near an X-ray tube, and the tube is then put in operation. The leaves of the electroscope fall, indicating that the X-rays ionize molecules in the air, thus increasing the conductivity of the air.

4–7 MILLIKAN'S EXPERIMENT

While the above properties of the cathode ray were known by 1895, it was another 15 years before Robert A. Millikan (1868-1953), at the University of Chicago, determined the magnitude of the charge on the electron. His experiments also established that the negative charge borne by an electron cannot be divided into smaller charges; the electron carries the smallest possible negative charge.

Figure 4.11(*a*) is a photograph of a modern version of Millikan's apparatus. The basic parts of the apparatus are two horizontal metal plates. They are visible in the central portion of Figure 4.11(*a*), and are shown in more detail in Figure 4.11(*b*). Figure 4.12 shows these plates and the electrical connections to them. The plates, *P* and *N* in Figure 4.12, were charged as shown, and small oil droplets were sprayed into the region between the plates, where they were observed through a microscope. Some of the oil drops became ionized during the spraying process; those which became negatively charged were attracted upward by the electric field. The potential difference between *P* and *N* was adjusted to cause some of these oil drops to remain stationary. Under these circumstances the electric force is equal to the gravitational force. The

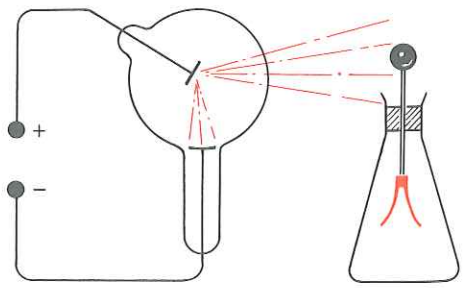
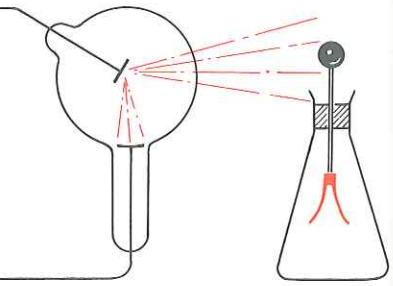


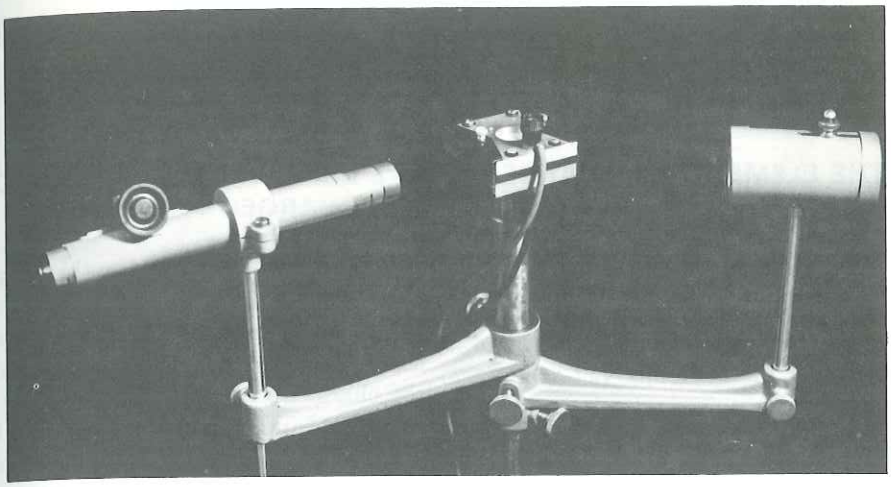
Fig. 4.10. An electroscope detects X-rays because the X-rays ionize the surrounding air.

-1953), at the University of Chicago, mined the magnitude of the charge e electron. His experiments also ished that the negative charge borne n electron cannot be divided into er charges; the electron carries the est possible negative charge.

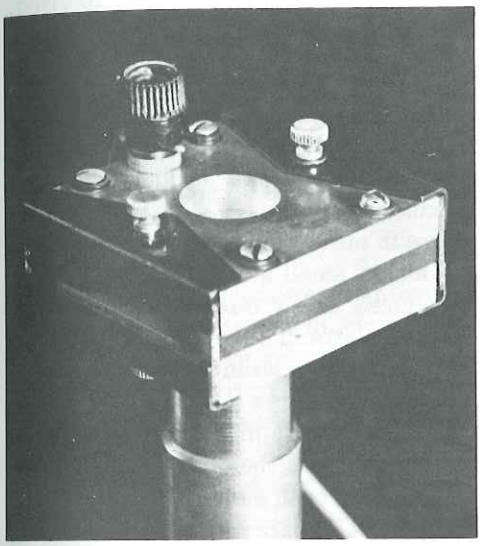
ure 4.11(*a*) is a photograph of a rn version of Millikan's apparatus. asic parts of the apparatus are two ntal metal plates. They are visible e central portion of Figure 4.11(*a*), re shown in more detail in Figure). Figure 4.12 shows these plates he electrical connections to them. lates, P and N in Figure 4.12, were ed as shown, and small oil droplets sprayed into the region between the , where they were observed through oscope. Some of the oil drops became d during the spraying process; those became negatively charged were ted upward by the electric field. otential difference between P and s adjusted to cause some of these ops to remain stationary. Under circumstances the electric force is to the gravitational force. The



0. An electroscope detects X-rays because ays ionize the surrounding air.



(a)



(b)

Physics Department, University of Western Ontario

Fig. 4.11. *(a)* A modern version of Millikan's apparatus. *(b)* A close-up view of the charged plates.

weight of an oil drop was calculated from observations of its rate of fall through the air when the P.D. between P and N was zero. Thus the electric force was known since it was equal to the gravitational force. The formula

$$QV = Fs$$

was then applied just as we did in Chapter 2; V, F and s are known and Q can be calculated.

The results of a great many experiments showed that, within the limits of experimental error, Q was always some integral multiple of 1.60×10^{-19} coulombs. Millikan, assuming that the negatively charged drop had an integral number of surplus electrons, concluded that the charge on the electron was an indivisible and fundamental quantity of charge equal to 1.60×10^{-19} coulombs. We refer to it now as the elementary charge.

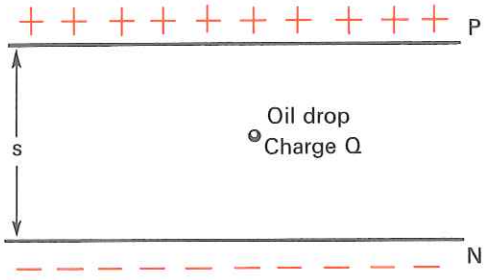


Fig. 4.12. The basic parts of the apparatus used by Millikan.

4–8 THE ELEMENTARY CHARGE

Electricity, then, comes in small bundles just as matter does; the charge on the electron is the "atom" of electricity. For many purposes in physics the charge on the electron—the elementary charge—is replacing the coulomb. Let us note some of the new units and relationships which result.

(*a*) Since the elementary charge is equal to 1.60×10^{-19} coulombs, then 1 coulomb $= \dfrac{1}{1.60 \times 10^{-19}}$ elementary charges $= 6.25 \times 10^{18}$ elementary charges. Therefore charges expressed in coulombs may be expressed in elementary charges if we multiply by 6.25×10^{18}.

(*b*) In the Coulomb's law formula

$$F = k\frac{qQ}{r^2}$$

and $k = 9.0 \times 10^9 \dfrac{\text{newton-(metre)}^2}{\text{(coulomb)}^2}$

If we change the coulombs to elementary charges, then

$$k = 2.3 \times 10^{-28} \frac{\text{newton-(metre)}^2}{\text{(elem. charge)}^2}$$

(*c*) A P.D. of 1 volt has been defined as 1 joule/coulomb. In terms of elementary charges, 1 volt $= 1.60 \times 10^{-19}$ joules/elementary charge.

(*d*) Reversing the definition of the volt,

1 coulomb-volt = 1 joule

Then 1 elementary charge-volt, more commonly called 1 electron-volt (abbreviated; ev), is a unit of energy as is the joule, and is equal to 1.60×10^{-19} joules. The electron-volt is widely used in atomic physics as an energy unit.

4–9 MEASURING THE MASS OF CHARGED PARTICLES

We mentioned earlier in this chapter that Sir J. J. Thomson was able to calculate the charge-mass ratio for the electron. Then, when Millikan determined the charge on the electron, the mass was calculated and found to be about 9×10^{-31} kg. Since the time of Millikan, other methods for independent measurement of the mass of the electron (and other charged particles) have been devised. One of these is described in the following paragraphs.

The basic part of the apparatus (Fig. 4.13) is essentially a diode-tube containing a cathode N and an anode P in an evacuated container. Let us consider first its use in measuring the mass of a hydrogen ion. A small amount of hydrogen is introduced into a box at the left of the charged plates. This hydrogen is ionized by a spark from an induction coil, and a few of the positive ions drift through a hole in the anode into the area between the plates. Here they are accelerated to the cathode, each acquiring a final speed v and kinetic energy $\frac{1}{2}mv^2$.

A hole drilled in the cathode allows some of the hydrogen ions to pass through the cathode into a tube about 0.5 metres long (Fig. 4.13). In the vacuum in this tube the ions travel at constant speed. We can calculate this speed if we use electronic apparatus (a cathode-ray oscilloscope, for example), to measure the time required for the ions to travel this 0.5 metres. Once the time of flight of the ions is known, their speed can be calculated. It remains then for us to calculate the mass of an ion.

9 MEASURING THE MASS OF CHARGED PARTICLES

Ve mentioned earlier in this chapter Sir J. J. Thomson was able to cal- te the charge-mass ratio for the elec- . Then, when Millikan determined charge on the electron, the mass was ulated and found to be about 9×10^{-31} Since the time of Millikan, other nods for independent measurement he mass of the electron (and other rged particles) have been devised. of these is described in the following graphs.

ne basic part of the apparatus (Fig.) is essentially a diode-tube contain- a cathode N and an anode P in an uated container. Let us consider first se in measuring the mass of a hydro- ion. A small amount of hydrogen is oduced into a box at the left of the ged plates. This hydrogen is ionized spark from an induction coil, and a of the positive ions drift through a in the anode into the area between plates. Here they are accelerated to athode, each acquiring a final speed d kinetic energy $\frac{1}{2} mv^2$.

hole drilled in the cathode allows e of the hydrogen ions to pass through athode into a tube about 0.5 metres (Fig. 4.13). In the vacuum in this the ions travel at constant speed. can calculate this speed if we use ronic apparatus (a cathode-ray oscil- pe, for example), to measure the time ired for the ions to travel this 0.5 es. Once the time of flight of the ions own, their speed can be calculated. mains then for us to calculate the of an ion.

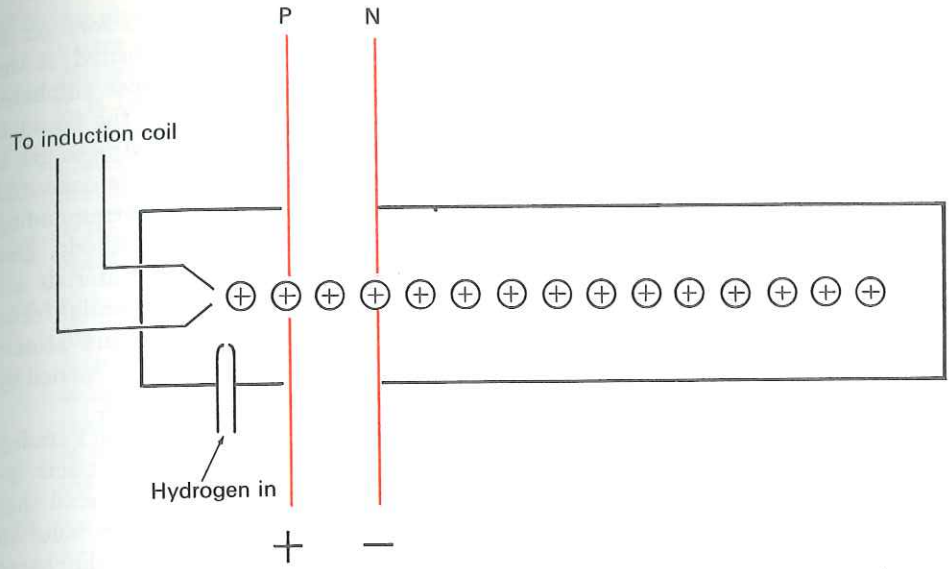


Fig. 4.13. Apparatus used to determine the speed, and hence the mass, of hydrogen ions and other charged particles.

As the ions are accelerated by the electric field between the anode and the cathode, they lose an amount of electric potential energy given by the formula $E = QV$ and gain an equal amount of kinetic energy .Thus

$$QV = \tfrac{1}{2} mv^2$$

where Q is the charge on each ion, V is the P.D. between the anode and cathode, m is the mass of each ion and v is the speed of each ion at the cathode, assuming that it was at rest at the anode.

We may not know the magnitude of the charge carried by a hydrogen ion, but let us assume that it is the elementary charge, equal in magnitude to the charge on an electron. As we will see later, this is a reasonable assumption. Experimental results such as the following are typical.

$Q = 1.60 \times 10^{-19}$ coulomb
$V = 100$ volts
$v = 1.36 \times 10^{5}$ m/sec

$$\therefore 1.60 \times 10^{-19} \times 100 = \tfrac{1}{2} \times m \times (1.36 \times 10^{5})^2$$
$$m = 1.7 \times 10^{-27} \text{ kg.}$$

Calculations from experiments in chemistry indicate that the mass of a hydrogen atom is 1.7×10^{-27} kg and we would expect a hydrogen ion to have approximately the same mass as the hydrogen atom. Apparently our assumption that the charge on the ion was one positive elementary charge, was correct.

In this and other experiments a hydrogen ion is always found to have only one elementary charge and its mass is only very slightly less than that of a hydrogen atom. The hydrogen ion is called a proton. Many experiments indicate that the proton, like the electron, is a "building block" for matter and that it is present in all atoms. Even the results of the one experiment which we have just described seem to indicate that a hydrogen atom consists

of a proton and an electron, the proton being much more massive than the electron.

The mass of the electron may be determined with this same apparatus if the oscilloscope is "fast" enough. The leads from the battery to the accelerating plates are reversed, and electrons are accelerated from what is now the cathode, to the anode. (The electrons are produced when the hydrogen is ionized, or may simply be obtained from a heated cathode in a vacuum tube.) The speed attained by the electrons is considerably higher than that of the protons under similar circumstances. The mass of the electron is now generally given as 9.11×10^{-31} kg, about $\frac{1}{2000}$ of that of the proton.

4–10 ELECTROLYSIS AND IONIC CHARGES

When we discussed the measurement of electric current, we described the special electrolytic cell called the silver voltameter (Fig. 3.4). The same type of cell may be used to investigate the conductivity of many liquids. If the electrodes are of platinum or carbon, so that they do not react with the liquid in the cell, the material deposited or liberated at the cathode depends on the nature of the liquid. If the liquid is a solution of silver nitrate, silver is deposited; if the liquid is a solution of copper sulphate, copper is deposited; and if the liquid is dilute hydrochloric acid, hydrogen gas is liberated. These materials are deposited or liberated because the corresponding ions are attracted to the cathode, give up their charges, and the materials are deposited or liberated in molecular form. If the necessary molecular and atomic weights are known, the charges carried by different ions can be compared.

The English scientist Michael Faraday investigated electroplating and electrolysis in detail. In 1833 he discovered that the charges carried by ions in solution were small integral multiples of the charge carried by the hydrogen ion. If we consider a hydrogen ion as being singly charged, then a silver ion is also singly charged, a copper ion doubly charged, an aluminum ion triply charged, etc. This was perhaps the first indication of the existence of an elementary charge. All of the charges carried by ions are now known to be multiples of the elementary charge determined by Millikan. In other words, there is only one elementary charge; ions have an integral surplus or deficit of electrons.

4–11 PROBLEMS

Where necessary, assume that
1 elementary charge $= 1.60 \times 10^{-19}$ coulombs
1 electron-volt $= 1.60 \times 10^{-19}$ joules
mass of electron $= 9.11 \times 10^{-31}$ kg
mass of proton $= 1.67 \times 10^{-27}$ kg
gravitational field strength $= 9.8$ newtons/kg

1. List the properties of cathode rays. Compare cathode rays and X-rays.
2. Summarize the experimental results which indicate that cathode rays (*a*) possess mass, (*b*) carry negative charge.

liquid. If the liquid is a solution of
er nitrate, silver is deposited; if the
id is a solution of copper sulphate,
per is deposited; and if the liquid is
te hydrochloric acid, hydrogen gas is
rated. These materials are deposited
iberated because the corresponding
are attracted to the cathode, give
heir charges, and the materials are
osited or liberated in molecular form.
he necessary molecular and atomic
hts are known, the charges carried by
rent ions can be compared.
he English scientist Michael Faraday
stigated electroplating and electroly-
n detail. In 1833 he discovered that
charges carried by ions in solution
small integral multiples of the charge
ed by the hydrogen ion. If we con-
r a hydrogen ion as being singly
ged, then a silver ion is also singly
ged, a copper ion doubly charged,
luminum ion triply charged, etc. This
perhaps the first indication of the
ence of an elementary charge. All of
harges carried by ions are now known
e multiples of the elementary charge
rmined by Millikan. In other words,
e is only one elementary charge; ions
an integral surplus or deficit of
rons.

$\times 10^{-19}$ coulombs
^{19} joules
$^{-31}$ kg
$^{-27}$ kg
9.8 newtons/kg

are cathode rays and X-rays.

hich indicate that cathode rays
e.

3. An electron is fired by an electron gun between the horizontal deflecting plates of a cathode ray tube. The initial speed of the electron is 3.0×10^8 cm/sec. A uniform force of 2.8×10^{-18} newtons acts on the electron when it is between the plates. The plates are 10 cm long. (*a*) Determine the amount of deflection which the electron undergoes as it passes between the plates. (*b*) Describe the path which the electron follows (*i*) between the plates, (*ii*) after emerging from between the plates.

4. A strong magnet is brought near an operating X-ray tube. The tube stops operating. Explain.

5. In a Millikan type experiment, the plates are horizontal and 2.5 cm apart. An oil droplet of mass 1.50×10^{-15} kg remains stationary between the plates when the P.D. is 460 volts, with the upper plate positive. (*a*) Calculate the magnitude of the electric field intensity between the plates. (*b*) Is the drop charged negatively or positively? (*c*) Calculate the magnitude of the charge on the drop. (*d*) How many surplus or missing electrons does the drop have?

6. An oil drop bearing a charge of 20 electrons and having a mass of 9.8×10^{-14} kg is suspended motionless by the electric field between two horizontal parallel plates 4.0 cm apart. (*a*) What is the P.D. between the plates? (*b*) What will happen to the drop when each of the following changes is made? (*i*) Five more elementary charges are placed on the drop. (*ii*) The mass of the drop becomes less because of the evaporation of some oil. (*iii*) The plates are moved closer together.

7. In a Millikan type experiment, a small plastic sphere carrying 2 negative elementary charges can be held motionless between two horizontal plates 3.0 mm apart by a P.D. of 270 volts. Calculate the mass of the sphere.

8. A conductor has a negative charge of 6.00×10^2 microcoulombs. How many surplus electrons does it have?

9. A pith ball carries a negative charge of 0.300 microcoulombs. How many surplus electrons does it have?

10. How many electrons must be removed from a neutral, insulated conductor in order to give the conductor a positive charge of 4.0×10^{-8} coulombs?

11. Calculate the quantity of charge on a conductor which has (*a*) 25.0×10^{12} surplus electrons, (*b*) 12.5×10^{14} too few electrons.

12. Express each of the following currents in elementary charges/sec: (*a*) 2.0 amperes, (*b*) 40 amperes, (*c*) 8 milliamperes.

13. Convert each of the following currents to amperes: (*a*) 6.25×10^{17} elementary charges/sec, (*b*) 12.5×10^{16} elementary charges/sec, (*c*) 5.0×10^{16} elementary charges/sec.

14. If 6.4×10^{17} electrons pass between the electrodes of a vacuum tube in 4 seconds, what is the average current?

15. If the current in a conductor is 0.25 amperes, (*a*) how many coulombs of charge, and (*b*) how many elementary charges pass through the conductor in 2 minutes?

16. Protons are accelerated in a cyclotron until the current is measured as 1.6×10^{-3} amperes. How many protons pass a point in the cyclotron each second?

17. Convert each of the following energies to electron volts: (*a*) 6.0×10^{-17} joules, (*b*) 8.0 joules, (*c*) 2.0×10^{-5} joules.

18. Express each of the following energies in joules: (*a*) 2.0 ev, (*b*) 5.0×10^{7} ev, (*c*) 6.0×10^{-2} ev.

19. Charged particles move between an anode A and a cathode C. Calculate the current between the electrodes in each of the following situations. (*a*) 2.0×10^{14} protons pass from A to C each second; (*b*) 2.0×10^{14} protons pass from A to C each second, and at the same time 2.0×10^{14} electrons pass from C to A each second.

20. A singly-charged ion is accelerated from rest through a potential difference of 100 volts. A "time of flight" measurement (Sect. 4–9) shows that the ion, after having been accelerated, travels a distance of 0.50 m in 4.0×10^{-5} sec. Calculate the mass of the ion.

21. By what factor would the "time of flight" in Problem 20 be changed if (*a*) the ion is doubly charged, (*b*) the P.D. is 75 volts?

22. What is the speed, in m/sec, of a proton which has 3.4×10^{-17} joules of kinetic energy?

23. The P.D. between two plates, A and B, is 3.0×10^{2} volts. An ion having one elementary charge starts from rest at A and arrives at B with a speed of 2.0×10^{5} m/sec. Calculate (*a*) the kinetic energy gained by the ion, (*b*) the mass of the ion.

24. Two parallel electrodes in a vacuum tube are maintained at a constant P.D. of 90 volts. A proton (hydrogen ion) starts from rest at the anode and is accelerated to the cathode. (*a*) How much kinetic energy, in joules, is imparted to the proton? (*b*) If the same tube were used to accelerate alpha particles (doubly charged helium ions) how much kinetic energy, in ev, would be imparted to each alpha particle?

25. Protons having negligible speed are released at the anode of an evacuated tube. The anode and cathode are parallel, and are 0.10 m apart. The P.D. between the plates is 45 volts. (*a*) Calculate (*i*) the work done on each proton as it moves from anode to cathode, (*ii*) the magnitude of the electric force. (*b*) What would the P.D. need to be if the speed of the protons arriving at the cathode is to be double that in (*a*)?

peres, (*a*) how many coulombs of
harges pass through the conductor

until the current is measured as
pass a point in the cyclotron each

o electron volts: (*a*) 6.0×10^{-17}
es.

oules: (*a*) 2.0 ev, (*b*) 5.0×10^{7} ev,

de A and a cathode C. Calculate
each of the following situations.
ach second; (*b*) 2.0×10^{14} protons
e same time 2.0×10^{14} electrons

rest through a potential difference
ement (Sect. 4–9) shows that the
a distance of 0.50 m in 4.0×10^{-5}

nt" in Problem 20 be changed if
). is 75 volts?

which has 3.4×10^{-17} joules of

$.0 \times 10^{2}$ volts. An ion having one
and arrives at B with a speed of
netic energy gained by the ion,

oe are maintained at a constant
starts from rest at the anode and
much kinetic energy, in joules, is
ube were used to accelerate alpha
ow much kinetic energy, in ev,
?

sed at the anode of an evacuated
and are 0.10 m apart. The P.D.
ulate (*i*) the work done on each
(*ii*) the magnitude of the electric
be if the speed of the protons
at in (*a*)?

26. Parallel conducting plates P and N in a vacuum tube are connected to a 100-volt battery. Positive ions are released at rest at the anode P. (*a*) Two positive ions have the same charge, but the mass of the second ion is double that of the first. What is the ratio of their speeds on arrival at N? (*b*) If a deuteron, consisting of one proton and one neutron, is accelerated from P to N, how much kinetic energy, in ev, does it acquire?

27. An electron is accelerated by a field of 1.82×10^{-26} newtons/elementary charge. Calculate its acceleration.

4–12 SUMMARY

Cathode rays possess negative charge, kinetic energy, and momentum. They are streams of electrons. They are used in many devices such as the cathode-ray oscilloscope and the X-ray tube. X-rays, as distinct from cathode rays, are a form of radiation.

Thomson determined the charge-mass ratio for electrons, and Millikan determined that 1 elementary charge equals 1.60×10^{-19} coulombs.

1 coulomb = 6.25×10^{18} elementary charges

1 volt = 1.60×10^{-19} joules/elementary charge

1 electron-volt = 1.60×10^{-19} joules

The constant in Coulomb's law = k = $2.3 \times 10^{-28} \frac{\text{newton-(metres)}^2}{\text{(elem. charge)}^2}$

The mass of a charged particle may be obtained from a "time of flight" measurement, with the aid of the relationship

$$QV = \tfrac{1}{2}mv^2$$

Chapter 5

The Nuclear Atom

5–1 INTRODUCTION

The discovery of electrons and protons, the fact that their charges were equal and opposite, and the measurement of their masses, led to the conclusion that all atoms were composed of these two particles. It was obvious that, since atoms are electrically neutral, the number of electrons was equal to the number of protons, but no evidence of the relative positions of the protons and electrons within the atom was available until 1909. Before 1909 scientists assumed that an atom consisted of a globule of positively charged fluid in which the electrons were embedded like raisins in a bun or seeds in a watermelon. The electrons were sometimes assumed to vibrate about average positions in the fluid, but the basic assumption was that the atom was uniformly dense throughout the space it occupied.

This model of the atom (the so-called Thomson atom) failed to account for several facts already known, notably the occurrence of line spectra. However, a better model seemed impossible with the information available at the time. Startling new information concerning the atom became available in the years 1909-1911, and it is with this information and later developments that this chapter is concerned. Before discussing these events, however, we should review the data concerning the atom, which were available from chemical experiments and theory.

5–2 EARLY INFORMATION CONCERNING THE ATOM

The concept of an element as a pure substance which cannot be broken down into, or synthesized from, other pure substances was pretty well established by the beginning of the nineteenth century. About one-third of the elements we now know had been identified as elements, and, in addition, many substances which

we now know to be compounds were then considered to be elements.

About the beginning of the 19th century the English scientist John Dalton (1766-1844) proposed a chemical atomic theory to explain the existence and chemical properties of elements. Dalton's basic assumptions were essentially as follows. (1) Matter is composed of indivisible atoms. (2) The atoms of different elements differ from one another, but all of the atoms of any one element are alike. (3) Atoms are not created nor destroyed during a chemical reaction; they are just rearranged. (4) A molecule of a compound contains a definite number of atoms of each of the elements which combine to form the compound.

The fourth assumption above explained very nicely two chemical laws which had been developed from experiment: the laws of definite and multiple proportions. When two elements unite to form one compound, they do so in definite proportions by weight; when they unite to form two or more compounds the weights of one element which combine with a given weight of the other element are in the ratio of small whole numbers. This is just what we would expect if elements take part in reactions in indivisible atomic units. These laws and Dalton's atomic theory led eventually to the calculation of atomic and molecular weights.

The French scientist, Joseph Gay-Lussac (1778-1850) noted that, when gases react under conditions of constant temperature and pressure, the volumes of the reactants and products are in the ratio of small whole numbers. This led the Italian physicist Amedeo Avogadro (1776-1856) to formulate the following hypothesis: Equal volumes of all gases, at the same temperature and pressure, contain equal numbers of molecules. The number of molecules in a given volume was eventually calculated by methods which we shall discuss very briefly later in this section.

The weights of atoms and molecules are small, and therefore we usually discuss, not their actual weights, but their relative weights. The basis of comparison is the atom of an isotope of carbon, $^{12}_{6}C$; the weight of this atom is arbitrarily taken as 12 exactly. We say that the atomic weight of this isotope of carbon is 12. The weight of any other atom or molecule, relative to this standard, is called the atomic or molecular weight of that element or compound. For example, since the weight of a nitrogen atom is $\frac{14}{12}$ times the weight of a carbon atom, the atomic weight of nitrogen is 14. Since the nitrogen molecule is diatomic, the molecular weight of nitrogen is 28. The gram-molecular weight (G.M.W.) of a substance is its molecular weight expressed in grams; the G.M.W. of nitrogen is therefore 28 gm.

The concept of gram-molecular weight has proven so useful in chemistry that gram-molecular weight is given a special name: the mole. Thus, when we speak of 3 moles of nitrogen, we mean 84 grams of nitrogen. All elements and compounds, whether in the solid, liquid or gaseous state, have the same number of molecules per mole. This number is called Avogadro's number.

Avogadro's number may be calculated if the mass of a molecule and the G.M.W. of the substance are known. Alternatively the charge transferred in an electrolytic cell per mole of an element liberated at an electrode, may be measured. If the charge on the ion of that element is known,

the number of ions per mole may be calculated and Avogadro's number determined. As a result of these and other methods, Avogadro's number has been determined to be 6.025×10^{23}. That is, a mole of any pure substance contains 6.025×10^{23} molecules.

One use to which a knowledge of Avogadro's number was put was to determine the approximate radius of an atom. For example, the volume of a mole of a gaseous element is found to be 22.4 litres at standard temperature and pressure; this is the volume occupied by 6.025×10^{23} molecules. The volume occupied by one molecule can then be calculated; the volume occupied by one of the atoms in the molecule guessed at, and the radius calculated. All atoms seem to have a radius of an order of magnitude of 10^{-8} cm, i.e., 10^{-10} metres.

One more fact concerning atoms must be mentioned before we return to the discussion of the structure of the atom itself. Dmitri Mendeléeff (1834-1907) organized the elements into what is now called the periodic table of the elements. To each position in the periodic table a number is assigned. This number is called the atomic number (symbol Z) of the element occupying this position, and we shall see that it has a direct relationship to the structure of the atom.

5–3 HOW TO EXPLORE ATOMS

Physicists usually explore atoms by "firing" other small particles at the atoms and then observing what happens to the particles after they interact with the atoms. This procedure requires first of all a source of particles of relatively high speed, and some means of detecting and counting these particles after they have been "fired" at the atoms. Alpha particles were the first atomic "probes" used; they are produced by radioactive disintegration.

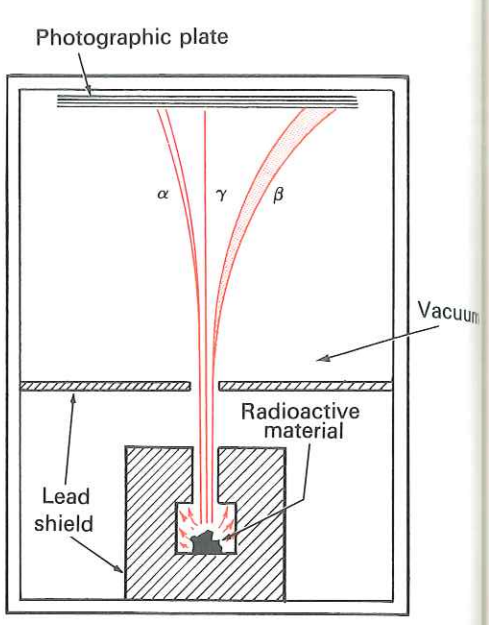
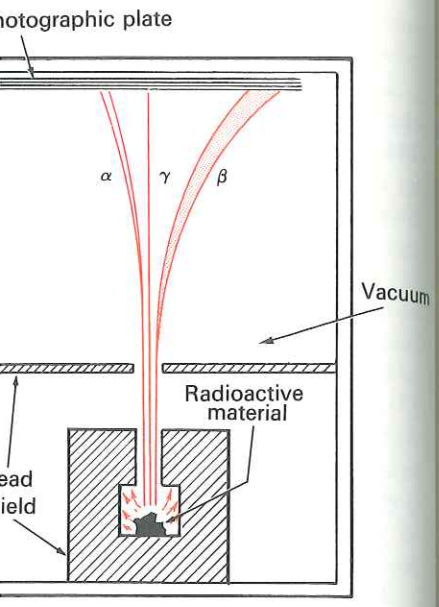


Fig. 5.1. The three types of radiation from a radioactive substance are separated by a magnetic field perpendicular to the plane of this diagram, and are detected by means of a photographic plate.

5–4 RADIOACTIVITY

In 1896 Becquerel, a French physicist, found that thorium and uranium emit radiant energy which can be detected by a photographic plate. In 1898 Madame and Pierre Curie isolated a few milligrams of radium, and eventually three types of radiation, alpha, beta and gamma rays, were detected. (See Figure 5.1.) Gamma (γ) rays are very penetrating rays similar to X-rays but having higher energy. Beta (β) rays are streams of electrons. Alpha (α) radiation is a stream of alpha particles which were eventually identified

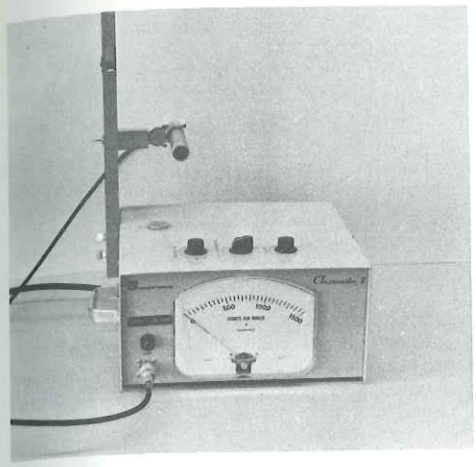


.1. The three types of radiation from a radio- substance are separated by a magnetic field ndicular to the plane of this diagram, and etected by means of a photographic plate.

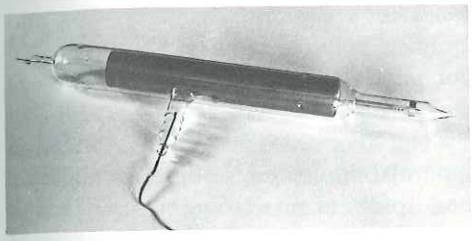
"fired" at the atoms. Alpha particles the first atomic "probes" used; they roduced by radioactive disintegration.

RADIOACTIVITY

1896 Becquerel, a French physicist, d that thorium and uranium emit nt energy which can be detected by otographic plate. In 1898 Madame Pierre Curie isolated a few milligrams dium, and eventually three types of tion, alpha, beta and gamma rays, detected. (See Figure 5.1.) Gamma ys are very penetrating rays similar -rays but having higher energy. (β) rays are streams of electrons. a (α) radiation is a stream of alpha cles which were eventually identified



(a)



(b)

Physics Department, University of Western Ontario

Fig. 5.2. Photographs of *(a)* a Geiger counter, and *(b)* a Geiger counter tube.

as ions of the element helium, each ion bearing two positive elementary charges.

The identification of alpha particles as doubly charged helium ions basically involved two phases. The magnitude of their deflection by a magnetic field (recall the method described in Section 4–4) indicated that their charge-mass ratio was the same as that for helium ions. Then a large number of alpha particles was collected in a discharge tube and the spectrum of the light produced by electrical discharge through the resulting gas was examined. The spectrum was that of helium.

Alpha particles can be stopped relatively easily by matter, but have sufficient energy to pass through thin metal foil. Their speed depends on the source; the speed of those emitted by the radioactive element polonium has been found to be 1.6×10^7 m/sec. The mass of an alpha particle is 6.6×10^{-27} kg.

5–5 DETECTION OF RADIATION

Several methods, some simple and some complex, have been devised for detecting different forms of radiation. As we have already noted (Sect. 4–6), X-rays ionize the gases through which they pass and as a result cause a charged metal-leaf electroscope to discharge.

One of the earliest detectors of alpha particles consisted of a zinc sulphide screen which fluoresces briefly when an alpha particle is incident on it; the brief flashes of light can be observed and counted with the aid of a microscope.

The Geiger counter (Fig. 5.2) is widely used in many branches of physics and chemistry. The basic principles of its construction are shown in Figure 5.3. A fine

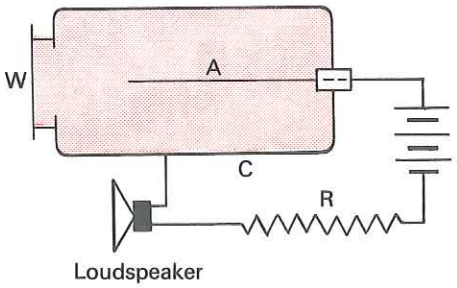


Fig. 5.3. The basic parts of a Geiger counter.

wire A is inserted into, and insulated from, a metal cylinder C. The cylinder contains gas at low pressure. The P.D. applied between A and C (with C negative), is slightly less than that necessary to produce an electrical discharge through the gas. An alpha particle entering the cylinder through the thin mica window W ionizes the gas in the cylinder, and electrons freed by this ionization are accelerated toward A. As these electrons travel through the gas, they collide with more gas molecules, freeing more electrons and so on. As a result there is a sudden surge of current which is amplified to produce a click in the loudspeaker. Because of the high resistance R in the circuit the current ceases very quickly, unless another alpha particle arrives to cause another click. The Geiger counter, therefore, may be used not only to detect, but also to count, alpha particles.

5–6 ALPHA PARTICLE DEFLECTION

We are now ready to discuss the experiments which were performed about 1910 under the direction of Ernest Rutherford. Lord Rutherford was a New Zealander who spent some time at McGill University in Montreal and then, in 1907, moved to Manchester University in England. It was in Manchester that the experiments for which he is most famous were carried out, mainly by two of his assistants, Geiger and Marsden.

In these experiments, alpha particles were fired at the atoms in a thin sheet of gold foil. The arrangement of the apparatus is shown in Figure 5.4. Alpha particles from a radioactive source were directed at the gold foil, and were detected and counted on the other side of the foil

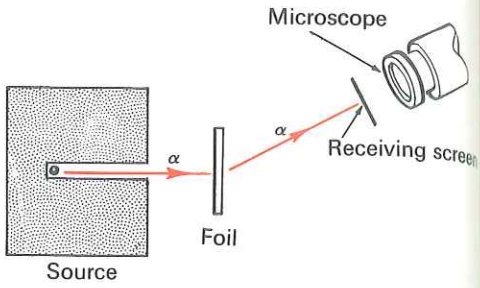
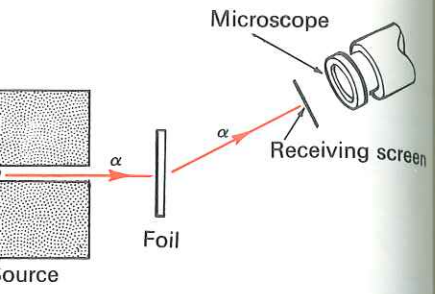


Fig. 5.4. Apparatus used by Rutherford in his experiment on the scattering of alpha particles. Only a very small percentage of the alpha particles were deflected to the extent shown in this drawing.

by viewing a zinc sulphide screen through a microscope. Geiger and Marsden spent hours in a darkened room noting and counting spots of light on the screen. No wonder Geiger later invented (and used) the Geiger counter.

Even the earliest observations with this apparatus indicated that the "raisin bun" model of the atom was incorrect. Although the gold foil was very thin, it consisted of several hundred layers of atoms—and yet most of the alpha particles went straight through the foil without being deflected at all. Most of the space occupied by an atom, then, is just space; the atom is not of uniform density throughout.

However, this information about the density of an atom was not the most startling conclusion to come from these experiments. Geiger and Marsden, by altering the position of the microscope and screen (Fig. 5.4), found that, although most of the alpha particles were deflected no more than one degree, if they were deflected at all, some were deflected through much larger angles. Indeed, about one alpha particle in 10,000 was deflected through more than 90°, and some were deflected as much as 180°,



5.4. Apparatus used by Rutherford in his ment on the scattering of alpha particles. a very small percentage of the alpha particles deflected to the extent shown in this drawing.

ewing a zinc sulphide screen through croscope. Geiger and Marsden spent s in a darkened room noting and ting spots of light on the screen. No ler Geiger later invented (and used) Geiger counter.

en the earliest observations with this ratus indicated that the "raisin bun" l of the atom was incorrect. Although old foil was very thin, it consisted veral hundred layers of atoms—and most of the alpha particles went ght through the foil without being ted at all. Most of the space occu- by an atom, then, is just space; the is not of uniform density throughout. wever, this information about the ty of an atom was not the most ing conclusion to come from these iments. Geiger and Marsden, by ng the position of the microscope reen (Fig. 5.4), found that, although of the alpha particles were deflected ore than one degree, if they were ted at all, some were deflected gh much larger angles. Indeed, one alpha particle in 10,000 was ted through more than 90°, and were deflected as much as 180°,

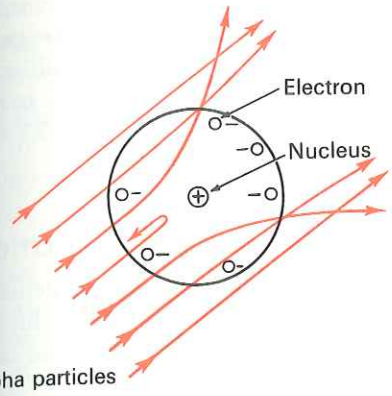


Fig. 5.5. Most alpha particles pass the nucleus with very little delfection; others are forced to turn completely around. The electrons have almost no effect on the alpha particles.

that is, they came straight back. It was in order to explain these large angle deflections that the Thomson model of the atom had to be discarded and a new model developed.

5–7 THE NUCLEAR ATOM

Rutherford realized that an atomic model which supposed that the mass of an atom was distributed thinly over its entire volume could not explain the fact that in some cases the alpha particles suffered very large deflections. He realized too that these deflections, although they occurred infrequently, were the ones which had to be explained. Consequently he assumed that an atom has a small positively charged nucleus, containing almost all of the mass of the atom and about which the electrons circle, somewhat as the planets circle the sun. Referring to Figure 5.5, let us see how this model explains the observations of Geiger and Marsden.

Since the volume of the nucleus is only a very small fraction of the volume of the atom, most of the alpha particles will not pass close enough to the nucleus to be affected by it. Moreover it is unlikely that even a series of encounters with electrons will have much effect on an alpha particle, for the alpha particle has a mass about 7500 times that of an electron. So most of the alpha particles will not encounter an obstruction and will not be deflected.

Now consider the situation which, though it will happen infrequently, is still a distinct possibility—an alpha particle travelling directly toward a nucleus (Fig. 5.5). Because both the alpha particle and the nucleus bear positive charges, the force of repulsion between the two causes the alpha particle to slow down, stop, and turn around. That is, it undergoes a deflection of 180°.

Alpha particles passing close to a nucleus but not travelling directly toward a nucleus will undergo an amount of deflection which depends on the aiming error. This situation is analyzed more fully in Figure 5.6. The aiming error b is the distance by which the alpha particle would miss the nucleus if it continued its original path; the scattering angle θ is the angle between the initial and final paths of the alpha particle. Obviously, as b increases, θ decreases. If we assume that Coulomb's inverse square law applies to the repulsion between the alpha particle and the nucleus (and Geiger and Marsden's results indicate that it does), the path of the alpha particle is a portion of an hyperbola.

If we use the idea of a potential hill which we discussed initially in Section 2–16, we can visualize the paths of alpha

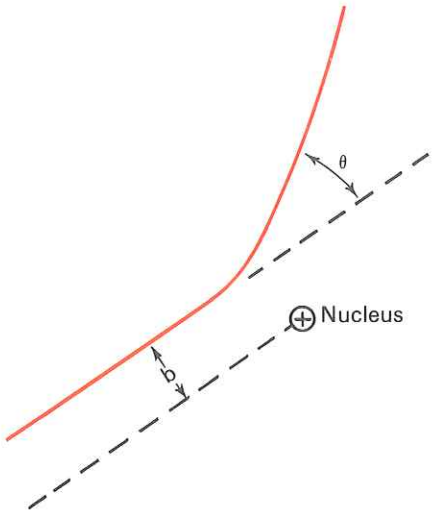


Fig. 5.6. The greater the aiming error b, the smaller the angle of deflection θ.

particles repelled by the nucleus. The paths of marbles rolled in the general direction of the "top of the hill" give a qualitative idea of what happens to alpha particles near a nucleus. Figure 5.7 shows three possible paths.

5–8 RUTHERFORD'S QUANTITATIVE DEDUCTIONS

By means of mathematical calculations which we shall not attempt to duplicate here, Rutherford came to several important conclusions.

(*a*) Since the experimental results agreed with predictions which he made on the assumption that Coulomb's law was valid in these cases, then apparently the inverse square law for electric forces applies at very small distances as well as

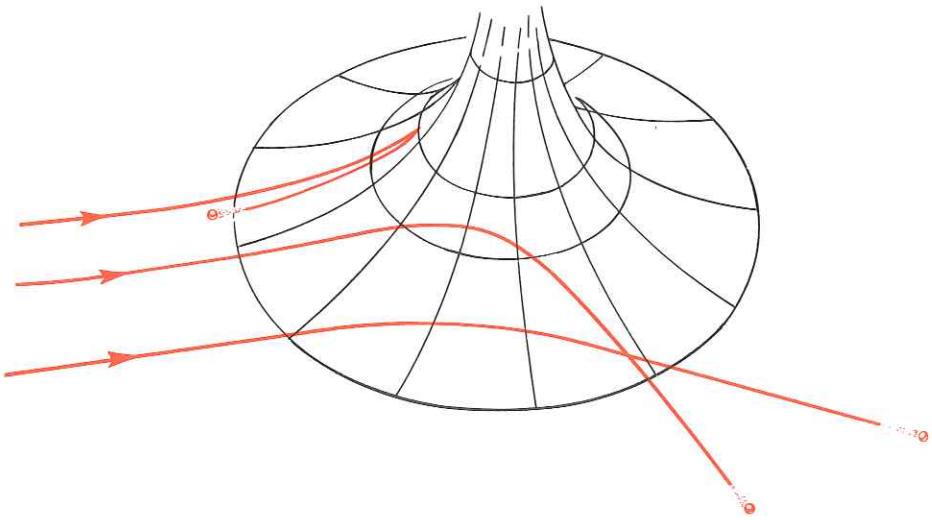


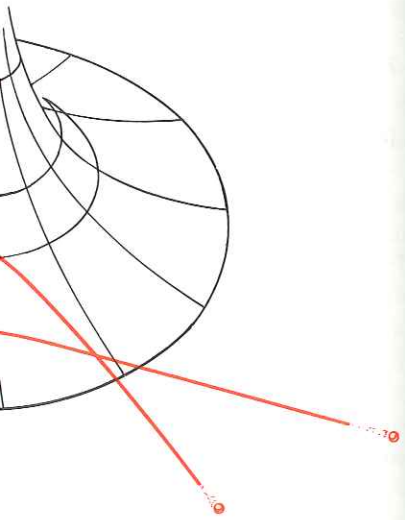
Fig. 5.7. As alpha particles approach a nucleus, they climb at least a portion of the potential hill surrounding the nucleus.

ticles repelled by the nucleus. The
ns of marbles rolled in the general
ction of the "top of the hill" give a
litative idea of what happens to alpha
ticles near a nucleus. Figure 5.7 shows
ee possible paths.

8 RUTHERFORD'S QUANTITATIVE DEDUCTIONS

y means of mathematical calculations
ch we shall not attempt to duplicate
e, Rutherford came to several impor-
t conclusions.

a) Since the experimental results
eed with predictions which he made
the assumption that Coulomb's law
valid in these cases, then apparently
inverse square law for electric forces
lies at very small distances as well as



climb at least a portion of the potential hill

at the larger distances which had been investigated by Coulomb.

(*b*) The radius of the nucleus varies from element to element, but is less than 4×10^{-14} metres. When we compare this number with the atomic radius of 10^{-10} metres, realizing that the latter is of the order of ten thousand times the former, we see that Rutherford's idea of the nucleus as a minute speck is vindicated. However, the nucleus contains well over 99% of the mass of the atom, and therefore the density of the nucleus is very high: of the order of 10^{11} kilograms per cubic centimetre.

(*c*) The charge on the nucleus varies from element to element, and is proportional to the atomic number Z of that element. We may get a qualitative idea of how this conclusion was arrived at by considering Figure 5.8. The scattering angle for an alpha particle depends not only on the particle's speed and aiming error, but also on the charge Q on the nucleus. The greater the scattering angle is observed to be, the greater Q must have been. Rutherford was able to show not only that the nuclear charge was proportional to the atomic number Z, but that the constant of proportionality was the elementary charge; that is,

$$Q = Ze$$

where e is the charge on the electron.

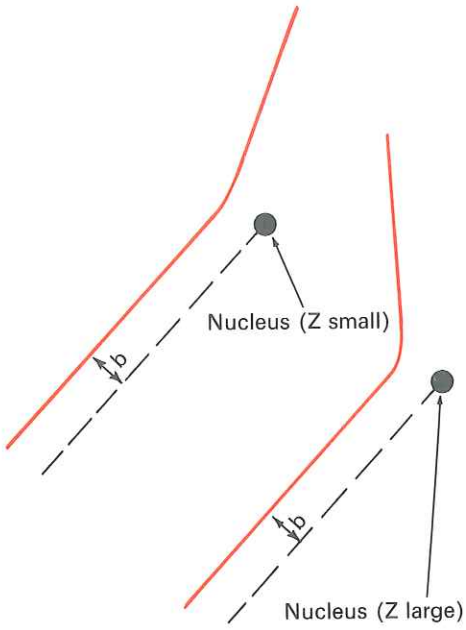


Fig. 5.8. For a given aiming error, the deflection of an alpha particle increases as the charge on the nucleus increases.

5–9 RUTHERFORD'S ATOMIC MODEL

Rutherford, then, pictured an atom as consisting of a very small (and very dense) nucleus around which electrons revolved much as planets do around the sun. The nucleus contains practically all of the mass of the atom. Since the number of positive elementary charges on the nucleus is equal to the atomic number of the element, then the number of electrons must also be equal to the atomic number. Notice that there is no mention here of electron orbits, shells, or energy levels, nor of the number of electrons in each shell. You may be familiar with these ideas already, but Rutherford was not; we shall see the evidence for them in Chapter 7.

This Rutherford atomic model, devised to explain alpha particle scattering, encountered difficulties. In order to appreciate one of the major difficulties, we need to pause briefly to consider electromagnetic radiation and the electromagnetic spectrum.

5–10 THE ELECTROMAGNETIC SPECTRUM

The visible spectrum forms but a very small part of a much broader spectrum called the electromagnetic spectrum (Fig. 5.9). This spectrum contains many forms of radiation not detected by the eye: infrared and ultraviolet, X-rays, gamma rays, and radio waves. All of these components exhibit wave properties, notably diffraction and interference, and the manner of their transmission presents the same theoretical difficulties that we encounter in connection with light. Their common speed is the speed of light. Their other properties show some variation which is dependent upon their wave lengths or frequencies. However it is not with the properties or the resulting uses of electromagnetic waves that we are concerned here, but with an important principle in connection with their production.

5–11 ELECTROMAGNETIC WAVES

In 1864 James Clerk Maxwell (1831-1879), professor of physics at Cambridge University, published his famous electromagnetic theory. Although this theory is complex, it is based essentially on two facts which, due to the experimental work of Faraday and others, were well known at the time. (*a*) A moving electric charge produces a magnetic field. (*b*) A changing magnetic field causes electric charge to move. Maxwell, treating these facts mathematically, developed a set of equations which correlated the pieces of experimental information. In addition, the form of the equations caused Maxwell to conclude that a disturbance in an electric or magnetic field would have an effect throughout space, though not instantaneously. The disturbance would be propagated as a wave motion having a finite speed of 3×10^{10} cm/sec. The theory predicted the existence of types of radiation which were not discovered until later, for Maxwell predicted radiation of electromagnetic waves over a very broad range of frequencies.

Physicists realized that, if Maxwell's predictions were correct, the oscillation (vibration) of charge in an electric circuit should cause electric energy to be radiated by means of an electromagnetic wave. The search for experimental verification

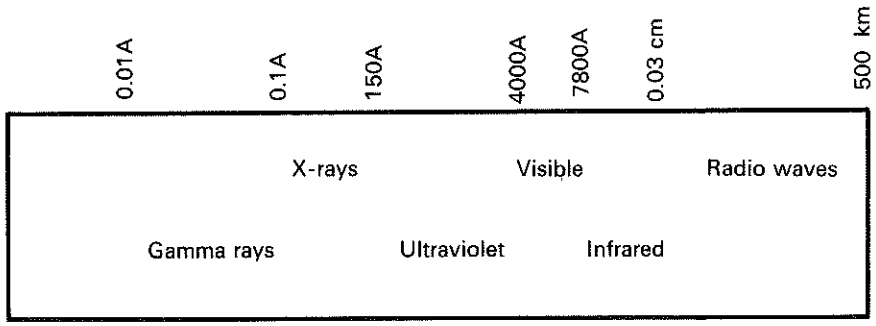
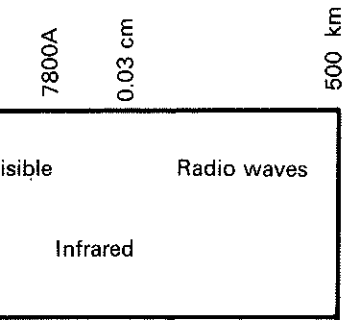


Fig. 5.9. The electromagnetic spectrum, and the approximate range of wave lengths for each component.

lex, it is based essentially on two
which, due to the experimental work
raday and others, were well known
e time. (*a*) A moving electric charge
ces a magnetic field. (*b*) A changing
etic field causes electric charge to
. Maxwell, treating these facts
ematically, developed a set of
ions which correlated the pieces of
imental information. In addition,
rm of the equations caused Maxwell
clude that a disturbance in an elec-
magnetic field would have an effect
ghout space, though not instanta-
ly. The disturbance would be propa-
as a wave motion having a finite
of 3×10^{10} cm/sec. The theory
ted the existence of types of radi-
which were not discovered until
for Maxwell predicted radiation of
omagnetic waves over a very broad
of frequencies.

ysicists realized that, if Maxwell's
tions were correct, the oscillation
tion) of charge in an electric circuit
d cause electric energy to be radiated
eans of an electromagnetic wave.
earch for experimental verification



imate range of wave lengths for each

of the existence of these waves became a race between Heinrich Hertz (1857-1894) and Sir Oliver Lodge. In 1887 Hertz obtained evidence of the existence of such waves and because of this fact the name Hertzian waves is applied to all electromagnetic radiation having wave lengths longer than those of infra-red. They are also called electric waves or radio waves and they include those used in domestic radio as well as in short wave radio, television waves, and even those produced by 60-cycle alternating current.

We shall not attempt to tell here the story of the development of radio and television. Most of us are at least vaguely aware of the general procedures employed. Oscillating circuits connected to the transmitting antenna cause the electrons in the antenna to vibrate. As the electrons vibrate, they are continually undergoing acceleration. Maxwell predicted that this acceleration of charge should cause an electromagnetic wave to be radiated. This electromagnetic wave, travelling past the receiving antenna, causes the electrons in the receiving antenna to vibrate. The present wide use of radio and television indicates that Maxwell's predictions were correct. The important point for us to realize here is that the success of wireless methods of communication confirms Maxwell's prediction that an accelerated electric charge radiates energy.

5–12 CONTRADICTIONS IN RUTHERFORD'S ATOMIC MODEL

Rutherford supposed that electrons travel in orbits around the nucleus of an atom. In doing so, the electrons undergo a central acceleration which, according to classical electromagnetic theory, should cause them to radiate energy. As a result of this energy loss, the electrons should spiral in toward the nucleus and the atom should collapse into the nucleus in a matter of 2×10^{-11} sec. But atoms do not normally emit radiation except under excitation of some sort, and atoms are normally quite stable.

If the emission of light from an excited atom (for example, an atom of hydrogen in a discharge tube) is a result of the electrons spiralling toward the nucleus, the spectrum of the light emitted should be continuous. For as the radius of the orbit decreases, the number of revolutions per second (i.e., the frequency) should increase continuously. Yet this is not the case. The spectrum of atomic hydrogen is not a continuous spectrum, but a line spectrum.

At this stage then we have two unresolved problems.

(*a*) How is light transmitted, by waves or by particles? This was the major question introduced and left unresolved in Book I of this series.

(*b*) What modifications must be made in the Rutherford model of the atom to accord with the facts discussed above? The answers to these questions necessitated a radical change in the thinking of physicists early in this century. We shall attempt to give some indication of the answers in the next two chapters.

5–13 PROBLEMS

Where necessary, assume that

(*a*) the mass of an alpha particle
$= 6.6 \times 10^{-27}$ kg

(*b*) 1 elementary charge $= 1.60 \times 10^{-19}$ coulombs

(*c*) the constant k in Coulomb's law

$$= 2.3 \times 10^{-28} \frac{\text{newton-(metre)}^2}{(\text{elementary charge})^2}$$

$$= 9.0 \times 10^{9} \frac{\text{newton-(metre)}^2}{(\text{coulomb})^2}$$

(*d*) the constant G in the law of gravitation

$$= 6.67 \times 10^{-11} \frac{\text{newton-(metre)}^2}{(\text{kg})^2}$$

(*e*) the mass of the gold nucleus $= 3.3 \times 10^{-25}$ kg

(*f*) the mass of an electron $= 9.11 \times 10^{-31}$ kg

(*g*) the speed of electromagnetic waves in space
$= 3.0 \times 10^{8}$ m/sec

1. One gram of radium emits approximately 3.7×10^{10} alpha particles/sec. Calculate the order of magnitude, in kg, of the mass of the helium which could be formed from the alpha particles emitted in one day.

2. What did Rutherford's experiment indicate about (*a*) the sign of the charge on the nucleus, (*b*) the magnitude of the charge on the nucleus, (*c*) the size of the nucleus?

3. Calculate the charge, in coulombs, on the nucleus of an atom of each of the following elements: (*a*) magnesium (atomic number 12), (*b*) copper (atomic number 29), (*c*) aluminum (atomic number 13), (*d*) gold (atomic number 79).

4. The radius of a gold atom is approximately 1.4×10^{-10} m and the radius of its nucleus is approximately 8.7×10^{-15} m. What percent of the atomic volume is in the nucleus?

5. What would be the effect on the trajectories of alpha particles of an atom in which the positive charge and mass were distributed over the whole volume of the atom?

6. What effect do the electrons in atoms have on alpha particles passing close to them? Justify your answer.

7. The nucleus of a gold atom carries 79 positive elementary charges. Calculate the force of repulsion between an alpha particle and a gold nucleus when the distance between them is 3.0×10^{-13} m (about 30 times the diameter of the nucleus).

$\times 10^{-19}$ coulombs
law
etre)^2
charge)^2
^{2}

gravitation
re)^2

$= 3.3 \times 10^{-25}$ kg
11×10^{-31} kg
waves in space

3.7×10^{10} alpha particles/sec.
f the mass of the helium which
mitted in one day.

about (*a*) the sign of the charge
arge on the nucleus, (*c*) the size

ucleus of an atom of each of the
number 12), (*b*) copper (atomic
13), (*d*) gold (atomic number 79).

y 1.4×10^{-10} m and the radius
m. What percent of the atomic

es of alpha particles of an atom
ere distributed over the whole

on alpha particles passing close

ve elementary charges. Calculate
article and a gold nucleus when
m (about 30 times the diameter

8. Refer to problem 7. (*a*) Calculate the gravitational force of attraction between the alpha particle and the gold nucleus when they are 3.0×10^{-13} m apart. (*b*) What is the ratio of the electric force to the gravitational force at this distance? (*c*) What would the ratio be if the distance were three times as great? Does the gravitational force need to be taken into account in the mathematical analysis of alpha particle deflection?

9. Refer to problem 7. Calculate the electric potential energy of the alpha particle at a distance of 3.0×10^{-13} m from the gold nucleus.

10. An alpha particle with a speed of 2.0×10^7 m/sec approaches a gold nucleus head-on. Assuming that the kinetic energy of the alpha particle is converted to electric potential energy, calculate the distance of closest approach of the alpha particle to the nucleus. What information does your answer give you about the dimensions of the nucleus?

11. An alpha particle approaches the nucleus of a cadmium atom (atomic number, $Z = 48$) with a kinetic energy of E joules. If the distance of closest approach is to be 6.0×10^{-14} m, calculate the value of E.

12. A neutron travelling at a speed of 1.0×10^5 m/sec, collides head-on with a stationary deuteron whose mass is double that of the neutron. The collision is elastic, and the particles do not stick together. Calculate the speed of each after collision.

13. Summarize the electromagnetic spectrum by listing for each type of radiation (*a*) the speed in a vacuum, (*b*) the approximate range of wave lengths in a vacuum, (*c*) the approximate range of frequencies.

14. Calculate the wave length in a vacuum of electromagnetic waves of each of the following frequencies: (*a*) 2.0×10^4 c/s, (*b*) 1.5×10^8 c/s, (*c*) 6.0×10^{14} c/s, (*d*) 3.0×10^{18} c/s.

15. Calculate the frequency for an electromagnetic wave, if the wave length, in a vacuum, is (*a*) 4.0×10^4 m, (*b*) 6.0×10^3 A, (*c*) 2.0×10^{-2} A.

16. A radar signal directed toward the planet Venus returned to earth 10 seconds after transmission. What is the order of magnitude, in metres, of the distance from the earth to Venus?

17. Incident and reflected electromagnetic waves form a standing wave pattern in space. The average distance between nodes in the pattern is 1.8 m. Calculate the frequency of the waves. What type of waves are they?

18. Should sound waves be included in the list of electromagnetic waves? Justify your answer.

19. Do electric transmission lines, transmitting 60-cycle alternating current, radiate energy?

20. An electron in an X-ray tube has a kinetic energy of 7.2×10^{-16} joules just before it strikes the target. Calculate (*a*) its speed, and (*b*) the P.D. through which it has been accelerated.
21. Calculate the frequency of X-rays of wave length 1.5 A.
22. The electric current flowing through an X-ray tube is 0.24 milliamperes. Calculate the number of electrons striking the target each second.

5–14 SUMMARY

Thomson's model of the atom was a "raisin-bun" model, assuming that the density of the atom was uniform. Rutherford's experiments with alpha particle deflection showed that the Thomson model was incorrect. The mass of the atom was found by Rutherford to be concentrated in a dense central nucleus whose diameter is 10^{-3} or 10^{-4} of the diameter of the atom. The atomic diameter is of the order of 10^{-10} m, the nuclear diameter is of the order of 10^{-14} m.

Rutherford's atomic model, which assumed that electrons travelled in circular orbits about the nucleus, was not in accord with classical electromagnetic theory. The orbiting electrons do not radiate energy, as centrally accelerated charges should.

c energy of 7.2×10^{-16} joules
(*a*) its speed, and (*b*) the P.D.

length 1.5 A.

-ray tube is 0.24 milliamperes.
the target each second.

diameter is 10^{-3} or 10^{-4} of the
ter of the atom. The atomic diam-
of the order of 10^{-10} m, the nuclear
ter is of the order of 10^{-14} m.
therford's atomic model, which as-
l that electrons travelled in circular
about the nucleus, was not in accord
lassical electromagnetic theory. The
ng electrons do not radiate energy,
trally accelerated charges should.

Chapter 6

The Dual Nature of Radiation and Matter

6–1 INTRODUCTION

The contradictions present in Rutherford's model of the atom seemed to indicate that some facts about matter could not be explained on the basis of nineteenth century physics. Some basic change in outlook seemed necessary. Moreover, the contradictions present in atomic models were paralleled by contradictions present in the wave model for light and other electromagnetic radiations. The first change of outlook, resulting in an extension of Newtonian mechanics but at the same time preserving the bases upon which it had been built, came as a result of experiments with light incident upon matter.

6–2 THE PHOTOELECTRIC EFFECT

About 1887 the German physicist, Heinrich Hertz, was investigating conduction in gases and the phenomenon of transfer of electric energy through them. Among other things, he found that a discharge between an anode and a cathode would take place more readily if the cathode was subjected to the ultraviolet radiation from a carbon arc lamp. Apparently the radiation made possible the emission of a greater number of electrons by the cathode. This type of electron emission is called photoelectric emission.

A more conclusive experiment than that of Hertz was devised by Thomson about 1900 and can be repeated with the equipment illustrated in Figure 6.1. A well cleaned zinc plate, about twenty centimetres square, is connected electrically to an electroscope. The plate is grounded to neutralize the system. The ground connection is removed and the radiation from a carbon arc lamp, without a lens or other glass intervening, is directed toward the

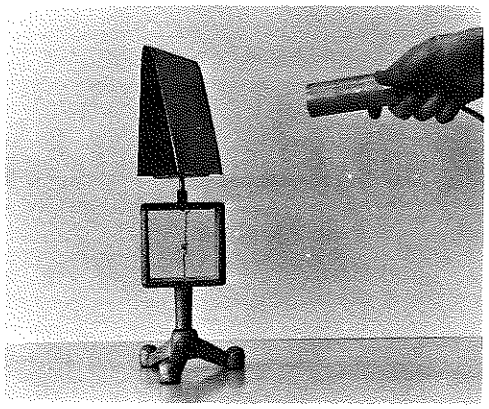


Fig. 6.1. The radiation from the ultraviolet source causes a negatively charged zinc plate to emit electrons.

zinc plate. The leaves do not show any response. The experiment is then repeated with the plate-electroscope system positively charged. Again, the leaves do not show any response. The experiment is repeated a third time with the zinc-electroscope system negatively charged. This time the leaves of the electroscope fall steadily, yet if the radiation is interrupted the fall of the leaves is arrested. This last test shows that electrons (negative charge) leave the zinc plate and that the radiation is the agent causing the emission.

6–3 THE MILLIKAN EXPERIMENT AND THE PHOTOELECTRIC EFFECT

The apparatus shown in Figure 6.2 may be used to show that light has some properties that are not characteristic of waves. The apparatus is similar to that used by Millikan. Charged oil drops "float" in the space between the plates; their motions may be observed through a microscope.

When the drops are illuminated by the light from an arc lamp (Fig. 6.2), the light energy incident on them may impart sufficient energy to electrons in the drops to cause photoelectric emission to occur. As a result a drop may be left sufficiently positively charged that it "jumps" upward toward the negative plate, because the upward electric force is then greater than the downward gravitational force.

Let us try to predict when these jumps occur, assuming that the light emitted by the arc and incident on the drops is transmitted by means of waves. A wave model for light assumes that the energy in the light beam is distributed uniformly along the wave front. Therefore all the drops in a given region should be affected in the same manner by the light, absorbing energy at the same rate. When each of the drops has absorbed sufficient energy to cause an electron to be emitted, all of the drops should jump simultaneously. Even in strong light, there should be a considerable and readily measurable time lag between the turning on of the light and the first jump or jumps. Moreover, the more intense the incident light, the sooner the drops should jump.

The experimental observations are entirely different from the results predicted above. Regardless of light intensity, the first jump in many cases is observed to occur almost immediately after the light is turned on. In other cases considerable time may elapse before a drop jumps, even in strong light. Experiments also indicate that the number of electrons ejected from the drops by strong light is greater than the number ejected by weak

tions may be observed through
ope.
the drops are illuminated by the
m an arc lamp (Fig. 6.2), the
rgy incident on them may im-
cient energy to electrons in the
cause photoelectric emission to
s a result a drop may be left
ly positively charged that it
upward toward the negative
cause the upward electric force
eater than the downward gravi-
force.
try to predict when these jumps
suming that the light emitted by
nd incident on the drops is trans-
y means of waves. A wave model
assumes that the energy in the
m is distributed uniformly along
front. Therefore all the drops
en region should be affected in
manner by the light, absorbing
t the same rate. When each of
s has absorbed sufficient energy
an electron to be emitted, all of
s should jump simultaneously.
strong light, there should be a
able and readily measurable time
een the turning on of the light
first jump or jumps. Moreover,
intense the incident light, the
e drops should jump.
perimental observations are en-
ferent from the results predicted
egardless of light intensity, the
p in many cases is observed to
ost immediately after the light
on. In other cases considerable
y elapse before a drop jumps,
strong light. Experiments also
that the number of electrons
om the drops by strong light is
an the number ejected by weak

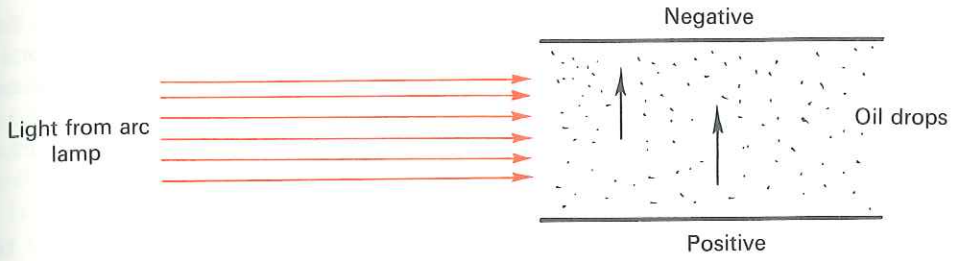


Fig. 6.2. A variation of Millikan's experiment. Light from the arc lamp, falling on the oil drops, may cause some of the oil drops to lose electrons and begin to drift upward.

light. Over a period of time the emission of electrons is uniform across the light beam. However, these are not the startling or important points. The important point is that, even in weak light, the first jump occurs much earlier than the wave model predicts.

6–4 PHOTONS

Since the wave model fails to explain random emission of electrons from oil drops, let us return to the particle model and see how well it fits the facts here. Ejection of an electron from one of the drops would occur when that drop is struck by a particle of light. The first strike causes the first drop to jump, and this may occur immediately after the light is turned on, whether the light is strong or weak. Moreover, even a dense beam of particles (a strong beam of light) might fail to make a direct hit on a drop for some time. Over a period of time, of course, the greater the number of particles in the beam, the greater will be the number of collisions, and the greater will be the number of electrons ejected.

A particle model seems to explain photoelectric emission in quite an accurate and convincing manner. In some ways light acts as if it came in bundles, just as matter is composed of atoms and electricity is composed of elementary charges. The bundles of light are called photons or quanta (singular: quantum). Many phenomena other than the photoelectric effect seem to indicate the existence of photons, for example, the action of light on a photographic film and the action of the television camera tube. But photons, if they exist, do not seem to solve the problem of how light is transmitted, for how, in terms of photons, can we explain interference of light?

6–5 INTERFERENCE EFFECTS IN EXTREMELY WEAK LIGHT

About 1910, Sir Geoffrey Taylor, at Cambridge University, set up an experiment to find whether interference resulted from the interaction of many photons, or whether single photons produced interference effects. The basic parts of the apparatus used in an experiment of the type carried out by Taylor, are shown in Figure 6.3. Light originating at a small source of low intensity passes through a very dark filter, thence through a single

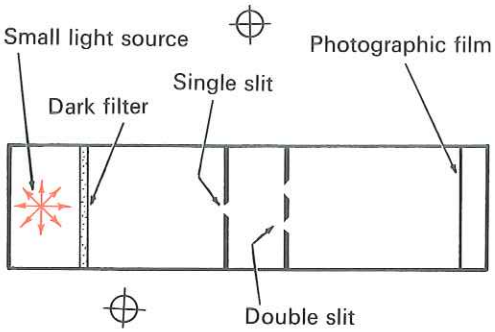


Fig. 6.3. Arrangement of apparatus for demonstrating interference effects in extremely weak light.

slit and a double slit, and falls on a photographic film. The complete apparatus is enclosed in a light-tight box. Taylor was able to calculate that there was never more than one photon of light in the box at a time; indeed the light reaching the photographic plate was so dim that a three month exposure was necessary. Yet the photograph showed the usual double slit interference pattern.

The above observations rule out the possibility of explaining interference on the assumption that photons reinforce or destroy one another. We might assume, then, that each photon splits into two halves, one half going through each slit. This assumption leads to the unlikely conclusion that by obstructing one of the slits we could obtain half-photons, each possessing half the energy of the original photon! This conclusion not only contradicts our basic ideas concerning photons, but is in fact never observed. The photon either arrives intact, or does not arrive at all. A photon never becomes divided, and yet, in order to produce interference phenomena with the apparatus shown in Figure 6.3, both slits must be open. Apparently "particles of light"—photons—possess wave properties. But how can they?

6–6 PROBABILITY AND THE PHOTON

In order to attempt to reconcile the particle and the wave aspects of photons we find it necessary to introduce some ideas concerning probability. We cannot treat the subject properly and fully here, but we can perhaps give some indication of how a series of random events can build up a regular pattern such as is associated with interference. If you toss a coin and record the occurrence of heads and tails, the first ten tosses may very well result in 8 heads and 2 tails. But if you persist for thousands of tosses, it is very probable that the number of heads will be very nearly equal to the number of tails.

Let us now try to apply this idea to the production of an interference pattern of light. Perhaps this attempt should not be made, for it is beginning to be obvious that light is not a wave and not a particle. Yet in our continued search for an analogy we continue to make the attempt. Apparently no single photon interferes with itself. Each photon arrives independently at the detector, and at a definite place. The probability of the arrival at some positions (the positions of the bright lines in the interference pattern) is high. The probability of photons arriving at other positions (the positions of the dark lines in the interference pattern) is low. At other positions the probability of arrival is between the maximum and minimum values. It is common to think of the probability of the photon's arrival at a

6.3, both slits must be open. Ap-
y "particles of light"—photons—
wave properties. But how can

ROBABILITY AND THE HOTON

der to attempt to reconcile the
and the wave aspects of photons
it necessary to introduce some
ncerning probability. We cannot
e subject properly and fully here,
can perhaps give some indication
a series of random events can
a regular pattern such as is asso-
with interference. If you toss a
l record the occurrence of heads
s, the first ten tosses may very
ult in 8 heads and 2 tails. But if
sist for thousands of tosses, it is
obable that the number of heads
very nearly equal to the number

now try to apply this idea to the
on of an interference pattern of
rhaps this attempt should not be
or it is beginning to be obvious
t is not a wave and not a particle.
ur continued search for an analogy
inue to make the attempt. Ap-
no single photon interferes with
ch photon arrives independently
etector, and at a definite place.
bability of the arrival at some
(the positions of the bright lines
terference pattern) is high. The
ity of photons arriving at other
(the positions of the dark lines
nterference pattern) is low. At
sitions the probability of arrival
en the maximum and minimum
It is common to think of the
ty of the photon's arrival at a

particular point as being fixed by the wave which accompanies the photon. The situation seems completely paradoxical, yet the ideas of probability give us some vague notion of how a pattern can be built up from random events.

In spite of the difficulties inherent in the above model, there is no doubt about the existence of the photon. Further facts concerning photons and their properties can be obtained from quantitative measurements associated with the photoelectric effect.

6–7 THE PHOTOELECTRIC CELL

In applications of the photoelectric effect, the electron emitter (the cathode) is contained in a tube of glass or of metal with a glass window. The tube is either evacuated or filled with an inert gas at low pressure. An anode (plate) collects the emitted electrons. This device is called a photoelectric cell, an example of which is illustrated in Figure 6.4. The cathode is a piece of thin metal, semi-cylindrical in shape and coated with cesium, cesium oxide and silver. The anode is a straight metal wire placed in front of the cathode.

The circuit illustrated in Figure 6.5 may be used to demonstrate the action of the cell. If the semi-cylindrical elec-

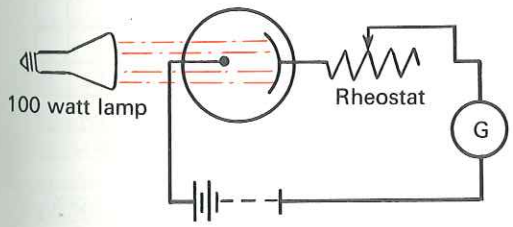


Fig. 6.5. Circuit to demonstrate the action of a photoelectric cell.

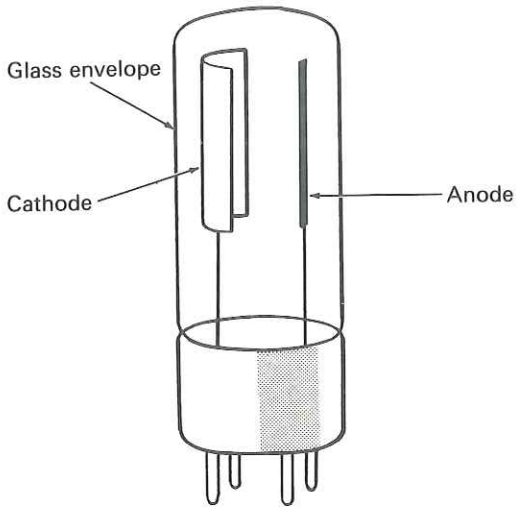


Fig. 6.4. A photoelectric cell.

trode is connected to the negative terminal of the battery, and if light is incident on that electrode, the galvanometer indicates that current is flowing in the circuit. The strength of the current increases when the intensity of the light is increased.

6–8 QUANTITATIVE FACTS ABOUT THE PHOTOELECTRIC EFFECT

The photoelectric cell described above employs a cathode coating which has been found to be suitable for many purposes. However, cesium-coated cathodes were not the first ones used, and experiments with other cathodes provide some startling information concerning the photoelectric effect.

(*a*) Suppose we use a cell having a copper cathode, connect it in a circuit as shown in Figure 6.5, and illuminate the cathode with light which contains a

high proportion of ultraviolet (for example, the light from a carbon arc). The galvanometer reading indicates that the cathode is emitting electrons, and, as we expect, the greater the intensity of the light, the greater the number of electrons emitted.

Now suppose we use green light, (light of longer wave length and lower frequency than the ultraviolet). This time, no matter how intense the light, no electrons are emitted by the cathode. Apparently the wave length of the incident light must be less than a certain threshold value, otherwise the photoelectric effect does not occur. For a copper cathode, the threshold is about at the violet end of the visible spectrum. If we use a cathode coated with potassium, we find that it has a threshold in the green portion of the spectrum. That is, light of wave lengths longer than the green, (red, for example), does not cause photoelectric emission from potassium. But light of wave lengths shorter than the green, (blue, for example), cause photoelectric emission, and the number of electrons emitted depends on the intensity of the light incident on the cathode. Each material has a different photoelectric threshold; the cesium-coated type of cathode is used in most cells because it has a threshold in the infrared and is therefore sensitive to all of the components of white light.

(*b*) The second unexpected fact concerning photoelectric emission may be obtained with a circuit of the type shown in Figure 6.6. This circuit is essentially the same as that shown in Figure 6.5 except that the battery connections are reversed, and that the magnitude of the P.D. between the electrodes can be varied. A low positive potential is applied to the cesium-coated electrode. Light is allowed to fall on this electrode, and we find, perhaps contrary to our expectations, that

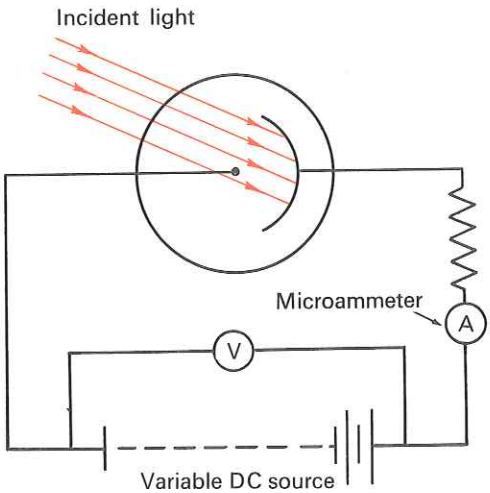


Fig. 6.6. Circuit for determining cuf-off potentials.

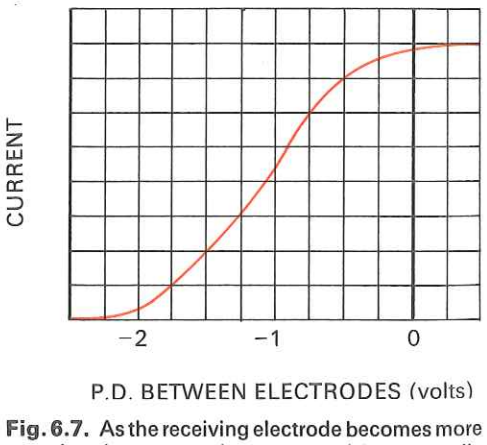
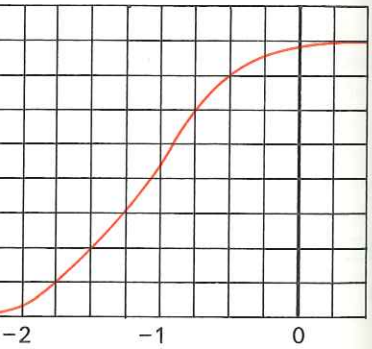


Fig. 6.7. As the receiving electrode becomes more negative the current decreases and is eventually cut off.

m. But light of wave lengths
han the green, (blue, for example),
notoelectric emission, and the
of electrons emitted depends on
sity of the light incident on the
Each material has a different
ectric threshold; the cesium-
ype of cathode is used in most
ause it has a threshold in the
and is therefore sensitive to all
mponents of white light.

he second unexpected fact con-
photoelectric emission may be
with a circuit of the type shown
e 6.6. This circuit is essentially
e as that shown in Figure 6.5
hat the battery connections are
and that the magnitude of the
ween the electrodes can be varied.
sitive potential is applied to the
oated electrode. Light is allowed
n this electrode, and we find,
ontrary to our expectations, that



D. BETWEEN ELECTRODES (volts)

s the receiving electrode becomes more
e current decreases and is eventually

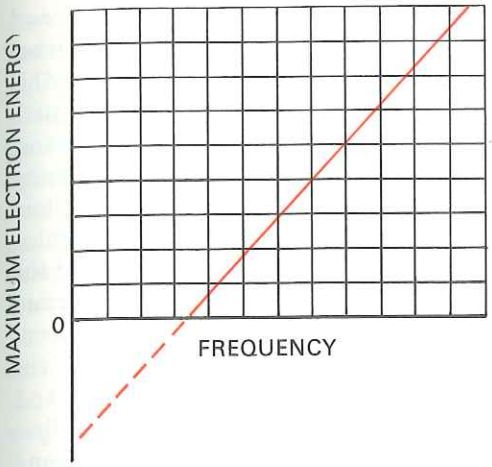


Fig. 6.8. The graph which results when the frequency of the incident radiation is plotted against the maximum kinetic energy of the emitted electrons.

there is current in the circuit. If the positive potential of this electrode is increased, the current decreases and is eventually cut off (Fig. 6.7); the P.D. between the electrodes when the current ceases is called the cut-off potential. Apparently at this potential even the most energetic electrons emitted are attracted back to the cesium electrode. The startling fact here is that the cut-off potential is independent of the intensity of the incident light, but does depend on the frequency or wave length of the light. The intensity of the incident light does, however, affect the number of electrons emitted.

Knowing the cut-off potential V and the charge Q on the electron, we can use the relationship $E = QV$ to calculate the maximum kinetic energy of the emitted electrons. If we do this for light of various frequencies, we can plot a graph of frequency ν against maximum electron energy E (Fig. 6.8). (In most articles on

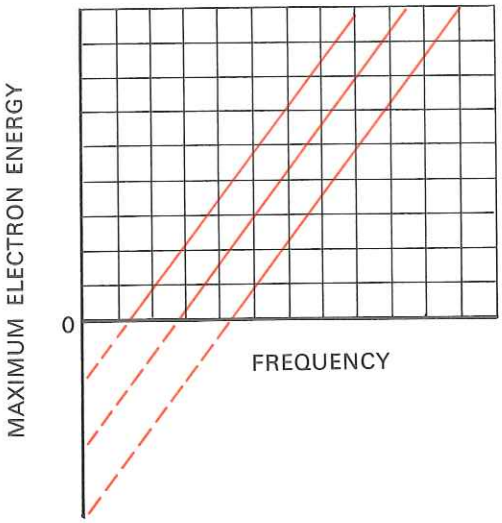


Fig. 6.9. The graphs of frequency versus maximum electron energy, for different surfaces, have equal slopes.

modern physics, the Greek letter ν—pronounced nu—is used rather than the letter f to represent frequency.) Since this graph is a straight line, $E \propto \nu$, and since the graph does not pass through the origin, the equation of the graph is of the form

$$E = h\nu - B$$

The variation constant h is the slope of the graph and is called Planck's constant, after the German physicist Max Planck (1858-1947). B is the intercept of the extended graph on the energy axis.

(*c*) Experiments with different types of surfaces on the emitting electrode yield a set of graphs of E against ν as shown in Figure 6.9. These graphs constitute a family of parallel lines; that is, they have equal slopes but different intercepts on the energy axis. Planck's constant h then seems to be a universal constant whose

value is independent of the type of emitting surface, whereas B is a constant whose value depends on the material from which the electrons are emitted.

(*d*) If the frequency of the light incident on the emitter is above the threshold frequency, even the feeblest light causes the photoelectric effect to begin immediately. This observation is similar to the observation which we have already discussed in connection with the emission of electrons from oil drops in the Millikan-type experiment.

6–9 EINSTEIN'S INTERPRETATION OF THE PHOTOELECTRIC EFFECT

We have already noted that certain facts about the emission of electrons from oil drops cannot be explained on the assumption that the incident light is in the form of waves. In order to explain the observed facts, we supposed that light came in small bundles called photons. This idea of a quantum or photon of radiation was first proposed by Planck in 1900 to explain emission and absorption of radiation. In 1905 Albert Einstein (1879-1955) applied the idea in a startlingly successful manner to explain many facts concerning light.

Einstein began by conceding that the wave model of light had been successful in explaining phenomena associated with the propagation of light. However, these phenomena were observed over relatively large time intervals, and perhaps the same model could not be expected to explain the emission and absorption of light—events which involved interaction of radiation and matter over very short time intervals. Planck had assumed the existence of photons or quanta of radiation only at the moments of emission and absorption; Einstein went one step further and assumed (as we did earlier in this chapter) that they were associated with the wave by which the radiation is transmitted. It is here that we have difficulty in setting up a visual model, for Einstein assumed that the energy being transmitted is not distributed evenly over the wave front but is concentrated in certain regions of the wave front. These concentrations or bundles of energy are the photons we discussed earlier, and Einstein's photon theory seems at first glance a curious mixture of wave and particle models—a mixture which is hard to visualize.

Planck's failure to assume the existence of photons during transmission made it difficult to see how energy could be absorbed only in definite quanta as Planck himself had assumed. Einstein simply said that during transmission the photons are normally so numerous that their individuality is obscured, just as the individual atoms are not in evidence when we observe a piece of matter. Moreover, Einstein's theory fits the facts discussed in Section 6–8 concerning the photoelectric effect; facts which, incidentally, were not observed quantitatively until after 1905.

As we have already seen, the photon idea accounts for the fact that the photoelectric effect begins almost immediately even in weak light. Einstein assumed that each photon has associated with it a quantum of energy $h\nu$, where h is Planck's constant, and ν is the frequency of the source which emitted the photon and the frequency of the electromagnetic wave associated with the propagation of the light. Thus the higher the frequency (and

the moments of emission and
ion; Einstein went one step further
umed (as we did earlier in this
) that they were associated with
e by which the radiation is trans-
It is here that we have difficulty
g up a visual model, for Einstein
l that the energy being trans-
s not distributed evenly over the
ont but is concentrated in certain
of the wave front. These concen-
or bundles of energy are the
s we discussed earlier, and
's photon theory seems at first
curious mixture of wave and
models—a mixture which is hard
lize.
k's failure to assume the existence
ons during transmission made it
to see how energy could be ab-
nly in definite quanta as Planck
ad assumed. Einstein simply said
ing transmission the photons are
so numerous that their indi-
is obscured, just as the indi-
toms are not in evidence when
've a piece of matter. Moreover,
's theory fits the facts discussed
on 6–8 concerning the photo-
effect; facts which, incidentally,
t observed quantitatively until
05.
have already seen, the photon
ounts for the fact that the photo-
effect begins almost immediately
veak light. Einstein assumed that
oton has associated with it a
of energy $h\nu$, where h is Planck's
, and ν is the frequency of the
hich emitted the photon and the
y of the electromagnetic wave
d with the propagation of the
us the higher the frequency (and

the shorter the wave length) of light, the more energy each photon possesses.

Using this idea of variation of photon energy with frequency, Einstein predicted that a photon meeting an electron in a cathode may or may not give to the electron enough energy for the electron to be emitted by the cathode. If the photon energy is too low, no electrons will be emitted no matter how numerous the photons are. But if the energy content of each photon is increased by increasing the frequency of the light, eventually the stage is reached where the photon energy is sufficient for electron emission. The required minimum frequency is of course what we have already called the threshold frequency; this threshold frequency varies from metal to metal because electrons are more tightly bound in some metals than in others.

We may get some idea of the energy necessary for electron release from the relationship $E = h\nu - B$. In this relationship E is the maximum kinetic energy of an emitted electron; presumably this is the energy of an electron released from very near the surface of the cathode, where the electrostatic attraction of nearby nuclei is at a minimum. An electron from deeper within the metal will be emitted with less energy if the P.D. between the electrodes is above a certain minimum value, but will not be emitted at or below what we have called the cut-off potential. In the relationship $E = h\nu - B$, then, $h\nu$ is the energy imparted to the electron by the photon, and B is the minimum energy expended by the electron in freeing itself from the metal; E is the maximum energy possessed by the electron when it is free of the cathode. The term B then depends solely on the material from which the electron is emitted, but the term $h\nu$ is independent of the nature of the surface and depends only on the frequency of the incident light.

All of these predictions were found to be in accord with observations in experiments carried out after Einstein's predictions were made—experiments from the results of which the graphs shown in Figures 6.8 and 6.9 were drawn. The values of the constants involved are of interest. Planck's constant h is now generally accepted as 6.62×10^{-34} joule-sec. This value may be determined in several ways; the most obvious is to find the slope of the graph represented by $E = h\nu - B$. The term B is called the work function of the metal from which the electrons are emitted. The work function of zinc is about 5.3×10^{-19} joules per electron; the work function of cadmium is about 6.3×10^{-19} joules per electron.

The energy $h\nu$ of a photon is the product of Planck's constant and the frequency of the radiation. For light of frequency 5×10^{14} cycles per second (i.e. of wave length 6000 A and therefore in the orange section of the visible spectrum) the energy of a photon is equal to $6.62 \times 10^{-34} \times 5 \times 10^{14}$ joules or 3×10^{-19} joules. The energy is perhaps better expressed in electron volts rather than in joules. Since 1 electron volt $= 1.60 \times 10^{-19}$ joules, the energy possessed by a photon of this frequency is about 2 electron volts.

6–10 THE COMPTON EFFECT

Another curious fact about photons was discovered by the American physicist A. H. Compton in 1923 and is known as the Compton effect. Compton discovered that an X-ray photon, after interacting

with an electron in a block of carbon, bounced off in another direction with slightly lower frequency ν (and therefore less energy $h\nu$), while the electron recoiled as if the photon had been a particle colliding with it. Similar phenomena have been observed in cloud chambers; the effect is as shown in Figure 6.10. To account for the observations shown, we may assume that momentum disappears in the collision, or we may say that the photon undergoes a change of momentum. The latter explanation is tempting. After all, we would like the law of conservation of momentum to apply on the subatomic scale as well as on much larger scales. But how can a photon—a bundle of disembodied energy—possess momentum?

Light has been found to exert a very small, but measurable, pressure on surfaces it strikes. If this pressure is produced by a "rain" of small particles striking the surface, then those particles must possess momentum. The actual pressures, though small, are measurable, and the momentum of a photon can then be calculated. The photon momentum p is given by the relationship

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

where c is the speed of light and λ is the wave length of the radiation.

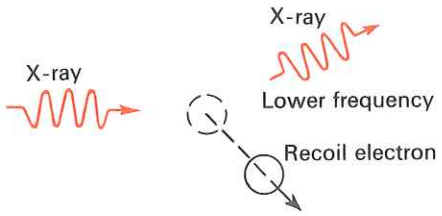


Fig. 6.10. An X-ray photon, striking an electron, behaves as if it had momentum.

It turns out that, if we use this relationship to calculate the momentum of the X-ray photon before and after collision, the change in momentum of the photon is equal to the change in momentum of the electron. If a photon, then, has momentum, does it also have mass?

6–11 THE PHOTON-WAVE DILEMMA

Light and all electromagnetic radiations seem to have some wave characteristics and some particle characteristics, and the two sets of properties frequently seem irreconcilable. The dilemma seems to arise because of the inadequacies of our vocabularies and our analogies. We are tempted sometimes to over-simplify the problem by saying that light shows wave characteristics as it travels, and particle characteristics as it interacts with matter. But this oversimplification does not solve the problem, nor will the discussion which follows. But it may help.

Experiments dealing with the transmission phenomena of reflection, refraction, diffraction and interference, usually involve radiation that is relatively intense. Thus the photons are so numerous that their individual characteristics are not evident and the effect is that of a wave. This is particularly true for the low frequency (radio wave) portion of the electromagnetic spectrum. The frequency ν is small, and therefore the photon energy $h\nu$ is also small. Moreover, the usual detectors of this radiation are not sensitive enough to operate unless the number of photons incident on them is so large as to give the impression of a continuous wave. But in the high frequency portion of the electro-magnetic spectrum, the

turns out that, if we use this re-
ship to calculate the momentum of
X-ray photon before and after col-
, the change in momentum of the
on is equal to the change in momen-
of the electron. If a photon, then,
nomentum, does it also have mass?

THE PHOTON-WAVE DILEMMA

ht and all electromagnetic radi-
seem to have some wave charac-
cs and some particle characteristics,
ne two sets of properties frequently
irreconcilable. The dilemma seems
se because of the inadequacies of
ocabularies and our analogies. We
empted sometimes to over-simplify
roblem by saying that light shows
characteristics as it travels, and
le characteristics as it interacts with
r. But this oversimplification does
olve the problem, nor will the dis-
n which follows. But it may help.
periments dealing with the trans-
on phenomena of reflection, refrac-
diffraction and interference, usually
ve radiation that is relatively intense,
the photons are so numerous that
individual characteristics are not
nt and the effect is that of a wave.
s particularly true for the low fre-
cy (radio wave) portion of the
omagnetic spectrum. The frequency
all, and therefore the photon energy
also small. Moreover, the usual de-
s of this radiation are not sensitive
h to operate unless the number of
ns incident on them is so large as
ve the impression of a continuous
But in the high frequency portion
e electro-magnetic spectrum, the

photon energy is much greater, the detectors are more sensitive, and the photon nature of the radiation is more evident.

The wave properties of an individual photon, as shown by Taylor's experiment (Sect. 6–5), are difficult to visualize unless, as we did earlier, we think of a wave accompanying the photon and fixing the probability of its arrival at a given position. The approach to understanding the nature of radiation then becomes a mathematical approach. This method constitutes a part of quantum mechanics and is beyond the scope of this text.

An attempt to interpret or to assign a meaning to photon mass may also lead us far afield. Certainly the mass of a photon is not its mass when at rest, but an apparent mass which it possesses because it is moving with the speed of light. The fact that the mass of an object apparently increases sharply as its speed approaches the speed of light is a portion of the theory of relativity and necessitates a revision of Newtonian mechanics for very high speeds. But we shall not pursue further the discussion of this very interesting aspect of modern physics, except to say that the necessary modifications of Newtonian mechanics do not in any way invalidate Newton's ideas as applied to objects of normal size and speed.

6–12 MATTER WAVES

By 1923 the discovery of photons and their properties had forced a major revision in the ideas of classical physics. The formerly tidy compartments, one containing waves and the other containing particles, had had their walls shattered. And their walls were shattered further in subsequent years by the experimental verification of an hypothesis first suggested by the French physicist Louis deBroglie. deBroglie's ideas were as follows.

(*a*) The momentum p of a photon is given by the relationship

$$p = \frac{h}{\lambda}$$

where λ is the wavelength of the radiation.

(*b*) Moving particles possess momentum. Might it be that they too have a wavelength given by the relationship $\lambda = \frac{h}{p}$? And if so, would particles not have wave characteristics, just as waves have particle characteristics? For example, could particles be used to produce interference patterns similar to those produced by light?

Let us analyse briefly the requirements of any experiment designed to verify deBroglie's ideas. Since h is very small, λ is negligible unless p is also very small. Then the particle used must have small mass or low speed or both. The use of low speed particles proves to be impractical and as a result we must use small particles, preferably subatomic. Moreover, if a definite value of λ is to be obtained, p must be the same for all of the particles. The particles must all have the same mass and speed. Even under these circumstances the momentum is relatively large and the wave lengths are short. Therefore any slits used to produce the interference effects must be very narrow and closely spaced.

6–13 DIFFRACTION OF PARTICLES

Diffraction and interference of particles have now been demonstrated with many types of particles: electrons, neutrons and even helium atoms, for example. But

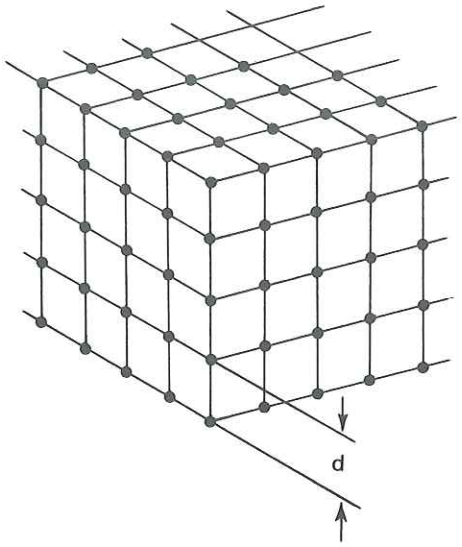


Fig. 6.11. In a crystal, successive planes of atoms are separated by a constant distance *d*. Therefore the crystal acts as a grating for particle diffraction.

electrons have proven to be the best for purposes of demonstration: they are easily obtainable; their speed and therefore their momentum can be controlled readily by altering the potential difference through which they are accelerated; and they produce excellent diffraction and interference patterns if a suitable grating can be found from which to reflect them or through which to pass them.

Mechanically ruled gratings or slits such as are used for visible light are too coarse to be of use for demonstrating particle diffraction. Various devices have been used; among the most satisfactory are crystals of various sorts. In 1912, Max von Laüe discovered that X-rays are regularly diffracted by crystals acting as diffraction gratings. By analysing the diffraction patterns von Laüe and others were able to infer what the structure of the crystal was, and to deduce the dis-

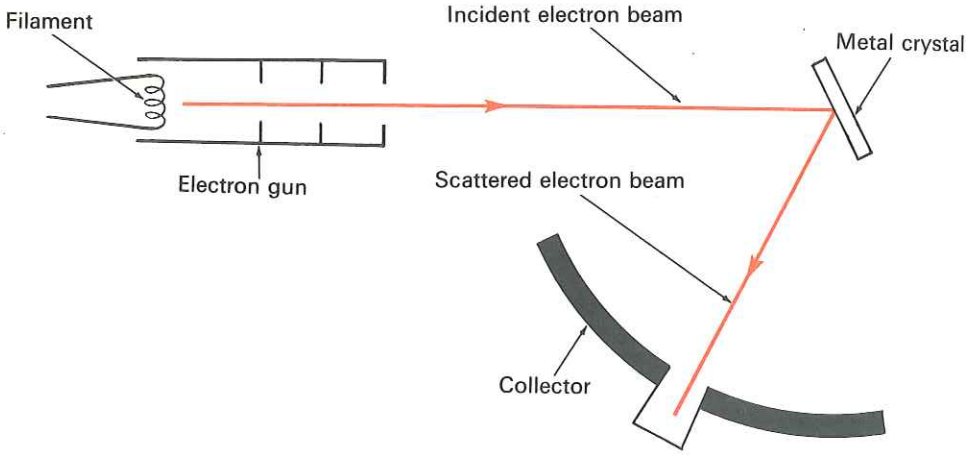
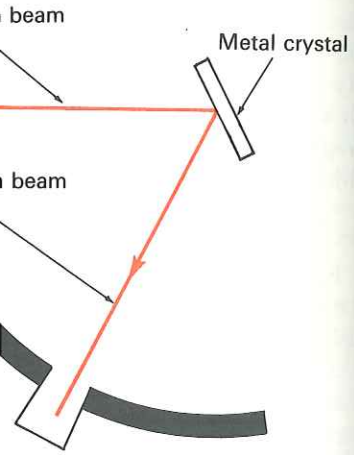


Fig. 6.12. The apparatus with which Davisson and Germer observed electron diffraction. The parts shown here were enclosed in a vacuum chamber.

have proven to be the best for
of demonstration: they are easily
le; their speed and therefore their
ım can be controlled readily by
he potential difference through
ey are accelerated; and they pro-
llent diffraction and interference
f a suitable grating can be found
ich to reflect them or through
pass them.
nically ruled gratings or slits
re used for visible light are too
be of use for demonstrating par-
raction. Various devices have
d; among the most satisfactory
als of various sorts. In 1912,
Laüe discovered that X-rays
rly diffracted by crystals acting
tion gratings. By analysing the
n patterns von Laüe and others
e to infer what the structure of
al was, and to deduce the dis-



observed electron diffraction. The

tance between successive planes of atoms in it. This distance d (Fig. 6.11), corresponding to the distance between successive lines in an optical diffraction grating, happens to be of the right order of magnitude to demonstrate particle diffraction.

Electron diffraction and interference were first detected in 1927, by C. J. Davisson and L. H. Germer in the United States, and by G. P. Thomson in England. Figure 6.12 shows, in simplified form, the apparatus used by Davisson and Germer at the Bell Telephone Laboratories. Slow electrons of known energy were fired from an electron gun at a metal crystal. The reflected electrons were detected by a collector which could be moved around an arc of a circle having the crystal as its centre. Davisson and Germer, by moving the electron collector around, found that the electrons were selectively scattered in certain directions.

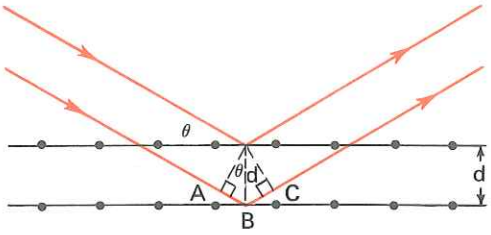
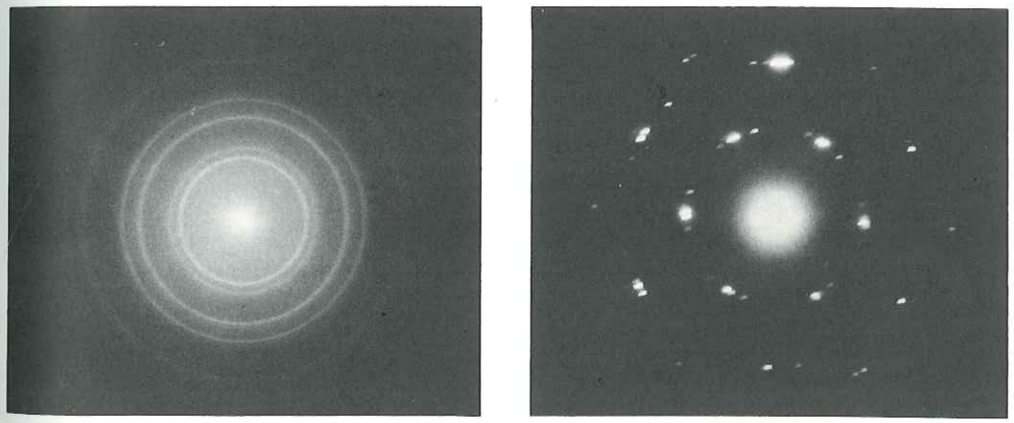


Fig. 6.13. The two reflected electron beams interfere constructively if $n\lambda = 2d \sin \theta$.

This selective scattering may be explained by assuming that the electrons have waves (deBroglie waves) associated with them. Some of the deBroglie waves associated with the electrons reflect from the first layer of the crystal, and some reflect from the second layer (Fig. 6.13). The extra distance travelled by those reflected from the second layer is ABC. If θ is the angle between the surface of the



(a) (b)

M. J. Curry

Fig. 6.14. Two photographs of the diffraction and interference patterns produced when electrons pass through thin crystals.

crystal and the direction of the incident electrons, and if d is the distance between the two layers, then $AB = d \sin \theta$, and the path difference for the two electron beams is $2d \sin \theta$. The reflected electron beams should then reinforce one another when θ is of such a size that the path difference is an integral number of deBroglie wave lengths, that is, when

$$n\lambda = 2d \sin \theta \qquad (n = 1, 2, 3, \ldots)$$

The values of θ calculated from this relationship, using values of λ calculated from the deBroglie relationship $\lambda = \frac{h}{p}$, agree very closely with the observed values. Thus deBroglie's hypothesis, which initially was no more than a guess, was verified.

Thomson demonstrated electron diffraction and interference by passing electrons through thin metal crystals. Figure 6.14(a) shows the effect of passing a beam of 10 kev* electrons through a face-centred cubic aluminum crystal. The transmitted electrons struck a fluorescent screen and the pattern on the screen was photographed. The bright spot at the centre of the pattern was caused by the undeflected beam. The bright circular rings were caused by constructive interference of the deBroglie waves associated with the electrons. The wave length was calculated to be 0.122 A.

Figure 6.14(b) shows the pattern produced by a 6 kev electron beam passing through a hexagonal graphite crystal. This pattern is similar to that produced when visible light passes through two successive diffraction gratings with their rulings at right angles to one another. The deBroglie wave length for these electrons was calculated to be 0.158 A.

*1 kev = 1000 ev

6–14 WORKED EXAMPLE

Calculate the deBroglie wave length of an electron which has been accelerated from rest through a potential difference of 10^2 volts.

SOLUTION

For the electron,

$$E_K = \tfrac{1}{2}mv^2 = 10^2 \text{ electron volts} = 1.60 \times 10^{-17} \text{ joules}$$

where v is its speed in m/sec

$$\text{But } m = 9.11 \times 10^{-31} \text{ kg}$$

$$\therefore v^2 = \frac{3.20 \times 10^{-17}}{9.11 \times 10^{-31}} \text{ m}^2/\text{sec}^2$$

$$\text{and } v = 5.9 \times 10^6 \text{ m/sec}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} =$$

$$\frac{6.62 \times 10^{-34} \text{ joule-sec}}{(9.11 \times 10^{-31} \text{ kg})(5.9 \times 10^6 \text{ m/sec})}$$

$$= 1.2 \times 10^{-10} \text{ m}$$

$$= 1.2 \text{ Angstrom units}$$

(Compare this wave length with the wave length range from 4000 A to 7800 A for visible light. Even for small particles of relatively low speed, the wave lengths are very short.)

gh a hexagonal graphite crystal. pattern is similar to that produced visible light passes through two ssive diffraction gratings with their s at right angles to one another. eBroglie wave length for these elec- was calculated to be 0.158 A.

WORKED EXAMPLE

culate the deBroglie wave length of ectron which has been accelerated rest through a potential difference volts.

TION

the electron,

$= \frac{1}{2}mv^2 = 10^2$ electron volts
$= 1.60 \times 10^{-17}$ joules
v is its speed in m/sec
ut $m = 9.11 \times 10^{-31}$ kg
$\therefore v^2 = \frac{3.20 \times 10^{-17}}{9.11 \times 10^{-31}}$ m^2/sec^2
and $v = 5.9 \times 10^6$ m/sec

$$\lambda = \frac{h}{p} = \frac{h}{mv} =$$

$$\frac{6.62 \times 10^{-34} \text{ joule-sec}}{11 \times 10^{-31} \text{ kg})(5.9 \times 10^6 \text{ m/sec})}$$

$= 1.2 \times 10^{-10}$ m
$= 1.2$ Angstrom units

pare this wave length with the wave range from 4000 A to 7800 A for e light. Even for small particles of vely low speed, the wave lengths ery short.)

6–15 PROBLEMS

Where necessary, assume that
(*a*) Planck's constant $= 6.62 \times 10^{-34}$ joule-sec
(*b*) Speed of electromagnetic radiation $= 3.0 \times 10^8$ m/sec
(*c*) Mass of an electron $= 9.11 \times 10^{-31}$ kg
(*d*) 1 elementary charge $= 1.60 \times 10^{-19}$ coulombs

1. Experiments on the photoelectric effect show that it requires 6.3 volts to remove an electron from platinum, and 5.0 volts to remove one from nickel. Insulated pieces of platinum and nickel are rubbed together. When these are separated, which of the two would you expect to be positively charged? Give a reason for your answer.
2. State how (*a*) the energy, and (*b*) the number of photoelectrons emitted by a surface are affected (*i*) if the intensity of the incident light is increased while its frequency is kept constant, (*ii*) if the frequency of the incident light is changed while its intensity is kept constant.
3. Calculate the value of Planck's constant in ev-sec.
4. Calculate the energy, in joules, of a photon of visible light of frequency 5.0×10^{14} c/s.
5. Calculate the frequency and the wave length of a photon having 1.0×10^{-19} joules of energy.
6. Calculate, in ev, the approximate limits of the range of photon energies in visible light.
7. What are the energies, in electron-volts, of photons of the following frequencies: (*a*) 1.0×10^{12} c/s (infrared), (*b*) 4.3×10^{14} c/s (red), (*c*) 7.5×10^{14} c/s (violet), (*d*) 1.0×10^{18} c/s (X-rays)?
8. What are the energies, in ev, of photons of the following wave lengths: (*a*) 1.5 A, (*b*) 3.0×10^3 A, (*c*) 6.0×10^3 A, (*d*) 1.0×10^7 A?
9. Show that $E = \frac{1.24 \times 10^4}{\lambda}$, where E is the photon energy in ev, and λ is the wavelength in Angstroms.
10. Use the relationship developed in Problem 9 to recalculate the answers for Problem 8.
11. Calculate the wave length for X-rays, each photon of which has 4.0×10^4 ev of energy.
12. A source of light radiates light energy in the visible range at a rate of 10^{-3} watts. Calculate the order of magnitude of the number of photons emitted per second.

13. A material having a work function of 0.5 ev is irradiated with light having a wave length of 6000 A. Calculate the maximum kinetic energy, in joules, of the emitted electrons.

14. A material having a work function of 2.0×10^{-19} joules/electron is irradiated with light of frequency 6.0×10^{14} c/s. Calculate the maximum kinetic energy of the emitted electrons.

15. In an experiment on the photoelectric effect, the light incident on the surface has a wave length of 4.0×10^{-7} m. The cut-off potential is found to be 1.0 volts. Calculate the work function of the surface.

16. The work function of potassium is 1.60 ev, of calcium is 3.33 ev, and of mercury is 4.50 ev. (*a*) Calculate the threshold frequency for each of the above elements. (*b*) Calculate the maximum kinetic energy of electrons emitted by each element, if the incident light is violet (frequency 7.5×10^{14} c/s).

17. Calculate the work function of nickel, in ev, from the following information. The cut-off potential is 1.2 volts when the wave length of the incident radiation is 2.0×10^3 A.

18. In a particular X-ray tube electrons are accelerated through a P.D. of 10^4 volts. As the electrons strike the target, they are decelerated and a few of them emit X-ray photons. Analysis of the X-ray spectrum shows that the spectrum is continuous, but that there is a definite minimum wave length. (*a*) Why is there a minimum wave length? (*b*) Calculate this minimum wave length.

19. Calculate the momentum of a photon of wave length 1.0×10^{-13} m.

20. The wave length of a photon is 5000 A. Calculate its momentum.

21. Calculate the momentum of a photon of frequency 3.0×10^{15} c/s.

22. The frequency of a photon is 7.4×10^{14} c/s. Calculate (*a*) its wave length in metres, (*b*) its energy in joules, (*c*) the magnitude of its momentum in kg-m/sec.

23. Show that the momentum p of a particle of mass m and kinetic energy E is given by the relationship $p = \sqrt{2mE}$.

24. Calculate the wave length of each of the following particles: (*a*) an electron having 20 ev of kinetic energy, (*b*) a 5.0×10^{-18} kg oil drop moving at 2.0 cm/sec, (*c*) a 1-ton car travelling at a speed of 30 mi/hr.

25. Calculate the deBroglie wave length for an electron (*a*) if its speed is 5.0×10^4 m/sec, (*b*) if its kinetic energy is (*i*) 1.0 ev, (*ii*) 2.0 ev, (*iii*) 4.0 ev.

26. What is the deBroglie wave length of a rifle bullet of mass 2.0 gm moving with a speed of 400 m/scc?

irradiated with light having
m kinetic energy, in joules,

10^{-19} joules/electron is ir-
/s. Calculate the maximum

the light incident on the
ne cut-off potential is found
the surface.

calcium is 3.33 ev, and of
d frequency for each of the
kinetic energy of electrons
violet (frequency 7.5×10^{14}

m the following information.
vave length of the incident

rated through a P.D. of 10^4
re decelerated and a few of
ay spectrum shows that the
nite minimum wave length.
b) Calculate this minimum

length 1.0×10^{-13} m.

ate its momentum.

ency 3.0×10^{15} c/s.

lculate (*a*) its wave length in
nitude of its momentum in

ss m and kinetic energy E is

ing particles: (*a*) an electron
10^{-18} kg oil drop moving at
d of 30 mi/hr.

electron (*a*) if its speed is
0 ev, (*ii*) 2.0 ev, (*iii*) 4.0 ev.

llet of mass 2.0 gm moving

27. An electron is accelerated from rest through a P.D. of 4.8×10^2 volts. Calculate (*a*) its kinetic energy (*i*) in electron-volts, (*ii*) in joules; (*b*) its speed; (*c*) its wave length.
28. Compute the wave length in metres for (*a*) a 5.0-ev photon, (*b*) a 5.0-ev electron.
29. Compute the momentum of (*a*) a 5.0-ev photon, (*b*) a 5.0-ev electron.

6–16 SUMMARY

Both radiation and matter have both wave and particle characteristics.

The photoelectric effect can be explained fully only by assuming the existence of photons. For a photon,

$$E = h\nu$$

and for the photoelectric effect,

$$E = h\nu - B$$

Photons appear to be guided by a wave which determines the probability of a photon's arrival at a particular point.

The Compton effect indicates that a photon has momentum. For a photon

$$p = \frac{h}{\lambda}$$

Particle diffraction experiments indicate that particles have a wave associated with them. For a particle

$$\lambda = \frac{h}{p}$$

Chapter 7

Atomic Structure

7–1 DIFFICULTIES

In Chapter 5 we saw why, as a result of alpha particle scattering experiments, Rutherford concluded that an atom consists of a dense positively charged nucleus with electrons orbiting around it. But Rutherford's atomic model left at least four questions unanswered. (1) If accelerated charge radiates energy, why do the centrally accelerated electrons not radiate energy and spiral in to the nucleus in a relatively short interval of time? In other words, how can a Rutherford atom be stable? (2) Why are the spectra of incandescent gases, line spectra rather than continuous spectra? (3) What are the positions of the electrons relative to the nucleus? That is, what are the radii of the electron orbits? (4) Why are the orbital radii not infinitely variable? That is, why are all of the atoms of a given element of the same size?

Rutherford readily admitted the shortcomings of his atomic model, and it was one of his co-workers, Niels Bohr (1885-1962) who developed a more satisfactory model. Bohr, a Danish physicist, joined Rutherford and his group in Manchester in 1911, after having worked for some time with J. J. Thomson. He was influenced by the work and ideas of Thomson and Rutherford, and also by the still unverified quantum theory advanced by Planck and Einstein. In addition, he exhibited remarkably daring intuition and a willingness to break with some of the established traditions of classical physics.

7–2 BOHR'S MODEL OF THE HYDROGEN ATOM

Rutherford's model of the hydrogen atom (Fig. 7.1) consisted of a nucleus containing one proton around which a single electron travelled in an orbit of unspecified (and presumably continuously variable) radius. Bohr accepted this general picture, and elaborated on it. His

his co-workers, Niels Bohr (1885-
who developed a more satisfactory
Bohr, a Danish physicist, joined
ford and his group in Manchester
, after having worked for some
ith J. J. Thomson. He was influ-
y the work and ideas of Thomson
therford, and also by the still un-
quantum theory advanced by
and Einstein. In addition, he
ed remarkably daring intuition
willingness to break with some of
tablished traditions of classical
.

OHR'S MODEL OF THE YDROGEN ATOM

herford's model of the hydrogen
(Fig. 7.1) consisted of a nucleus
ing one proton around which a
electron travelled in an orbit of
ified (and presumably continuous-
able) radius. Bohr accepted this
picture, and elaborated on it. His

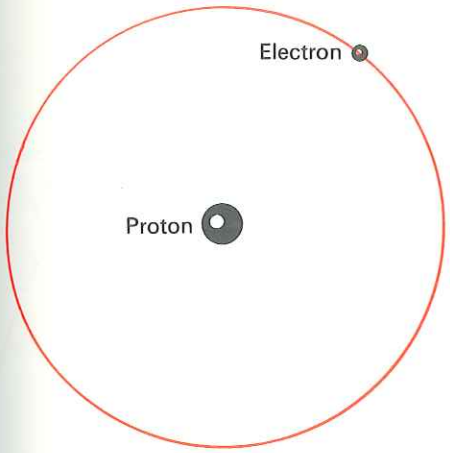


Fig. 7.1. Rutherford's model of the hydrogen atom.

novel and daring method of getting around the fact that an orbiting electron does not radiate energy was simply to accept it. In other words, he assumed that an orbiting electron normally does not radiate energy; that the laws which apply to electromagnetic radiation for large circuits do not apply to oscillators of atomic size. He assumed that the radius of rotation of the electron about the nucleus normally remains constant; the electron normally has a constant amount of energy and the atom is stable.

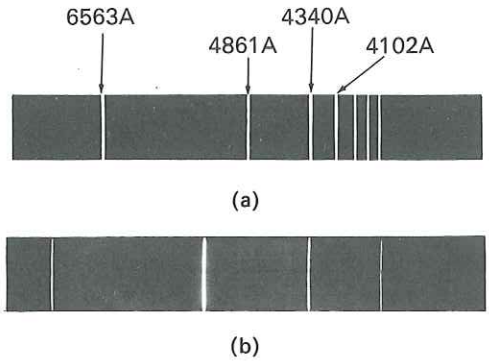
Perhaps the key word in the last sentence above is the word "normally". Hydrogen in a discharge tube certainly radiates energy and as it does so it produces a line spectrum. Bohr accounted for this radiation by assuming that the energy absorbed by a hydrogen atom as it was excited in a discharge tube caused the electron to move to an orbit of greater radius and higher energy. Later, the electron may return to its original orbit, radiating the energy that it absorbed when excited. The energy ΔE lost by the electron upon its return to its original orbit is radiated as a photon whose energy is $\Delta E = h\nu$. Therefore the frequency ν of the radiation is given by the relationship

$$\nu = \frac{\Delta E}{h}$$

But the frequency ν can have only those values associated with the lines in the spectrum of atomic hydrogen (Fig. 7.2). Therefore ΔE can have only certain values, and as a result the electron orbits can have only certain radii.

Bohr's method of arriving at this atomic model was more detailed (and more quantitative) than that outlined above, but the conclusions were those we have given. Let us recapitulate, remembering that we are still referring to the simplest atom, that of hydrogen.

(*a*) The electron may orbit the nucleus in any one of a large number of orbits



Physics Department, University of Western Ontario

Fig. 7.2. The bright line emission spectrum of atomic hydrogen. The drawing in *(a)* shows 4 lines in the visible portion, and three in the ultraviolet. The four lines in the visible portion can be seen in the photograph in *(b)*.

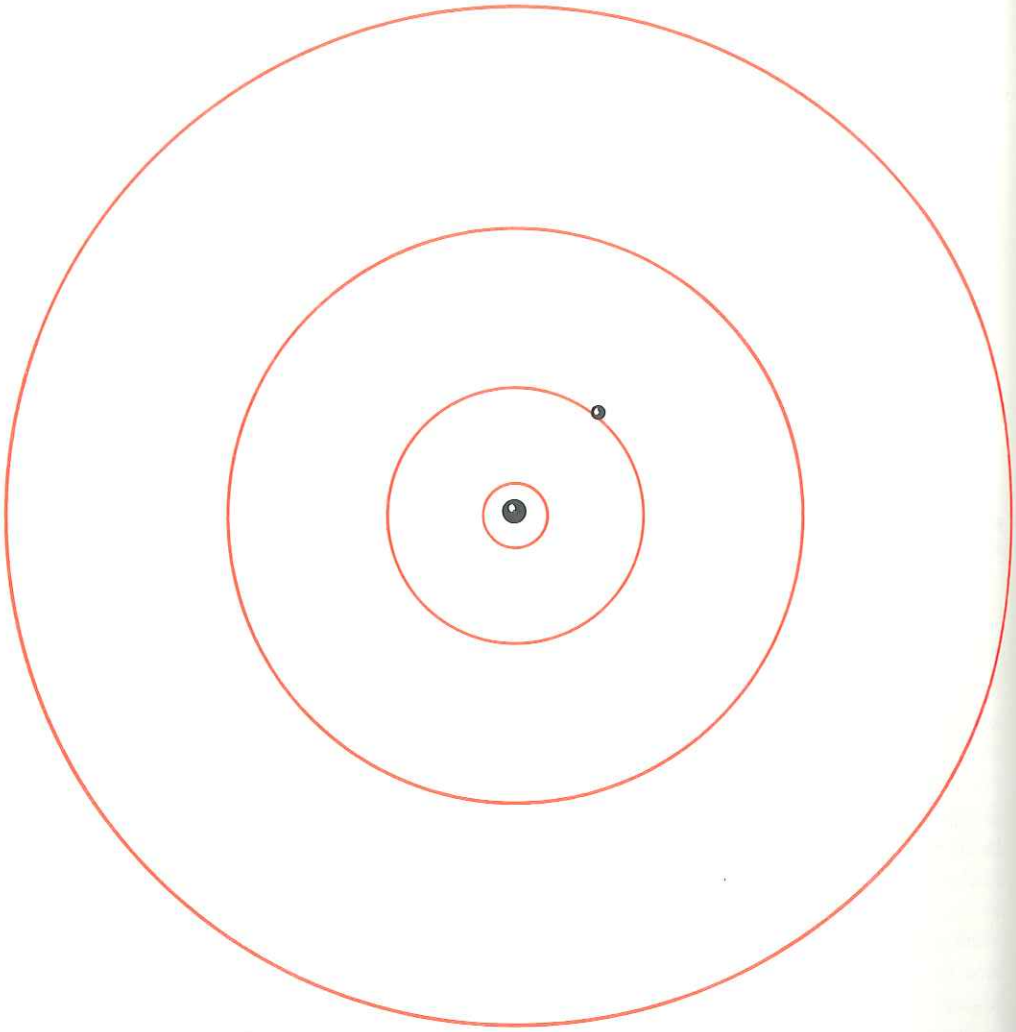
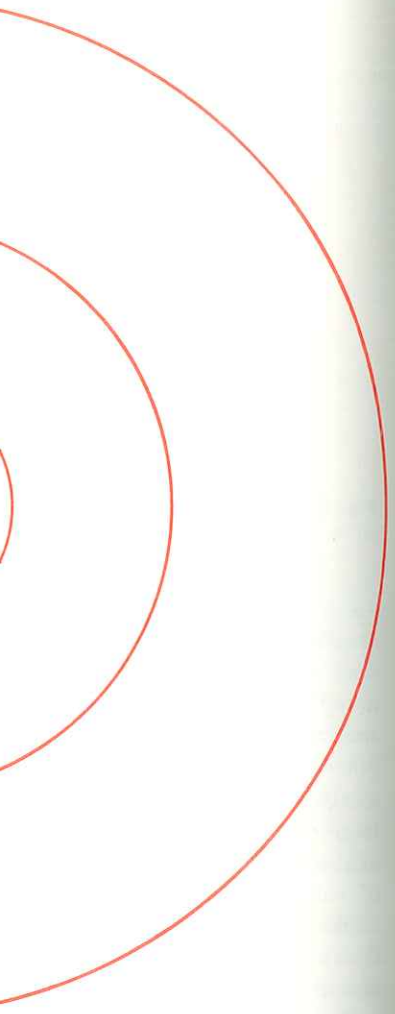


Fig. 7.3. Bohr's model of the hydrogen atom. Four of the permitted electron orbits are shown.

(Fig. 7.3). Bohr calculated that, if n is the number of the orbit and r the radius of the orbit, $r \propto n^2$. That is, the radii of the orbits, beginning with the smallest, are in the ratio 1 : 4 : 9 : 16 : 25—etc. More specifically, Bohr calculated that $r = 0.52 \times 10^{-10} \times n^2$ where r is the radius in metres. The radius of an unexcited hydrogen atom ($n = 1$) should therefore be 0.52×10^{-10} metres. This checked extremely well with the value of 0.5×10^{-10} metres calculated in other ways. This result seemed to indicate the validity of Bohr's assumptions. Moreover, it explained why all atoms of hydrogen are normally of the same size.



permitted electron orbits are shown.

hydrogen atom ($n = 1$) should
e be 0.52×10^{-10} metres. This
extremely well with the value of
10^{-10} metres calculated in other
This result seemed to indicate the
of Bohr's assumptions. Moreover,
ained why all atoms of hydrogen
mally of the same size.

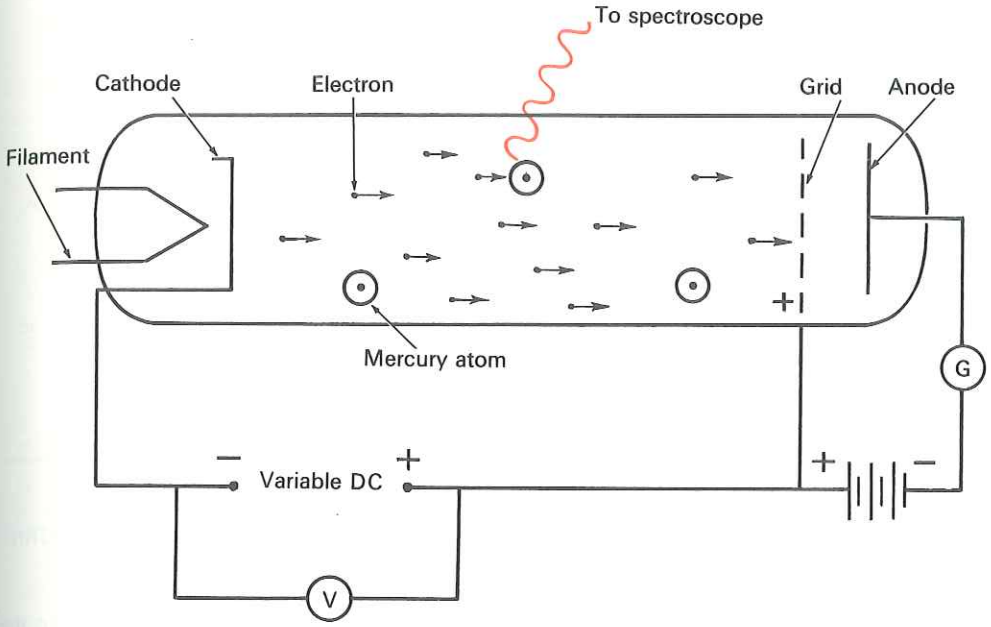


Fig. 7.4 (a). Arrangement of apparatus used in the Franck-Hertz experiment.

(*b*) Each orbit has associated with it a definite electron energy; the greater the radius of the orbit, the greater the energy. Since only certain orbital radii are permitted, the electron may possess only certain definite energies. This implies that an electron may accept energy only in certain definite amounts—those amounts necessary to raise it from its present energy level to some other permitted level. We shall see later how the Franck-Hertz experiments verified this prediction. Bohr's theory also predicted that the electron may radiate energy only in certain definite amounts—those amounts which it loses when "dropping" from an excited energy level to some lower energy level in an orbit of smaller radius. We have already had some indication of how this idea explains line spectra, and we shall examine the quantitative details of these spectra later.

7–3 THE FRANCK-HERTZ EXPERIMENT

James Franck and Gustav Hertz in Germany in 1914, and other physicists elsewhere in later years, carried out experiments which verified Bohr's assumption that atoms could accept energy only in sharply defined amounts.

Figure 7.4(*a*) is a simplified diagram of the apparatus used by Franck and Hertz for an experiment on the energy levels of mercury. Electrons are emitted by a heated cathode and given a known amount of kinetic energy by the potential difference applied between the cathode

and the grid. The grid is a fine wire screen, and most of the electrons pass through it. These electrons may also have enough energy to overcome the small retarding potential difference between the grid and the anode. They reach the anode, and their flow through the circuit is recorded by the galvanometer G.

The tube contains mercury vapour at about 250°C. Some of the electrons collide with mercury atoms. If the kinetic energy of the electrons is low, the collisions are elastic, and the electrons continue with undiminished kinetic energy and eventually reach the anode. Under these circumstances, the anode current increases as the potential difference between grid and cathode is increased (Fig. 7.4*b*).

When the kinetic energy of the electrons is about 4.9 ev, the anode current drops sharply. Apparently collisions between electrons and mercury atoms remove energy from the electrons, with the result that they can no longer overcome the retarding voltage between grid and anode.

If the kinetic energy of the electrons is gradually increased further by increasing the accelerating voltage between cathode and grid, the anode current increases again, but dips sharply each time the accelerating voltage reaches a multiple of 4.9 volts (Fig. 7.4*b*). In these cases, the electron collides several times with mercury atoms, surrendering 4.9 ev of energy at each collision.

Apparently the smallest amount of energy that the mercury atom can accept is 4.9 ev. If it is offered less than this amount, it refuses it. Moreover, if it is offered slightly more than 4.9 ev of energy, it takes the 4.9 ev and refuses

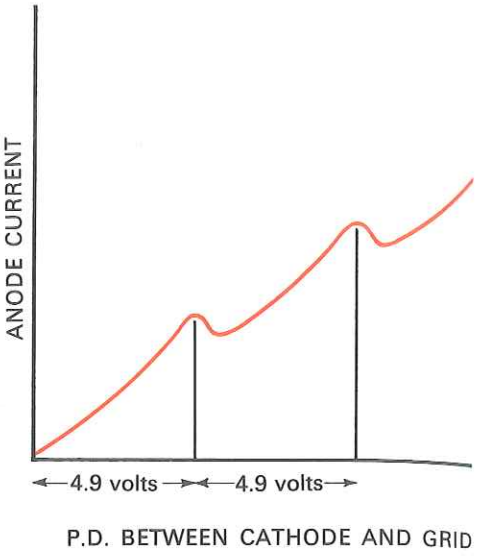


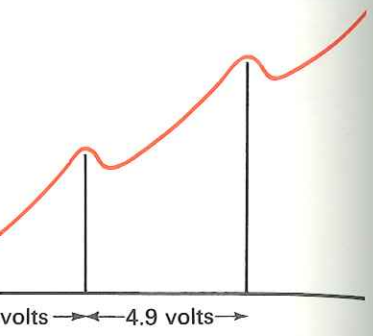
Fig. 7.4 (b). In the Franck-Hertz experiment, the anode current dips each time the accelerating voltage reaches a multiple of 4.9 volts.

the rest. For example, if the initial electron energy is 5.5 ev, the final electron energy is 0.6 ev.

The spectroscope (Fig. 7.4*a*) indicates the presence of radiation of wave length 2.5×10^3 A at the time the first dip in anode current occurs. We shall discuss the significance of this observation in Sections 7–4 and 7–5.

Further experiments show that 4.9 ev is not the only bundle of energy that the mercury atom will accept. If the initial electron energy is 6.7 ev or more, the atom will accept either 4.9 ev or 6.7 ev. If the initial energy is increased still further, it is found that there are still larger bundles of energy that the mercury atom will absorb.

Atoms other than those of mercury also absorb energy only in discrete



BETWEEN CATHODE AND GRID

b). In the Franck-Hertz experiment, the rent dips each time the accelerating aches a multiple of 4.9 volts.

For example, if the initial elec-rgy is 5.5 ev, the final electron 0.6 ev.

pectroscope (Fig. 7.4*a*) indicates ence of radiation of wave length 0^3 A at the time the first dip in urrent occurs. We shall discuss ificance of this observation in 7–4 and 7–5.

er experiments show that 4.9 ev e only bundle of energy that the atom will accept. If the initial energy is 6.7 ev or more, the l accept either 4.9 ev or 6.7 ev. itial energy is increased still fur-s found that there are still larger of energy that the mercury atom rb.

other than those of mercury sorb energy only in discrete

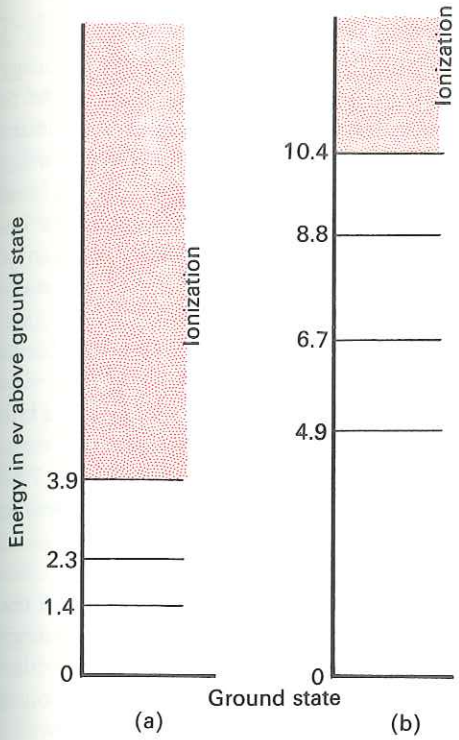


Fig. 7.5. Some of the energy levels for *(a)* cesium, *(b)* mercury.

amounts, but the amounts are not the same as for mercury; for each type of atom there are characteristic amounts of energy which it will accept. The internal energy of an atom can change, but the changes occur in definite steps. The resulting energy contents of the atom are called energy levels. For the minimum energy level, the atom is said to be in its ground state, and this energy level is usually taken as the zero of energy. The next energy level is reached when the atom accepts the smallest possible amount of energy. This added bundle of energy is called the first excitation energy and the atom is then in the first excited state.

In terms of Bohr's atomic model, the ground state of the hydrogen atom is associated with the smallest permitted orbital radius, the first excited state with the second permitted radius, etc. This orbital picture is now considered by many scientists to be too naive a picture of the hydrogen atom. It is perhaps better to describe the atom mathematically or graphically in terms of energy levels. Certainly the orbital model becomes complicated for atoms possessing many electrons. However, the idea of permitted orbits frequently assists us in our attempts to visualize the structure of the atom. Moreover, it helps us to understand one further observation from the Franck-Hertz experiment.

Referring again to mercury atoms, if the energy of the bombarding electrons is increased above 10.4 electron volts, the mercury atoms ionize, ejecting electrons and producing positive ions. In terms of the Bohr atomic model, sufficient energy has been supplied to remove an electron so far from the nucleus that the electron frees itself from the electrostatic attraction of the nucleus and no longer follows any orbit. The 10.4 electron volts is called the ionization energy for mercury. Each type of atom has its own ionization energy as well as its own energy levels. Figure 7.5 shows several of the energy levels for cesium and for mercury, and the ionization energy for each. An atom will accept any amount of energy greater than the ionization energy, for the electron liberated when an atom ionizes may have any amount of energy whatsoever.

Energy level diagrams such as those in Figure 7.5 are likely better pictures of atomic structure than diagrams showing a variety of orbits. And it is quite likely

that still better pictures will be devised as our information concerning atoms grows. But Bohr's model and the energy level idea combine to explain very well the production of emission and absorption spectra.

7–4 WORKED EXAMPLE

Calculate the photon energy for radiation of wave length 25×10^2 A.

SOLUTION

$$\lambda = 25 \times 10^2 \text{ A} = 25 \times 10^{-8} \text{ m}$$

$$c = 3.0 \times 10^8 \text{ m/sec}$$

$$\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{25 \times 10^{-8}} \text{ cps} = 12 \times 10^{14} \text{ cps}$$

$$E = h\nu$$

$$= 6.62 \times 10^{-34} \times 12 \times 10^{14} \text{ joules}$$

$$= \frac{6.62 \times 10^{-34} \times 12 \times 10^{14}}{1.60 \times 10^{-19}} \text{ ev}$$

$$= 4.9 \text{ ev}$$

7–5 ATOMIC EMISSION SPECTRA

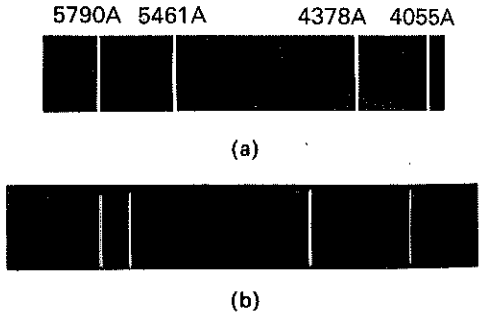
When an atom is excited (for example, in a discharge tube), its energy level increases from the ground state to one of the higher permitted states. But the atom remains in the excited state for only a short time (of the order of 10^{-8} sec) and then the energy drops to the ground state. The energy lost is radiated as a photon of radiation. Since the downward energy shifts are limited to certain permitted steps, the photon energy can have only several permitted values. That is, the radiation involves only a limited number of frequencies, and a line spectrum results.

Let us be more specific. A line of wave length 2537 A occurs in the ultraviolet portion of the spectrum of mercury. The worked example (Sect. 7–4) shows that a photon of about this wave length has an energy of about 4.9 ev. But this is the smallest amount of energy that a mercury atom can absorb—the energy that excites it from the ground state to the first excited state. This same amount of energy is reradiated as a photon when the atom returns to its ground state. Here we have an excellent correspondence between experimental result and theory. You can (and should) predict the wave lengths of several other lines in the spectrum of mercury, using the energy levels shown in Figure 7.5. Can you find any of these lines in Figure 7.6? Do you find any lines in this figure which do not correspond to the energy levels in Figure 7.5?

In general, the following relationship holds for emission spectra:

$$E = h\nu = E_i - E_f$$

where E or $h\nu$ is the energy of each of the radiated photons, E_i is the initial energy of the excited atom, and E_f is the final energy of the atom, frequently the ground state.



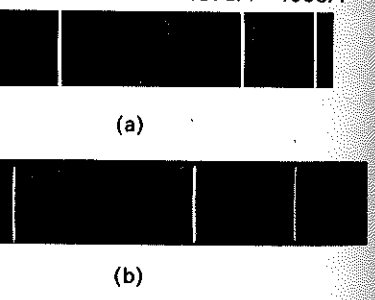
Physics Department, University of Western Ontario

Fig. 7.6. *(a)* A drawing of the visible portion of the spectrum of mercury. *(b)* A photograph of the visible portion of the mercury spectrum. You should calculate the energy of a photon of each of the wave lengths shown.

lest amount of energy that a atom can absorb—the energy tes it from the ground state to excited state. This same amount is reradiated as a photon when returns to its ground state. Here an excellent correspondence be- xperimental result and theory. (and should) predict the wave f several other lines in the spec- nercury, using the energy levels Figure 7.5. Can you find any ines in Figure 7.6? Do you find in this figure which do not corre- the energy levels in Figure 7.5? eral, the following relationship emission spectra:

$$E = h\nu = E_i - E_f$$

or $h\nu$ is the energy of each of the photons, E_i is the initial energy cited atom, and E_f is the final the atom, frequently the ground



artment, University of Western Ontario

a) A drawing of the visible portion of m of mercury. *(b)* A photograph of the on of the mercury spectrum. You should e energy of a photon of each of the ns shown.

7–6 ATOMIC ABSORPTION SPECTRA

Incandescent sodium vapour removes from the continuous spectrum of white light passing through the vapour those frequencies normally occurring in the emission spectrum of sodium. (See Figure 7.7.) Here again the explanation in terms of energy levels is relatively simple. The sodium atoms absorb from the white light those photons whose energy is the difference between two of the energy levels for sodium. It is true that the atoms whose energy levels are thus raised later reradiate the energy. However, they reradiate it in all directions. As a result, the intensity of the energy of this wave length, radiated in the direction of the spectroscope, is so small as to pass unnoticed.

In general, the following relationship holds for absorption spectra:

$$E_f = E_i + h\nu$$

where E_i is the energy level of the atom before the photon is absorbed, E_f is the energy level of the atom after the photon is absorbed, and $h\nu$ is the energy of the photon.

7–7 THE SPECTRUM OF ATOMIC HYDROGEN

The discussion of energy levels and their connection with spectra should lead you to suspect that the lines in an emission spectrum are not scattered haphazardly throughout the spectrum. In fact, they form definite series. These series were studied by Balmer, Rydberg and others in the early part of this century. Formulae were developed connecting the frequencies or wave lengths of the lines in any series, mainly for elements of low atomic weight, and particularly for hydrogen. Then in 1913 Bohr showed the relationships between his planetary model of the atom and these series of spectral lines. Bohr developed the following formula from theoretical considerations and previously known universal constants. For hydrogen

$$E = -\frac{13.6}{n^2} \text{ ev}$$

where E is the energy of an electron in the n^{th} permitted orbit for hydrogen, and it is assumed that the electron energy is

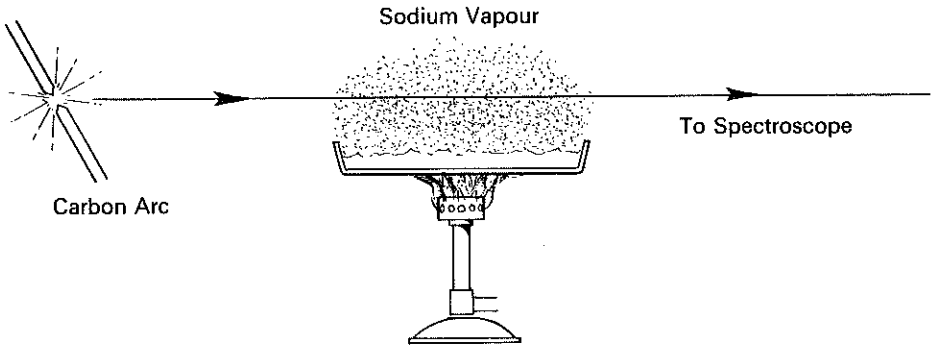


Fig. 7.7. A dark line absorption spectrum is produced when light from a carbon arc passes through incandescent sodium vapour.

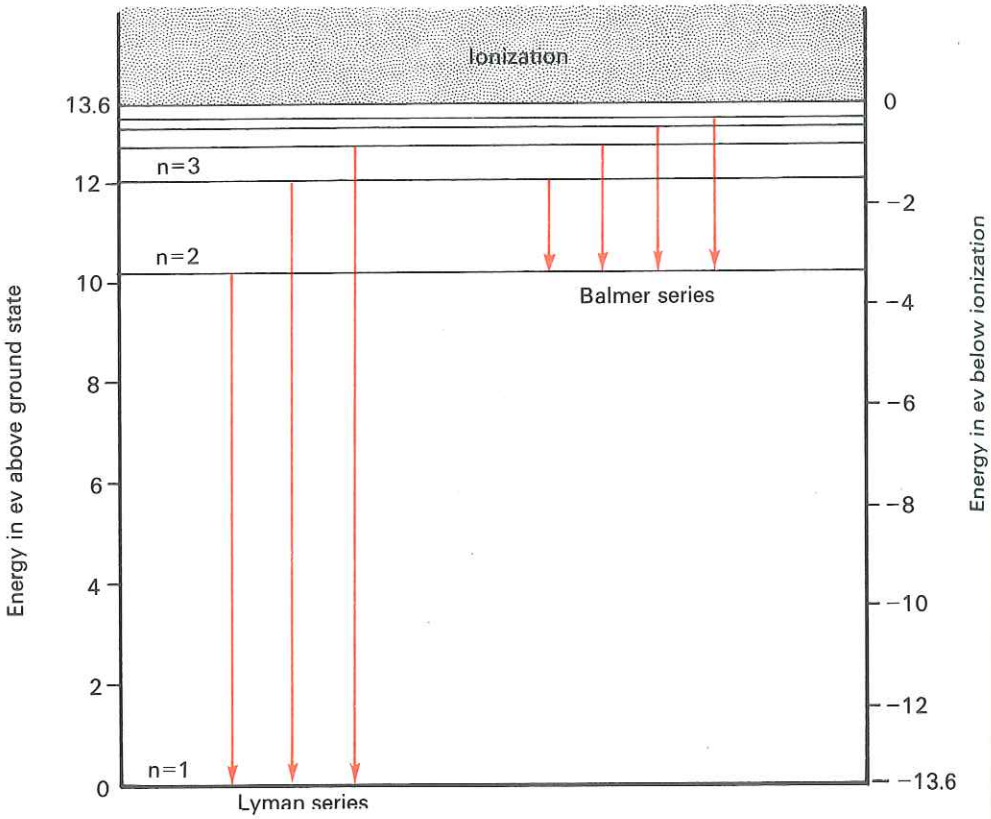


Fig. 7.8. The energy levels of hydrogen. Changes from higher energy levels to the ground state produce the Lyman series of spectral lines. Changes from higher energy levels to the second energy level produce the Balmer series, the four visible lines of which were shown in Figure 7.2.

zero at an infinite distance from the nucleus. (Recall that we made this assumption in Section 2–16. The negative sign in the above formula means that work has to be done on the electron to move it further from the nucleus. Therefore its energy increases, and if it is increasing toward zero, it must be negative.)

We will show later in this chapter how this formula may be developed. At the present time, let us consider some predictions which we can make from it.

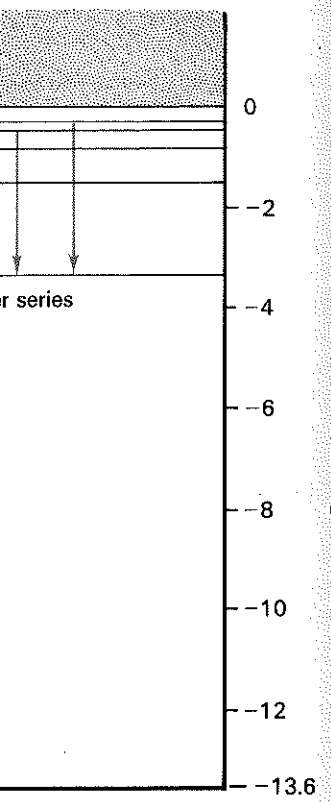
(*a*) If $n = 1$, $E = -13.6$ ev. That is, the energy level of the ground state is -13.6 ev.

(*b*) If $n = 2$,

$$E = \frac{-13.6}{4} \text{ ev} = -3.4 \text{ ev}$$

That is, the second energy level is 10.2 ev above the ground state. Similarly other energy levels may be calculated.

(*c*) If n becomes very large, E becomes very small. In the limit, when the electron



energy levels to the ground state
energy levels to the second energy
were shown in Figure 7.2.

$n = 1$, $E = -13.6$ ev. That
ergy level of the ground state
ev.

$= 2$,

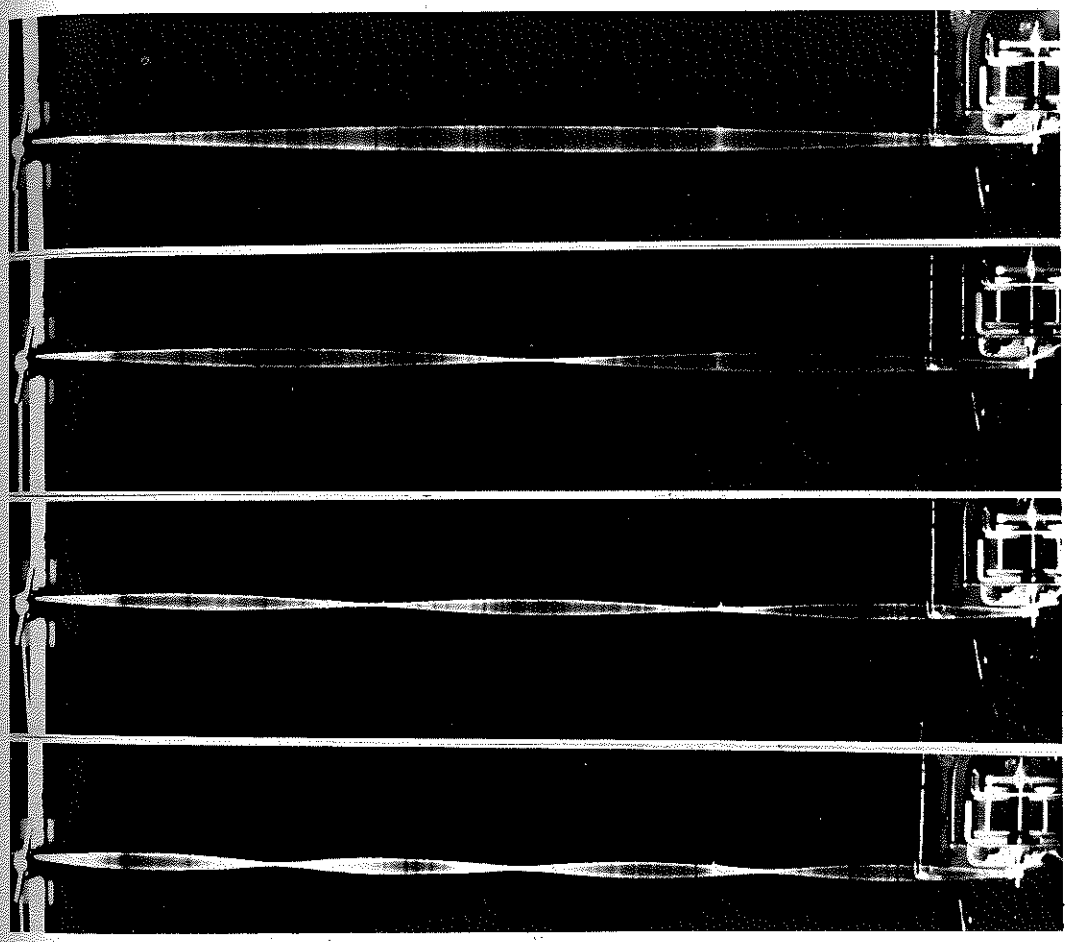
$$= \frac{-13.6}{4} \text{ ev} = -3.4 \text{ ev}$$

he second energy level is 10.2 ev
ground state. Similarly other
vels may be calculated.
becomes very large, E becomes
. In the limit, when the electron

is at an infinite distance from the nucleus, that is, when the hydrogen atom is ionized, the electron energy is zero. That is 13.6 ev above the ground state. Therefore the ionization energy of hydrogen should be 13.6 ev.

We have plotted some of these energy levels, and the ionization energy, in Figure 7.8. In this figure the zero of energy is the ground state, and the other energy levels are shown in electron volts above the ground state. Experiments of the Franck-Hertz type verify these predictions with amazing accuracy, and Bohr's atomic model is a tremendous success in this respect.

If we calculate the wave lengths of the radiation produced in energy level changes



Physics Department, University of Western Ontario

Fig. 7.9. Standing wave patterns produced by the interference of incident and reflected waves on a string.

from the second, third, fourth, etc. energy levels to the ground state (see Fig. 7.8) we obtain a set of wave lengths which belong to a series of lines in the spectrum of atomic hydrogen. This series is called the Lyman series, and had been identified, though it had not been explained, long before the energy level idea was developed. Another series of lines, the Balmer series (see Fig. 7.8 again), can be accounted for by energy level changes from the third, fourth, fifth, etc. energy levels to the second energy level. Again Bohr's model was a success, and after these successes the model was used constantly for many purposes. For example, because of the proven connection between energy levels and line spectra, considerable information about the structure of an atom can be obtained from the corresponding atomic spectrum.

7–8 AN EXPLANATION OF ENERGY LEVELS

Up to this point we have simply stated or observed experimentally that the orbital radii or energy levels can increase only by definite steps. Eventually we are bound to ask why. A reasonable explanation can be given with the help of deBroglie's concept of matter waves.

When standing waves are produced on a string (Fig. 7.9), the number of possible patterns is limited. The wave length of the incident and reflected waves must be such that the half wave length fits into the length of the string an integral number of times. That is, $L = n(\frac{1}{2}\lambda)$, where L is the length of the string, λ is the wave length, and $n = 1, 2, 3$, etc. Thus λ cannot be greater than $2L$, that is, the wave length has a maximum which it cannot exceed. Therefore the frequency has a minimum below which standing waves are impossible and above which standing waves are possible for a limited number of permitted values. We might be tempted to say that the string can vibrate in a "ground state" and in certain higher "excited states". For these states, the incident and reflected waves reinforce one another, resonance occurs, and the energy remains constant except for frictional losses.

Now consider the electron in a hydrogen atom. If the electron is moving at constant speed about the nucleus in a circular orbit of constant radius, its momentum must be constant. Therefore it must have a constant deBroglie wave length, given by the relationship $\lambda = \frac{h}{p}$. If we imagine this matter wave travelling along the circumference of the orbit, we realize that it cannot reinforce itself unless the circumference is an integral multiple of the deBroglie wave length (Fig. 7.10). Under these circumstances, and only under these circumstances, can the deBroglie wave maintain its energy; thus the permitted orbits and energy states are accounted for. Moreover, there is a maximum permitted wave length and a minimum permitted frequency. Thus there is a minimum electron energy and the ground state is accounted for.

The above explanation is oversimplified. The wave is actually three dimensional and is best described mathematically. Moreover, the electron does not necessarily follow a circular orbit, (or any orbit, for that matter). The visual model of the atom becomes more and more blurred as time goes on, and the mathematical description becomes more and more precise. And yet, assuming a

mum below which standing
mpossible and above which
ves are possible for a limited
permitted values. We might
to say that the string can
"ground state" and in cer-
"excited states". For these
ncident and reflected waves
e another, resonance occurs,
rgy remains constant except
l losses.

der the electron in a hydrogen
electron is moving at constant
the nucleus in a circular orbit
adius, its momentum must be
herefore it must have a con-
glie wave length, given by the
$\lambda = \frac{h}{p}$. If we imagine this
e travelling along the circum-
he orbit, we realize that it
orce itself unless the circum-
an integral multiple of the
ave length (Fig. 7.10). Under
stances, and only under these
es, can the deBroglie wave
energy; thus the permitted
energy states are accounted
er, there is a maximum per-
ve length and a minimum
frequency. Thus there is a
ectron energy and the ground
unted for.

ve explanation is over-
The wave is actually three
and is best described mathe-
Moreover, the electron does
rily follow a circular orbit,
t, for that matter). The visual
he atom becomes more and
d as time goes on, and the
al description becomes more
recise. And yet, assuming a

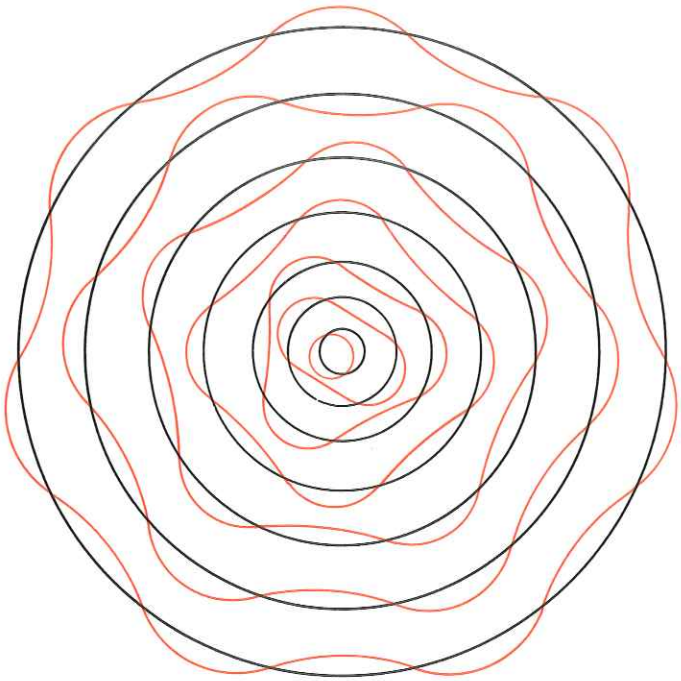


Fig. 7.10. deBroglie standing wave patterns for an electron on each of the first 7 Bohr orbits for hydrogen.

simple planetary model, assuming circular orbits, and using relatively simple mathematics, we can develop the relationship that we have already used so successfully for hydrogen, namely

$$E = \frac{-13.6}{n^2} \text{ ev}$$

The development of this formula, shown in the next section, is not quite as precise or mathematically rigid as Bohr's method, but it yields a satisfactory result. And perhaps it provides as good a summary and as fitting a conclusion for this series of books as it is possible to find, for it employs and ties together deBroglie's ideas, Bohr's ideas, Coulomb's law, the centripetal force formula, and our concepts of standing waves.

7–9 CALCULATING ENERGY LEVELS AND ORBITAL RADII

When an electron rotates about a nucleus, the centripetal force necessary to keep the electron in orbit is supplied by the electrostatic force of attraction between the electron and the nucleus. Thus

$$\frac{mv^2}{r} = \frac{kqQ}{r^2}$$

or

$$mv^2 = \frac{kqQ}{r} \qquad (1)$$

where m is the mass of the electron,
v is its linear speed,
r is its orbital radius,
q and Q are the charges on the electron and the nucleus respectively,
and k is the constant in Coulomb's law.

In order that the associated deBroglie waves may form a standing wave,

$$n\lambda = 2\pi r \qquad (n = 1, 2, 3, \ldots)$$

But $$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\therefore \frac{nh}{mv} = 2\pi r \qquad (2)$$

From (2) $$m = \frac{nh}{2\pi rv}$$

Substitute in (1)

$$\frac{nh}{2\pi rv} \cdot v^2 = \frac{kqQ}{r}$$

whence $$v = \frac{2\pi kqQ}{nh} \qquad (3)$$

This equation (3) permits us to calculate the possible speeds of an orbital electron. Suppose we do so for the hydrogen atom. Both q and Q in this case are 1 elementary charge, $k = 2.3 \times 10^{-28}$ newton-(metres)^2/(elementary charge)^2; and $h = 6.62 \times 10^{-34}$ joule-sec. Substituting these values in (3), we obtain

$$v = \frac{2.18 \times 10^6}{n} \text{ m/sec}$$

Thus the electron speed in the first orbit is 2.18×10^6 m/sec; and for all orbits the speed varies inversely as the number of the orbit.

Let us next compute the permitted orbital radii. From equation (2)

$$v = \frac{nh}{2\pi rm}$$

From equation (3)

$$v = \frac{2\pi kqQ}{nh}$$

$$\therefore \frac{nh}{2\pi rm} = \frac{2\pi kqQ}{nh}$$

$$\therefore r = \frac{n^2h^2}{4\pi^2kqQm} \qquad (4)$$

For the hydrogen atom we substitute the previously stated values for k, q, Q, and h, and 9.11×10^{-31} kg for the mass m of the electron. Then

$$r = 5.3 \times 10^{-11} \times n^2 \text{ metres}$$

We have already discussed the significance of this prediction in Sections 5–2 and 5–8.

Next we may compute electron energy levels. The energy of the electron is partly kinetic and partly potential; its total energy is the sum of the kinetic and potential energies. Therefore

$$E = \tfrac{1}{2}mv^2 - \frac{kqQ}{r} \qquad (5)$$

But, from (3), $$v = \frac{2\pi kqQ}{nh}$$

and from (4) $$r = \frac{n^2h^2}{4\pi^2kqQm}$$

Substituting these values in (5) we obtain

$$E = -\frac{(2\pi kqQ)^2m}{2n^2h^2} \qquad (6)$$

Therefore for the hydrogen atom

$$E = \frac{-2.17 \times 10^{-18}}{n^2} \text{ joules}$$

But one joule $= 6.25 \times 10^{18}$ ev

Therefore $$E = -\frac{13.6}{n^2} \text{ ev}$$

This, of course, is the relationship which we discussed and used earlier in this chapter.

ion (3)

$$v = \frac{2\pi kqQ}{nh}$$

$$\therefore \frac{nh}{2\pi rm} = \frac{2\pi kqQ}{nh}$$

$$\therefore r = \frac{n^2h^2}{4\pi^2kqQm} \quad (4)$$

rogen atom we substitute the tated values for k, q, Q, and $\times 10^{-31}$ kg for the mass m of . Then

$.3 \times 10^{-11} \times n^2$ metres

lready discussed the signifi- is prediction in Sections 5–2

may compute electron energy nergy of the electron is partly partly potential; its total he sum of the kinetic and ergies. Therefore

$$E = \tfrac{1}{2}mv^2 - \frac{kqQ}{r} \quad (5)$$

3), $v = \dfrac{2\pi kqQ}{nh}$

) $r = \dfrac{n^2h^2}{4\pi^2kqQm}$

; these values in (5) we obtain

$$E = -\frac{(2\pi kqQ)^2m}{2n^2h^2} \quad (6)$$

or the hydrogen atom

$$\frac{-2.17 \times 10^{-18}}{n^2} \text{ joules}$$

le $= 6.25 \times 10^{18}$ ev

$$E = -\frac{13.6}{n^2} \text{ ev}$$

rse, is the relationship which ed and used earlier in this

7–10 PROBLEMS

Where necessary assume that
(a) Planck's constant $= 6.62 \times 10^{-34}$ joule-sec
(b) the mass of the electron $= 9.11 \times 10^{-31}$ kg
(c) the constant in Coulomb's law

$$= 2.3 \times 10^{-28} \frac{\text{newton-(metre)}^2}{\text{(elementary charge)}^2}$$

1. How do the energy changes in an atom differ from the kinetic energy changes of a car speeding up or slowing down?
2. Refer to Figure 7.5. What are the possible energy changes when (a) cesium is bombarded with 3.0 ev electrons, (b) mercury is bombarded with 9.5 ev electrons?
3. What energy level changes within an atom will give rise to radiation of wave length (a) 1240 A, (b) 6200 A, (c) 62000 A?
4. What energy level changes within an atom will give rise to radiation of frequency (a) 2.0×10^{14} c/s, (b) 4.0×10^{15} c/s, (c) 1.0×10^{15} c/s?
5. What radiation wave length corresponds to an energy level change of (a) 1.0 ev, (b) 2.5 ev, (c) 5.0 ev?
6. What radiation frequency corresponds to an energy level change of (a) 2.0 ev, (b) 6.0 ev, (c) 8.0 ev?
7. If the energy levels of an atom are known, its line spectrum can be predicted. Is the converse necessarily true?
8. Consider the relationship $E = -\dfrac{13.6}{n^2}$. (a) What is the effect on E of changing n by a factor of 3? (b) Is there any limit to the number of energy levels? What is true of the energy levels for large values of n?
9. Use the relationship $E = -\dfrac{13.6}{n^2}$ ev to compute the first 6 energy levels for the hydrogen atom. Draw the energy level diagram, showing two scales, one with zero at ionization and one with zero at the ground state. Then calculate the wave length, in Angstroms, of the first three spectral lines produced by transitions from higher energy levels to (a) $n = 1$ (the Lyman series), (b) $n = 2$ (the Balmer series), (c) $n = 3$ (the Paschen series). In what portion or portions of the electromagnetic spectrum does each series fall?
10. Use your answers to problem 9 to list wave lengths that you would expect to be absorbed when white light is passed through incandescent hydrogen.
11. Calculate the radii, in metres, of the first 5 Bohr orbits for hydrogen.

12. The nucleus of the hydrogen atom is thought to be about 1.5×10^{-13} cm in diameter. If the nucleus were magnified to have a diameter of 0.10 mm (the size of a speck of dust), how far away would the ground state electron be?
13. Calculate the speed, in m/sec, of an electron in each of the first 5 Bohr orbits for hydrogen.
14. Use your answers to problem 11 to calculate the circumference of each of the first 2 Bohr orbits for hydrogen. Then use your answers to problem 13 to calculate the wave length in each of these orbits. Show that, in each case, the circumference is an integral multiple of the electron wave length.
15. Calculate the kinetic energy of an electron in each of the first two Bohr orbits for hydrogen.
16. Calculate the electric potential energy of an electron in each of the first two Bohr orbits for hydrogen.
17. (*a*) Calculate the total energy (kinetic plus potential) for an electron in each of the first two Bohr orbits for hydrogen. (*b*) Calculate the binding energy for an electron in each of these two orbits.
18. Refer to Figure 7.5(*b*). What effect would you predict when mercury vapour is bombarded by (*a*) a 7.0 ev electron, (*b*) an 11.4 ev electron, (*c*) a 7.0 ev photon, (*d*) an 11.4 ev photon.

7–11 SUMMARY

Associated with each type of atom there are definite and discrete energy levels. These energy levels may be explained in terms of deBroglie standing waves for the orbital electrons. The permitted orbits are such that their circumferences are integral multiples of the electron wave lengths.

Atomic emission spectra are produced when the electrons in an atom, after the atom has been excited, drop to a lower energy level and emit photons of radiation. In such a case,

$$E = h\nu = E_i - E_f$$

Atomic absorption spectra are produced when white light passes through an incandescent gas. Electrons in atoms of the gas are excited by the incident light, and absorb energy from the light. In these cases,

$$E_f = E_i + h\nu$$

For atomic hydrogen,

$$E = -\frac{13.6}{n^2} \text{ ev}$$

tomic, 51
nolecular, 51
ction, 71

X

1
oe, 41

ANSWERS

Chapter 2—Section 2-17, page 24

1. F changes by a factor of (*a*) 3 (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{9}$ (*d*) $\frac{1}{6}$
2. (*a*) 9.0×10^{-6} newt (*b*) 5.8×10^{-6} newt (*c*) 4.0×10^{-6} newt
3. (*a*) 1.5×10^{-5} newt (*b*) 1.2×10^{-4} newt (*c*) 1.0×10^{-5} newt
4. 9
5. 0.577
6. 3.6×10^{-4} newt
7. 1.8×10^{-2} newt
8. 34 newt
9. 3.3×10^{-6} coul
10. 4.5×10^{-6} newt
11. 2.0×10^{2} newt/coul
12. 1.3×10^{2} m
13. 3.0×10^{3} newt/coul
14. 2.0×10^{2} volts
15. 1.0×10^{-4} coul
16. 1.5 joules; 5.0 newt
17. 10^{3} volts/m
18. 8.0×10^{3} newt/coul
19. (*b*) 1.28×10^{-16} joules
20. 3.0×10^{3} volts
21. 5 volts
22. (*a*) 3.2×10^{-4} joules (*b*) 6.4×10^{-4} newt (*c*) 3.2×10^{-4} joules (*d*) 8 m/sec
23. 1.6×10^{-13} joules; 1.4×10^{7} m/sec
24. (*a*) 7.3×10^{14} m/sec^2 (*b*) 6.6×10^{-16} newt (*c*) 83 volts
25. 5.3×10^{7} m/sec
26. 26 volts
27. 9.2×10^{6} m/sec
29. V changes by a factor of (*a*) 10 (*b*) 5
30. (*b*) (*i*) 2.7 joules (*ii*) 1.35 joules (*iii*) 0.90 joules
31. (*b*) (*i*) 4.8×10^{4} volts (*ii*) 3.6×10^{4} volts (*iii*) 2.4×10^{4} volts

Chapter 3—Section 3-11, page 33

1. 6
3. 0.25 amp
4. 1.5 min
5. (*a*) 600 (*b*) 3.75×10^{21}
6. (*a*) 1 coul (*b*) 2.5 amp (*c*) 5 min
7. (*a*) 7.2×10^{4} joules (*b*) 120 watts
8. (*a*) (*i*) 0.50 amp (*ii*) 1.0 amp (*b*) 2.4×10^{2} watts (*d*) 2.16×10^{5} joules
9. 0.51
10. 3.2×10^{-15} joules; 0.96 watts
11. (*a*) 4.8×10^{2} watts (*b*) 88 watts (*c*) 1.6×10^{2} coul

Chapter 4—Section 4-11, page 46

3. (*a*) 1.7×10^{-3} m
5. (*a*) 1.8×10^{4} newt/coul (*b*) negatively (*c*) 8.0×10^{-19} coul (*d*) 5 surplus
6. (*a*) 1.2×10^{4} volts 7. 2.9×10^{-15} kg
8. 3.75×10^{15}
9. 1.88×10^{12}
10. 2.5×10^{11}
11. (*a*) -4.00×10^{-6} coul (*b*) 2.00×10^{-4} coul
12. (*a*) 1.3×10^{19} (*b*) 2.5×10^{20} (*c*) 5×10^{16}
13. (*a*) 0.100 (*b*) 0.02 (*c*) 8.0×10^{-3}
14. 2.6×10^{-2} amp

15. (*a*) 30 (*b*) 1.9×10^{20} 16. 1.0×10^{16}
17. (*a*) 3.8×10^{2} (*b*) 5.0×10^{19} (*c*) 1.25×10^{14}
18. (*a*) 3.2×10^{-19} (*b*) 8.0×10^{-12} (*c*) 9.6×10^{-21}
19. (*a*) 3.2×10^{-5} amp (*b*) 6.4×10^{-5} amp 20. 2.1×10^{-25} kg
21. (*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\frac{2}{\sqrt{3}}$ 22. 2.0×10^{5}
23. (*a*) 3.0×10^{2} ev (*b*) 2.4×10^{-27} kg
24. (*a*) 1.4×10^{-17} (*b*) 1.8×10^{2} ev
25. (*a*) (*i*) 45 ev (*ii*) 7.2×10^{-17} newt (*b*) 1.8×10^{2} volts
26. (*a*) $\frac{1}{\sqrt{2}}$ (*b*) 100 27. 2×10^{4} m/sec^2

Chapter 5—Section 5-13, page 60

1. 10^{-11}
3. (*a*) 1.92×10^{-18} (*b*) 4.64×10^{-18} (*c*) 2.08×10^{-18}
(*d*) 1.26×10^{-17} 4. $2.4 \times 10^{-11}\%$ 7. 0.40 newt
8. (*a*) 1.6×10^{-36} newt (*b*) 2.5×10^{35} (*c*) 2.5×10^{35}
9. 1.2×10^{-13} joules 10. 2.8×10^{-14} m 11. 3.7×10^{-13}
12. -3.3×10^{4} m/sec; 6.7×10^{4} m/sec
14. (*a*) 1.5×10^{4} m (*b*) 2.0 m (*c*) 5.0×10^{3} A (*d*) 1.0 A
15. (*a*) 7.5×10^{3} c/s (*b*) 5.0×10^{14} c/s (*c*) 1.5×10^{20} c/s
16. 10^{9} 17. 8.3×10^{7} c/s
20. (*a*) 4.0×10^{7} m/sec (*b*) 4.5×10^{3} volts
21. 2.0×10^{18} c/s 22. 1.5×10^{15}

Chapter 6—Section 6-15, page 77

3. 4.14×10^{-15} 4. 3.3×10^{-19}
5. 1.5×10^{14} c/s; 2.0×10^{-6} m 6. $1\frac{1}{2}$ to 3
7. (*a*) 4.1×10^{-3} (*b*) 1.8 (*c*) 3.1 (*d*) 4.1×10^{3}
8. (*a*) 8.3×10^{3} (*b*) 4.1 (*c*) 2.1 (*d*) 1.2×10^{-3}
11. 0.31 A 12. 10^{15}
13. 2.5×10^{-19} 14. 2.0×10^{-19} joules
15. 2.1 ev/electron
16. (*a*) 3.9×10^{14} c/s; 8.0×10^{14} c/s; 1.1×10^{15} c/s
(*b*) 1.5 ev; no emission; no emission
17. 5.0 18. (*b*) 1.24 A
19. 6.6×10^{-21} kg-m/sec 20. 1.3×10^{-27} kg-m/sec
21. 6.6×10^{-27} newt-sec
22. (*a*) 4.1×10^{-7} (*b*) 4.9×10^{-19} (*c*) 1.6×10^{-27}

16. 1.0×10^{16}
(*c*) 1.25×10^{14}
(*c*) 9.6×10^{-21}
20. 2.1×10^{-25} kg
22. 2.0×10^{5}

(*b*) 1.8×10^{2} volts
27. 2×10^{4} m/sec^2

(*c*) 2.08×10^{-18}
7. 0.40 newt
(*c*) 2.5×10^{35}
11. 3.7×10^{-13}

^{3} A (*d*) 1.0 A
(*c*) 1.5×10^{20} c/s

(*d*) 4.1×10^{3}
(*d*) 1.2×10^{-3}

oules

g-m/sec

(*c*) 1.6×10^{-27}

24. (*a*) 2.7 A (*b*) 6.6×10^{-15} m (*c*) 10^{-38} m
25. (*a*) 1.5×10^{-8} m (*b*) (*i*) 1.2×10^{-9} m
(*ii*) 8.5×10^{-10} m (*iii*) 6.0×10^{-10} m
26. 8.3×10^{-34} m
27. (*a*) (*i*) 4.8×10^{2} ev (*ii*) 7.7×10^{-17} joules
(*b*) 1.3×10^{7} m/sec (*c*) 5.6×10^{-11} m
28. (*a*) 2.5×10^{-7} (*b*) 5.5×10^{-10}
29. (*a*) 2.7×10^{-27} kg-m/sec (*b*) 1.2×10^{-24} kg-m/sec

Chapter 7—Section 7-10, page 93

3. (*a*) 10.0 ev (*b*) 2.0 ev (*c*) 0.20 ev
4. (*a*) 0.83 ev (*b*) 17 ev (*c*) 4.1 ev
5. (*a*) 1.2×10^{4} A (*b*) 5.0×10^{3} A (*c*) 2.5×10^{3} A
6. (*a*) 4.8×10^{14} c/s (*b*) 1.4×10^{15} c/s (*c*) 1.9×10^{15} c/s
8. (*a*) E changes by a factor of $\frac{1}{9}$
9. (*a*) 1.2×10^{3}; 1.0×10^{3}; 9.7×10^{2}
(*b*) 6.5×10^{3}; 4.9×10^{3}; 4.3×10^{3}
(*c*) 1.9×10^{4}; 1.3×10^{4}; 1.1×10^{4}
11. 5.3×10^{-11}; 2.1×10^{-10}; 4.8×10^{-10}; 8.5×10^{-10}; 1.3×10^{-9}
12. about 3 m
13. 2.18×10^{6}; 1.09×10^{6}; 7.3×10^{5}; 5.5×10^{5}; 4.4×10^{5}
15. 2.2×10^{-18} joules; 5.4×10^{-19} joules
16. -4.3×10^{-18} joules; -1.1×10^{-18} joules
17. (*a*) -2.2×10^{-18} joules; -5.5×10^{-19} joules
(*b*) 2.2×10^{-18} joules; 5.5×10^{-19} joules