Assignment 6

1a) Density $\frac{N}{V} = \frac{P}{k_B T}$

$$= \frac{1 \times 10^5 \text{ Pascal}}{1.38 \times 10^{-23} \text{ J/K} \cdot 273 \text{ K}}$$

$$= 2.7 \times 10^{25} \text{ particles/m}^3$$

$$= 2.7 \times 10^{19} \text{ part./cm}^3$$

b) $n(1 \text{ torr}) = \frac{2.7 \times 10^{19}}{760}$

$$= 3.6 \times 10^{16} \text{ cm}^{-3}$$

$n(10^{-7} \text{ torr}) = 3.6 \times 10^{9} \text{ cm}^{-3}$

2) $\frac{c_p - c_V}{R} = \left[1 + \frac{a N_A^2}{PV^2}\left(\frac{2 N_A b}{V} - 1\right)\right]^{-1}\left(1 + \frac{a N_A^2}{PV^2}\right)$

$$= \left[1 + 5.5 \frac{l^2 \text{ atm}}{\text{mole}^2} \frac{1}{100 \text{ atm}} \frac{1}{(0.3 \, l/\text{mole})^2}\right.$$

$$\left. \cdot \left(2 \frac{0.030 \, l/\text{mole}}{0.3 \, l/\text{mole}} - 1\right)\right]^{-1} \cdot 1.61$$

$$= \left(1 + 0.61(-0.8)\right)^{-1} \cdot 1.61$$

$$= 3.15$$

3) $C_V = 33.3 \frac{J}{\text{mole } ^\circ C}$

$R = 8.314 \frac{J}{\text{mole } ^\circ C}$

$\therefore C_V = 4R$

$C_{V \text{ monatomic gas}} = \frac{3}{2} R \quad \therefore$ gas is polyatomic.

b) $C_V = \frac{R}{2} \cdot f$ where f = # degrees of freedom.

$\therefore f = 8.$

4) From graph $\frac{\Delta V}{V} = \left(\frac{\partial V}{\partial T}\right)_P \Delta T$

at $T = 6\,^\circ C$ + $\Delta T = 2\,^\circ C$ $\Rightarrow \frac{\partial V}{\partial T} \simeq \frac{1 \times 10^{-4}}{6} \cdot 2 = 3.3 \times 10^{-5}$

at $T = 10\,^\circ C$ + $\Delta T = 2\,^\circ C$ $\Rightarrow \frac{\partial V}{\partial T} \simeq \frac{3 \times 10^{-4}}{4} \cdot 2 = 7.5 \times 10^{-4}$

Ocean height $\frac{\Delta L}{L} = \frac{\Delta V}{V}$ where L = average ocean depth ~ 3 km.

a) $T = 6\,^\circ C \quad \Delta L = 3.3 \times 10^{-5} \cdot 3\text{km} = 0.10$ m.

b) $T = 10\,^\circ C \quad \Delta L = 7.5 \times 10^{-4} \cdot 3\text{km} = 0.23$ m.

5) Consider a rectangular volume.

$$V = l_x \, l_y \, l_z$$

$$\frac{\partial V}{\partial T} = \frac{\partial V}{\partial l_x}\frac{\partial l_x}{\partial T} + \frac{\partial V}{\partial l_y}\cdot\frac{\partial l_y}{\partial T} + \frac{\partial V}{\partial l_z}\cdot\frac{\partial l_z}{\partial T}$$

$$\frac{1}{V}\frac{\partial V}{\partial T} = \frac{1}{l_x}\frac{\partial l_x}{\partial T} + \frac{1}{l_y}\frac{\partial l_y}{\partial T} + \frac{1}{l_z}\frac{\partial l_z}{\partial T}$$

$$\beta = \alpha_x + \alpha_y + \alpha_z$$

$$= 4 \times 10^{-6} \; {}^\circ C^{-1}$$

6a) $$p^2 \, T^{-1/3} \, e^{aV} = b$$

$$2p\,dp\,T^{-1/3} e^{aV} - p^2 \frac{1}{3} T^{-4/3} dT\, e^{aV} + p^2 T^{-1/3} a e^{aV} dV = 0$$

$$\frac{2\,dp}{pb} - \frac{1}{3}\frac{dT}{Tb} + \frac{a\,dV}{b} = 0.$$

b) $$\kappa \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$= \frac{-1}{V}\left(\frac{-2}{Pa}\right)$$

$$\kappa = \frac{2}{aPV}$$

c) $$\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

$$= \frac{1}{3aVT}$$