Assignment 11

1. At a temperature of 20 C, compute the ratio of occupation numbers for bosons & fermions when:
   a) $\varepsilon - \mu = 0.002$ eV
   b) $\varepsilon - \mu = 0.02$ eV
   c) $\varepsilon - \mu = 0.2$ eV

2. Consider helium gas at room temperature and atmospheric pressure.
   a) Show whether or not classical statistics would be appropriate.
   b) At what temperature would the number of accessible quantum state and the number of He atoms become about equal if the particle density remained unchanged?

3. Consider the conduction electrons in Cu. Suppose each Cu atom contributes one electron to the conduction electrons. Cu has atomic weight of 64 and a density of 8.9 gm/cm^3.
   a) What is the density of conduction electron?
   b) Can these conduction electrons be treated using classical statistics at room temperature?

4. The hydrogen atom is made of two spin ½ particles: a proton and an electron.
   a) When the electron is in the ground state, what are the possible values for the total angular momentum of the atom?
   b) The atmosphere of a certain star is atomic hydrogen gas. Is this a boson or fermion gas?
   c) Deeper in the star's interior, the hydrogen atoms are stripped of their electrons due to the high temperatures and pressure. Are the leftover protons (or electrons) a gas of bosons or fermions?

5. In a star such as the sun, the gravitational attraction of the star on itself is countered by the thermal gas pressure produced by burning hydrogen to produce helium. When the supply of hydrogen is exhausted, the star implodes and becomes either: a White Dwarf, Neutron Star or Black Hole. During the implosion, the outer layers may be ejected in an enormous explosion called a supernovae. A supernovae in 1087 in the Crab nebula produced sufficient light to rival the moon in brightness for several weeks.

   A White Dwarf may be thought of as a Fermi gas of electrons. The electrons occupy all energy levels up to the Fermi energy given by:

$$E_F = \begin{cases} \dfrac{\hbar^2 p_F^2}{2m_e} & \text{Nonrelativistic case} \\[2ex] p_F c & \text{Relativistic case} \end{cases}$$

where the Fermi momentum $p_F = \left(\frac{3\pi^2 \hbar^3 N}{V} \right)^{1/3}$ $\frac{N}{V}$ = electron density

Adding the gravitational attraction results in the following expression for the total energy.

$$E_{TOT} = \begin{cases} \dfrac{N^{5/3} (9\pi \hbar^3/4)^{2/3}}{2 m_e R^2} - \dfrac{GM^2}{R} & \text{Nonrelativistic case} \\[2ex] \dfrac{N^{4/3} (9\pi \hbar^3/4)^{1/3}}{R} - \dfrac{GM^2}{R} & \text{Relativistic case} \end{cases}$$

a) Show the total energy only has a minimum for the nonrelativistic case for which the star's radius

$$R = \frac{N^{5/3} (9\pi \hbar^3/4)^{2/3}}{G M^2 m_e}$$

b) Hence, a white dwarf only occurs if the mass does not exceed a critical value M_{crit} when the electron gas becomes relativistic. Derive the following result for this critical mass by substituting m_e c for the electron Fermi momentum and equating the result for R to that found in a.

$$M_{crit} = \sqrt{\frac{9\pi}{64}} \left(\frac{\hbar c}{G} \right)^{3/2} \frac{1}{m_p^2}$$

c) Compare the result for M_{crit} to the sun's mass. A more detailed analysis shows M_{crit} = 1.44 M_{Sun} which is called Chandrasekhar's limit. Stars exceeding this mass cannot become white dwarfs.