Phys 4050 Assignment 4

1. Kinetic energy of electron gas. Show that the kinetic energy of a three dimensional gas of N free electrons at 0 K is $U_o = 3/5\ N\ \varepsilon_F$.

2. Fermi gases in astrophysics.
   a) Estimate the number of electrons in the sun.

   b) In a white dwarf star this number of electrons may be ionized and contained in a sphere of radius 2 x 10^9 cm. Find the Fermi energy of the electrons in electron volts.

   c) The energy of an electron in the relativistic limit $\varepsilon >> mc^2$ is related to the wavevector as $\varepsilon = pc = h/2\pi\ kc$. Show that the Fermi energy in this limit is $\varepsilon_F = h/2\pi\ (N/V)^{1/3} c\,(3\pi^2)^{1/3}$

   d) If the above number of electrons were contained within a pulsar of radius 10 km, show that the Fermi energy would be 10^8 eV. This value explains why pulsars are believed to be composed largely of neutrons rather than of protons and electrons, for the energy release in the reaction $n \rightarrow p + e-$ is only 0.8 x 10^6 eV, which is not large enough to enable many electrons to form a Fermi sea. The neutron decay proceeds only until the electron concentration builds up enough to create a Fermi level of 0.8 x 10^6 eV, at which point the neutron, proton and electron concentrations are in equilibrium.

3. Liquid He^3. The atom He^3 has spin ½ and is a fermion. The density of liquid He^3 is 0.081 gm/cm^3 near absolute zero. Calculate the Fermi energy ε_F and the Fermi temperature T_F.

4. Frequency dependence of electrical conductivity. Use the equation $m\ dv/dt + mv/\tau = -eE$ for the electron drift velocity v and τ is the mean time between collisions to show that the conductivity at frequency ω is

$$\sigma(\omega) = \sigma(0)\ \frac{1 + i\omega\tau}{1 + (\omega\tau)^2}$$

   where $\sigma(0) = ne^2\tau/m$.

5. Static magnetoconductivity tensor. Show that the static current density can be written in matrix form as

$$\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} = \frac{\sigma_o}{1+(\omega_c\tau)^2} \begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & 1+(\omega_c\tau)^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$